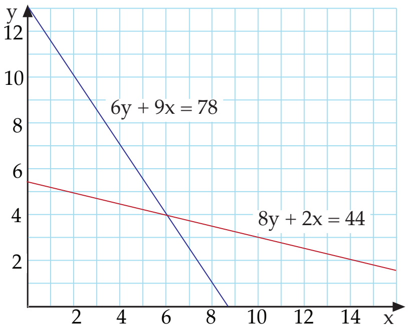
Answers

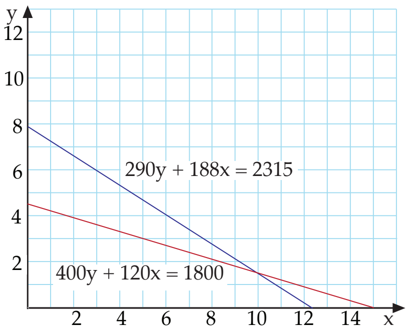
Page 5

1.



(6, 4)

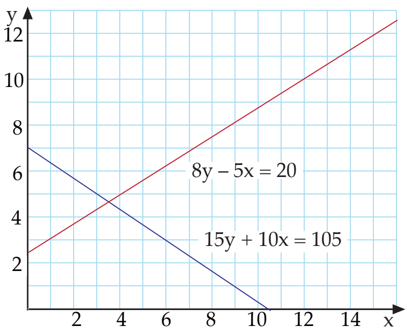
2.



(10, 1.5)

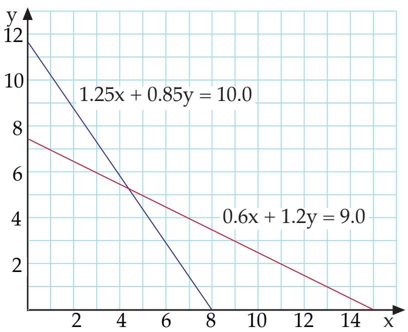
Page 6

3.



(3.484, 4.677) (4 sf)

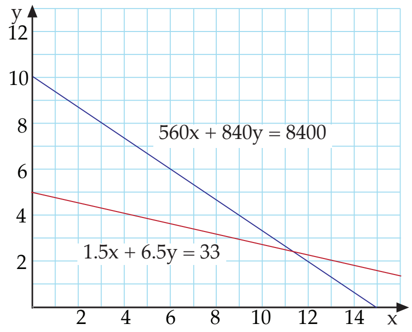
4.



(4.393, 5.303) (4 sf)

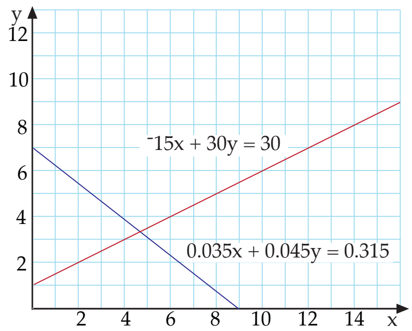
Page 6 cont...

5.



(11.29, 2.471) (4 sf)

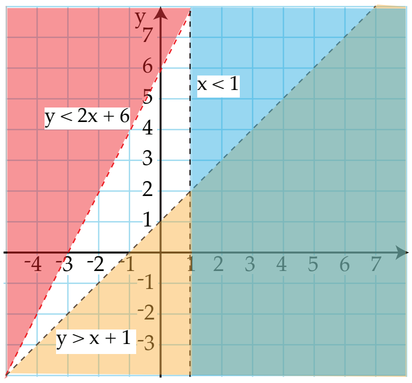
6.



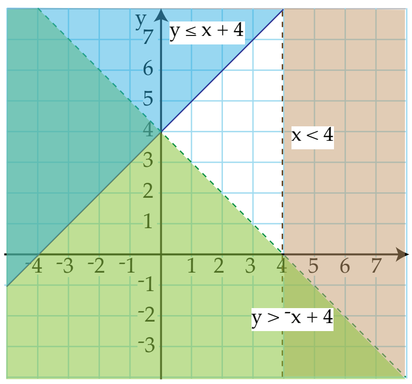
(4.696, 3.348) (4 sf)

Page 10

7.

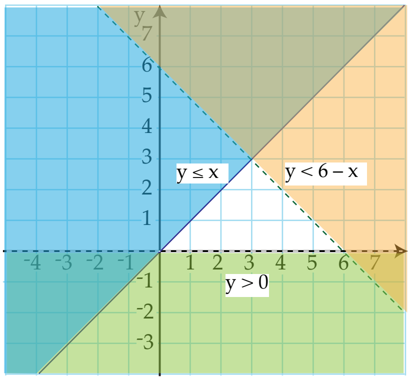


8.

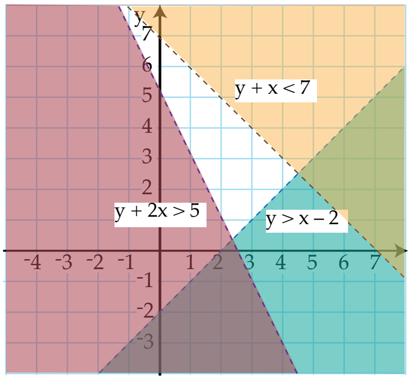


Page 10 cont...

9.

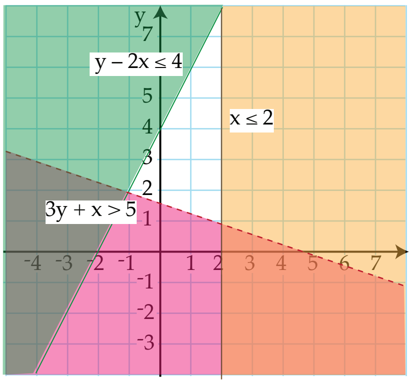


10.

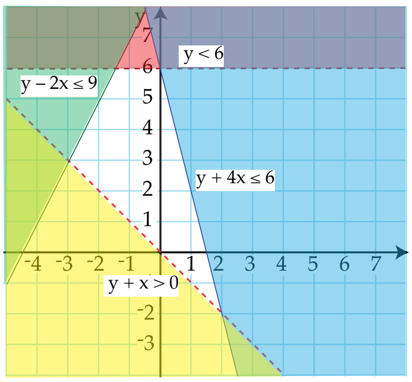


Page 11

11. (2, 8), (2, 1) and $(^-1, 2)$

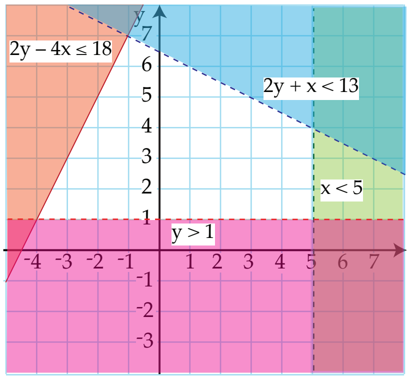


12. $(2, ^-2)$, (0, 6), $(^-1.5, 6)$ and $(^-3, 3)$



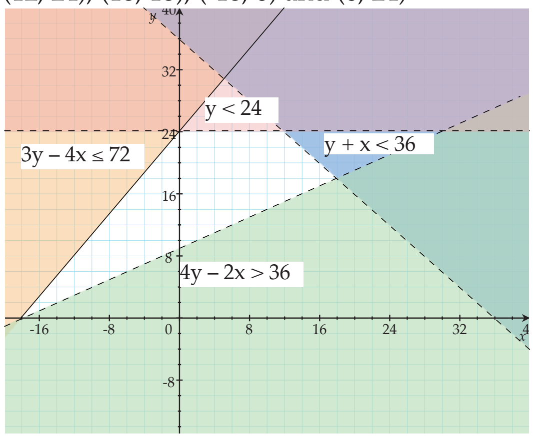
Page 11 cont...

13. (5, 1), (5, 4), $(^-1, 7)$ and $(^-4, 1)$

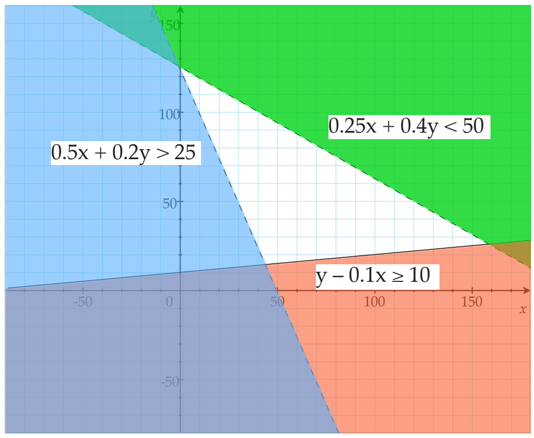


Page 12

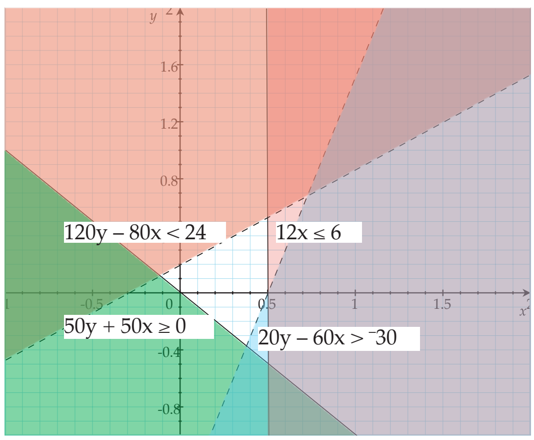
14. (12, 24), (18, 18), $(^-18, 0)$ and (0, 24)



15. (158.6, 25.8), (0, 125) and (44.2, 14.4)



16. (0.5, 0), (0.5, 0.533), $(^-0.12, 0.12)$ and $(0.375, ^-0.375)$



Page 16

17. At A (1, 4) F = $^{-}3$

At B (2.5, 1) F = $^{-}16.5$

At C ($^{-}5$, 1) F = 36

Answer C where F = 36

18. At A (0, 0) F = 21.2

At B (0, 4.333) F = 8.2

At C (6.5, 6.5) F = $^{-}30.8$

Answer A where F = 21.2

19. At A ($^{-}2$, 5) F = $^{-}11$

At B (4, 8) F = 4

At C (4, 2) F = 10

At D ($^{-}3$, 2) F = $^{-}11$

Answer C where F = 10

20. At A (0, 0) F = 0

At B (0, 6) F = 12

At C (3, 8) F = 14.5

At D (5, 2) F = 1.5

At E (3, 0) F = $^{-}1.5$

Answer C where F = 14.5

Page 17

21. At A (2, 1) G = $^{-}0.3$

At B (0.1428, 6.5714) G = $^{-}0.3$

At C (3, 8) G = 20.7

At D (4.5, 3.5) G = 20.7

Answer – all the points on the line from C to D as rearranging the function shows it has a gradient of $^{-}3$, the same as the line from C to D and both these end points return maximum answers.

22. At A ($^{-}1$, 4) G = 1.3

At B (1, 1) G = 3.9

At C (4, 3) G = 3.9

At D (3, 8) G = 0.5

Answer – all the points on the line from B to C as rearranging the function shows it has a gradient of 0.667, the same as the line from B to C and both these end points return maximum answers.

Page 19

23. At A ($^{-}2$, 9) F = $^{-}24$

At B $\left(2\frac{1}{3}, \frac{1}{3}\right)$ F = $6\frac{1}{3}$

or B (2.333, 0.333) F = 6.333

At C (4.5, 2.5) F = 8.5

Max. at C where F = 8.5

24. At A $\left(3\frac{2}{3}, \frac{2}{3}\right)$ or (3.667, 0.667)

F = $^{-}4.283$ (4 sf)

At B $\left(\frac{3}{4}, 6\frac{1}{2}\right)$ or (0.75, 6.5)

F = 1.9875

At C $\left(1\frac{4}{5}, 7\frac{1}{5}\right)$ or (1.8, 7.2)

F = 0.99

At D (6, 3) F = $^{-}6.15$

Max. at B where F = 1.9875

25. At A (3, 2)

F = 16

At B $\left(^{-}2\frac{1}{2}, 2\right)$ or ($^{-}2.5$, 2)

F = $^{-}6$

At C $\left(\frac{^{-}2}{5}, 6\frac{1}{5}\right)$ or ($^{-}0.4$, 6.2)

F = 15

At D $\left(1\frac{1}{3}, 5\frac{1}{3}\right)$ or (1.333, 5.333)

F = 19.333

Max. at D where F = 19.333

Page 20

26. At A (1, 8) F = $^{-}28$

At B (0.476, 2.762) F = $^{-}8.10$

At C (2.571, 1.714) F = 22.28

Min. at A where F = $^{-}28$

27. At A (1.2, $^{-}0.8$) F = $^{-}1000$

At B ($^{-}3$, 2) F = 20 000

At C (0.333, 8.667) F = 16 667

At D (5.5, 3.5) F = $^{-}11$ 750

Max. at B where F = 20 000

28. At A ($^{-}2$, 2) F = $^{-}550$

At B ($^{-}4.8$, 4.8) F = $^{-}1040$

At C (6.857, 7.714) F = 3 550

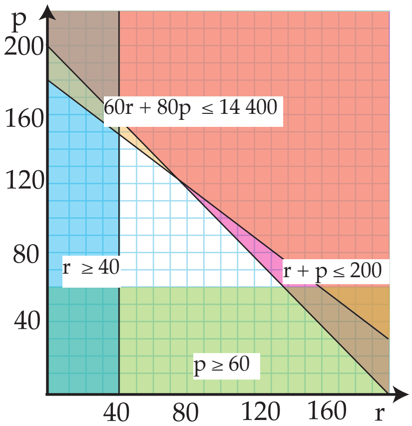
At D (2.8, $^{-}0.4$) F = 710

Max. at C where F = 3 550

Page 23

29.a) Let r = Ryder and p = Performa

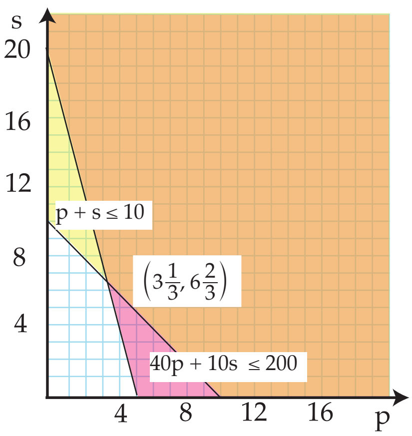
$r \geq 40$ and $p \geq 60$
$r + p \leq 200$
$60r + 80p \leq 14\,400$



b) Maximum profit at (80, 120) profit is \$9200.

30. a) Let p = Pinot and s = Sauv.

$p \geq 0$ and $s \geq 0$
$p + s \leq 10$ and $40p + 10s \leq 200$



b) At (3.333, 6.667) or $\left(3\frac{1}{3}, 6\frac{2}{3}\right)$ profit is \$56 667

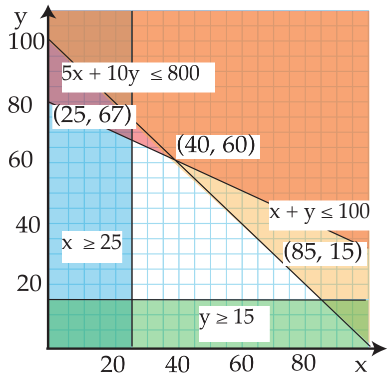
c) $3\frac{1}{3}$ Ha of Pinot Noir and

$6\frac{2}{3}$ Ha of Sauvignon Blanc.

Page 24

31. a) Let x = small and y = large

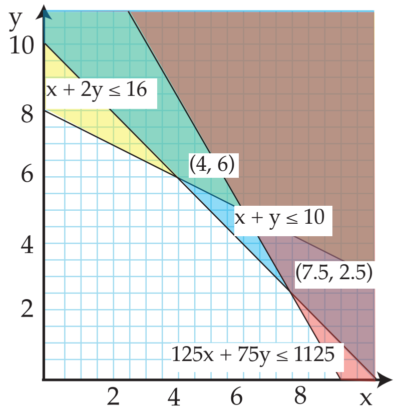
$x \geq 25$, $y \geq 15$, $x + y \leq 100$ and $5x + 10y \leq 800$



b) At (40, 60) maximum profit is \$1800 while at (25, 15) the minimum profit is \$675.

32. a) Let x = Ha of corn and y = Ha of pumpkin

$x \geq 0$, $y \geq 0$ and $x + y \leq 10$
$x + 2y \leq 16$ and $125x + 75y \leq 1125$



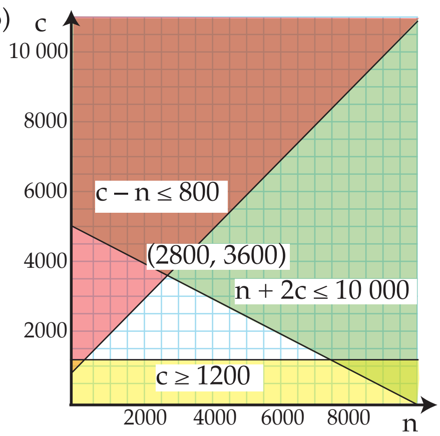
b) At (4, 6) the profit is \$2760 and at (7.5, 2.5) the profit is \$2550 so 4 ha of corn and 6 ha of pumpkin.

Page 30

33. a) Let c = chicken and n = nuggets

$c \geq 1200$, $c - n \leq 800$ and $n + 2c \leq 10\,000$

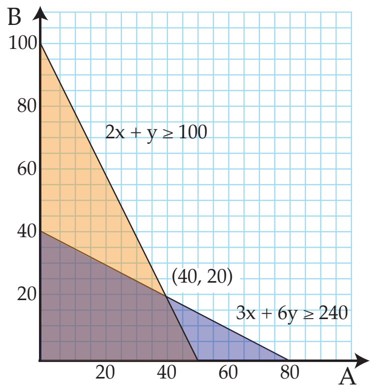
b)



c) At (2800, 3600) profit \$596.

Page 30 cont...

34. a) $2A + B \geq 100$
$3A + 6B \geq 240$
$A \geq 0$
$B \geq 0$



b) For (0, 100)
$C = 2500(0) + 2750(100)$
$= \$275\ 000$

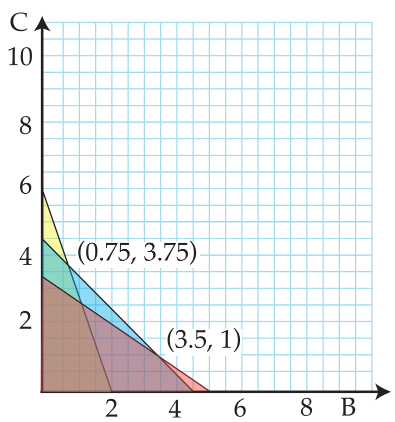
For (80, 0)
$C = 2500(80) + 2750(0)$
$= \$200\ 000$

For (40, 20)
$C = 2500(40) + 2750(20)$
$= 155\ 000$

Minimum cost \$155 000 for 40 hours Mine A and 20 hours Mine B.

Page 31

35. a) $200B + 300C \geq 1000$
$60B + 20C \geq 120$
$2B + 2C \geq 9$



b) For (0, 6), C = \$2.40

For (5, 0), C = \$4.00

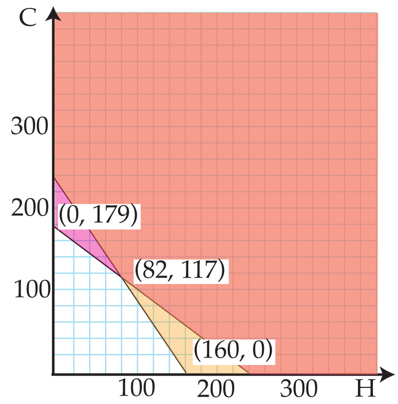
For (0.75, 3.75), C = \$2.10

For (3.5, 1), C = \$3.20

c) Minimum cost \$2.10 per pig per day for 0.75 kg of bean mash plus 3.75 kg of maize husks.

Page 31 cont...

36. a) $50H + 67C \leq 12000$
$50H + 33C \leq 8000$

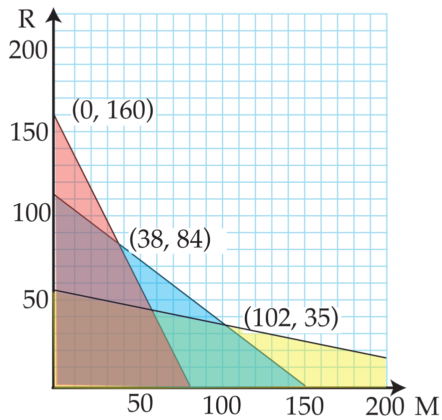


b) Integer solution as whole packets, round down.
(0, 179), Profit = \$537
(160, 0), Profit = \$640
(82, 117), Profit = \$679

Maximum profit \$679 with 82 packets of 'half n half' and 117 packets of 'chocolate spots'.

Page 32

37. a) $2M + R \geq 160$
$3M + 4R \geq 450$
$M + 5R \geq 280$



b) Integer solutions.
(280, 0), Cost = \$140 000

(102, 36), Cost = \$65 400

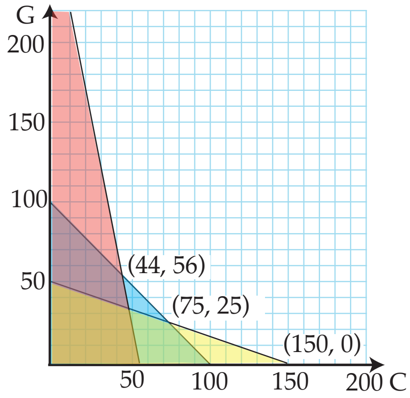
(38, 84), Cost = \$52 600

(0, 160), Cost = \$64 000

c) Minimum cost is \$52 600 with 38 tonnes of Middle East crude oil and 84 tonnes of Russian crude oil.

Page 32 cont...

38. a) $300C + 900G \geq 45\ 000$
$300C + 300G \geq 30\ 000$
$1000C + 200G \geq 55\ 000$

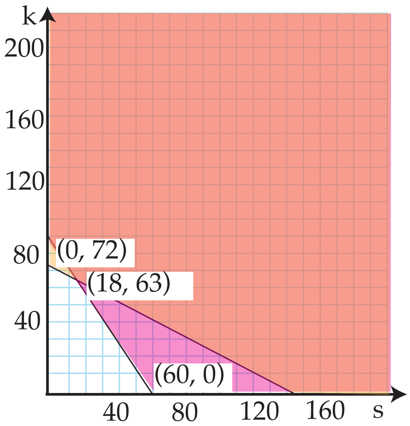


b), c)
Integer solution as whole days so round up.
(150, 0), Days = 150 so cost = \$150 000
(75, 25), Days = 100 so cost = \$100 000
(44, 56), Days = 100 so cost = \$100 000
(0, 275), Days = 275 so cost = \$275 000
(44, 56) gives:
Snapper 63 600 kg or 18 600 kg over quota.
Hoki 55 200 kg or 200 kg over quota.
(75, 25) gives:
Hoki 80 000 kg or 25 000 kg over quota.

Page 33

39. a) $\frac{s}{2} + \frac{k}{3} \leq 30$ or $3s + 2k \leq 180$

$\frac{s}{6} + \frac{k}{3} \leq 24$ or $s + 2k \leq 144$



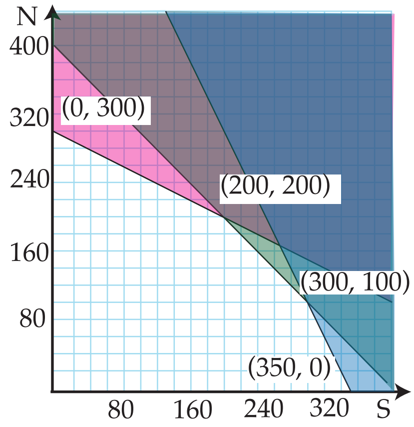
b) (60, 0) = \$1500, (0, 72) = \$1800 and (18, 63) = \$2025.

c) Sales at (18, 63) = \$2025 with a net return of \$1417.50. Profit = \$769.50.

d) Hourly rate for 54 hours of \$14.25 per hour.

Page 33 cont...

40. a) $0.1N + 0.1S \leq 40$
$0.4N + 0.2S \leq 120$
$0.2N + 0.4S \leq 140$

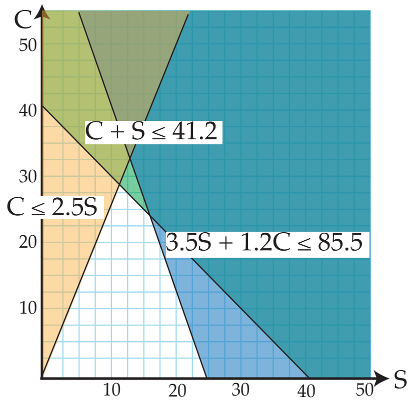


b) (0, 300) = \$2400
(200, 200) = \$3600
(300, 100) = \$3800
(350, 0) = \$3500
Best answer is 300 kg of sweet bars and 100 kg of nut bars.

c) Remaining 20 kg of nuts and raisins

Page 38

41. a) $C \leq 2.5S$, $C + S \leq 41.2$ and $3.5S + 1.2C \leq 85.5$



b) $P_1 = (11.77, 29.43)$
$P_2 = (15.68, 25.52)$
$P_3 = (24.43, 0)$

c) Profit P_1 = \$79 515.60
Profit P_2 = \$80 610.40
Profit P_3 = \$52 035.90
Squash = 15.68 ha and Corn = 25.52 ha gives \$80 610.40

d) Try other points near the vertex. See that as you get further from the vertex the profit drops.

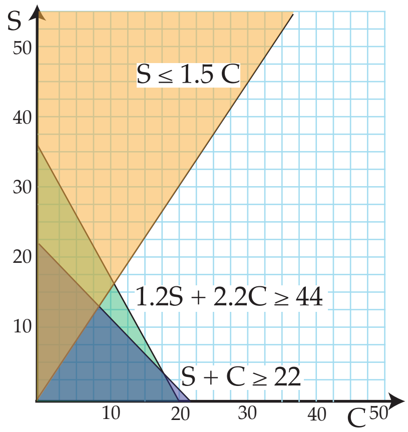
e) P_1 = \$85 814.30
P_2 = \$84 211.20
Squash = 11.77 ha and Corn = 29.43 ha gives \$85 814.30

Page 38 Q41 cont...

41. f) If squash also paid \$2200/ha profit all the points along the line $C + S = 41.2$ would be solutions (provided they met the other inequalities).

Page 39

42. a) S = Shell rock and C = Crushed concrete
$S \leq 1.5C$, $S + C \geq 22$ and $1.2S + 2.2C \geq 44$



b) $P_1 = (11, 16.5)$
$P_2 = (17.6, 4.4)$
$P_3 = (22, 0)$

c) $P_1 = \$4939.00$
$P_2 = \$4030.40$
$P_3 = \$4070.00$

The point P_2 of 17.6 m^3 of concrete and 4.4 m^3 of shell rock is the minimum solution.

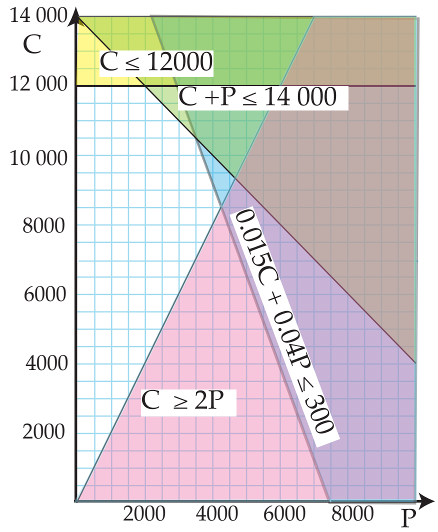
d) Note: Only points between P_1 and P_2 involve both shell rock and concrete. Price of concrete is 1.833 times that of the cost of shell rock,

i.e. Concrete = 1.833x or Concrete = $\frac{11}{6}x$

so points along the line (11, 16.5) to (17.6, 4.4) yield multiple solutions.

Page 40

43. a) C = Crabs, P = Prawns
$C \leq 12\,000$
$C + P \leq 14\,000$
$0.015C + 0.04P \leq 300$
$C \geq 2P$



b) (2 000, 12 000), (3 600, 10 400), (4 285, 8 571) while (0, 12 000) and (0, 0) are ignored.

c) Value = 30 x 0.085P + 24 x 0.060C or Value = 2.55P + 1.44C.

(2 000, 12 000) = \$22 380, (3 600, 10 400) = \$24 156
(4 285, 8 571) = \$23 269 (all 0 dp)

Best solution is 10 400 crabs and 3600 prawns.

d) Gradient from (3 600, 10 400) to (4 285, 8 571) is $m = {}^{-}2.67$ so the price for one individual prawn has to be 2.67 times the price of crabs.

Each prawn = 2.67 x 1.44
= 3.84

At 85 g each this means price must be \$45.18 per kilogram.

e) Deducting the processing costs means that the return on crab meat is now \$20.15 per kg or \$1.21 each while prawns are \$14.24 per kg or also \$1.21 each. Therefore any solution on the line C + P = 14 000 will give the same return. Any solution between (2 000, 12 000) to (3 600, 10 400).

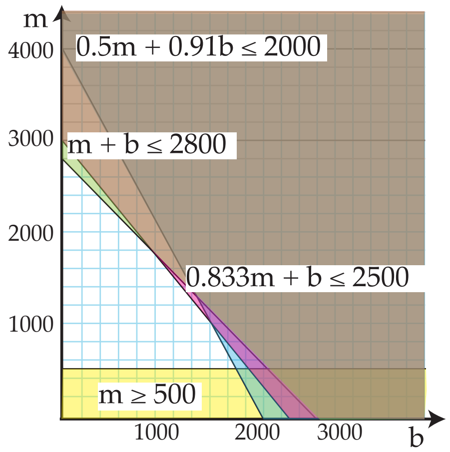
Page 41

44. a) $0.50m + 0.91b \le 2000$

$0.83m + b \le 2500$

$m + b \le 2800$

$m \ge 500$



b) (0, 2800), (1004, 1796), (1612, 1066), (1923, 500)

c) Total return = 12Rm + 10 x 1.2Rb where R is the return per kilogram for money maker and m and b the number of each type of tomato.

(0, 2800) = 33 600R, (1004, 1796) = 33 600R

(1612, 1066) = 32 136R, (1923, 500) = 29076R

All solutions on the line from (0, 2800) to (1004, 1796) give a return of 33 600R.

d) With a return of R/kg for money maker and a return for beef steak of 1.25R/kg then

Return = 12Rm + 12.5Rb

(0, 2800) = 33 600R, (1004, 1796) = 34 102R

(1612, 1066) = 32 942R, (1923, 500) = 30 038.5R

Best return with 1796 money maker and 1004 beef steak tomatoes.

e) Gradient from (1004, 1796) to (1612, 1066) is m = $^{-}1.20066$ so making m the subject of the return formula

Return = 12Rm + 10 x kRb where k is the constant ratio of beef steak to money maker.

$$m = \frac{\text{Return}}{12R} - \frac{10k}{12}b$$

The gradient of the line was $^{-}1.20066$ so k = 1.44079 and beef steak must be 1.44079 times the price of money maker or 44.1% above the price. Results rounded.
Return = 12Rm + 14.4079Rb

(0, 2800) = 33 600R, (1004, 1796) = 36 018R

(1612, 1066) = 36 018R, (1923, 500) = 33 706R

All points on the line from (1004, 1796) to (1612, 1066) are now solutions to the problem.

Pages 42 – 45

Practice Assessment – Linear Programming.

Part A

Inequalities

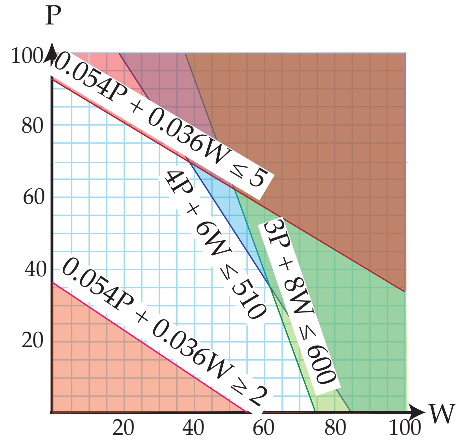
Number patu = P and the number of waka = W

$0.054P + 0.036W \le 5$

$0.054P + 0.036W \ge 2$

$4P + 6W \le 510$

$3P + 8W \le 600$



Income

Income = 85P + 170W

Checking the closest whole number solution to each vertex.

(0, 92.6) checking (0, 92)

Income = 85 x 92 + 170 x 0

Income = $7820

(41.9, 64.7) checking (41, 64)

Income = 85 x 64 + 170 x 41

Income = $12 410

(62.1, 34.3) checking (62, 34)

Income = 85 x 62 + 170 x 34

Income = $13 430

(75, 0) Income = 85 x 0 + 170 x 75

Income = $12 750

Best solution is 62 waka and 34 patu with $13 430.

This would use 4.1 m^3 (1 dp) of kauri a month and require 598 hours of carving. The carvers would be nearly fully employed and less than the maximum amount of kauri would be used each month.

Practice Assessment Pages 42 – 45 cont...

Part B – Multiple solutions

We start from the vertex (62, 34) as we are only increasing the price of patu slightly.

We need multiple solutions along the line 4P + 6W = 510 as this line passes through (62, 34) and increasing the price of just the patu will result in income rising when we have more patu and less waka.

The gradient of 4P + 6W = 510 is given by

$$4P + 6W = 510$$
$$P = {}^{-}1.5W + 127.5$$

If the equation that generates the income has the same gradient we will have multiple incomes.

Our income (with patu increased by factor *k*) has the equation

$$\text{Income} = k85P + 170W$$

Making P the subject to get the gradient gives

$$P = \frac{{}^{-}2W}{k} + \frac{\text{Income}}{85k}$$

If the gradients are equal then

$$^{-}1.5 = \frac{^{-}2}{k}$$
$$k = 1.333$$

k = 1.333 and when patu is sold at $113.33 there will be multiple solutions from (41.9, 64.7) to (62.1, 34.3).

Using whole number coordinates of (41, 64) and (62, 34) then k = 1.4. **Patu is sold at $119 each and the income is $14 586.**

At 41 waka and 64 patu the maximum of 5 cubic metres of kauri is used but only 520 hours of work is needed. At 62 waka and 34 patu nearly all 600 hours are worked but only 4.1 cubic metres of kauri are needed. If profit is important then (41, 64) results in less working hours (less pay). If providing work in the community is important, then (62, 34) uses nearly all the hours and it uses less of the precious kauri.

Practice Assessment – Linear Programming Marking

	Evidence/Judgements for Achievement	Evidence/Judgements for Achievement with Merit	Evidence/Judgements for Achievement with Excellence
	The student has applied linear programming methods in solving problems.	The student has applied linear programming methods, using relational thinking, in solving problems.	The student has applied linear programming methods, using extended abstract thinking, in solving problems.
	This involves selecting and using linear programming methods, demonstrating knowledge of concepts and terms and communicating using appropriate representations.	The student has connected different concepts or representations. The student has related findings to the context or has communicated thinking using appropriate mathematical statements.	The student has identified relevant concepts in context. The student has used correct mathematical statements or communicated mathematical insight.
	Example of possible student responses:	Example of possible student responses:	Example of possible student responses:
Evidence 1	Finding the equation of at least two linear inequalities.	Defining P and W and getting all inequalities.	Defining P and W and getting all inequalities.
Evidence 2	Identifies the feasible region as a graph as per their inequalities.	Graphs the feasible region correctly.	Graphs the feasible region and correctly identifies the points of intersection.
Evidence 3	Identifies the best answer as (62.1, 34.3) or (62, 34).	Identifies the best answer as 62 waka and 34 patu and the amount at $13 430.	Identifies the best answer as 62 waka and 34 patu and the amount at $13 430 and demonstrates other answers are less.
Evidence 4			**Multiple solutions.** Calculates the increased price of patu ($119) to get multiple solutions for k of 1.333 or 1.4. Identifies the implication of the possible answers on employment and kauri use.

Final grades will be decided using professional judgement based on a holistic examination of the evidence provided against the criteria in the Achievement Standard.