

Answers

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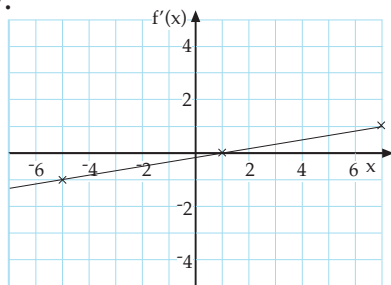
1. At $x = -2, m = -2$
At $x = 2, m = 2$
2. At $x = -4, m = 3$
At $x = -1, m = -2$
At $x = 1, m = 0$

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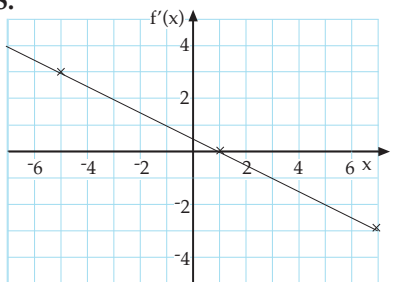
3. a) $m = 1$ at $(3, 2.5)$
b) $m = -2$ at $(-3, 5.5)$
4. a) $m = 2$ at $x = -3.5$ approx.
and $x = 3.5$ approx.
b) $m = 0$ at $x = -2$ and $x = 2$
5. a) Increasing
 $x < -4$ or $x > -1$
b) Decreasing
 $-4 < x < -1$
6. At $x = -4, -1$ and 4

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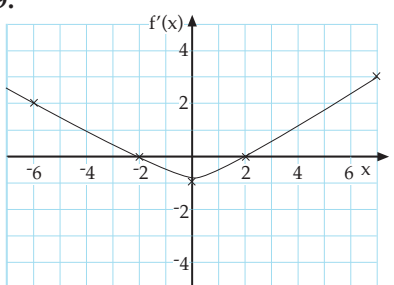
7.



8.

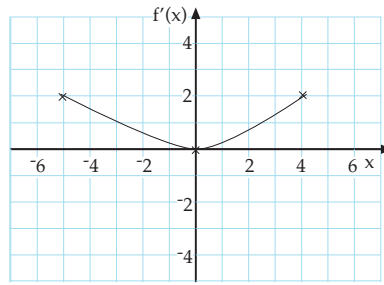


9.



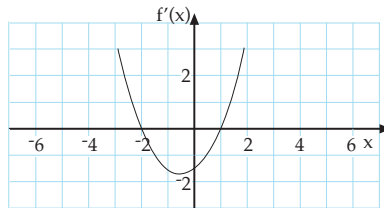
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10.

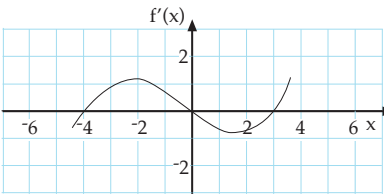


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11.



12.



13.

Description	Looks like	Gradient function
Increasing		Always +ve
Decreasing		Always -ve
Maximum		Is zero. Gradient goes from +ve through 0 to -ve
Minimum		Is zero. Gradient goes from -ve through 0 to +ve

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14. Average rate of change = 4
15. Average rate of change = 3
16. Average rate of change = -2
17. Average rate of change = -4
18. Average rate of change = 4
19. Average rate of change = 3
20. Average rate of change = 2.4
21. Average rate of change = 2.01
22. Average rate of change = 2.001
23. Average rate of change = 2.0001
24. Avg. rate of change = 2.000 01
25. Avg. rate of change = 2.000 001
26. Expect the gradient to be 2

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27. $f'(x) = 2x + 2$
28. $f'(x) = 2x - 6$
29. $f'(x) = 5 - 2x$
30. $f'(x) = 2x + 3$

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31. $f'(x) = 3x^2$
32. $f'(x) = -3$
33. $f'(x) = -4x - 1$
34. $f'(x) = 6x^2 - 1$
35. $f'(x) = 20x$
36. $f'(x) = 8x - 5$
37. $f'(x) = 2x + 1$
38. $f'(x) = 2x - 7$

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39. $f'(x) = 15x^2$
40. $f'(x) = 9$
41. $f'(x) = 0$
42. $f'(x) = 2x + 3$
43. $f'(x) = 4x$
44. $f'(x) = 10x - 10x^4$
45. $f'(x) = 15x^2 + 4x$
46. $f'(x) = 10x + 10$
47. $f'(x) = 55x^{10} - 45x^4$
48. $f'(x) = x - 2$
49. $f'(x) = \frac{1}{2}x - \frac{1}{5}$
50. $f'(x) = 2x^2 - \frac{1}{4}$
51. $f'(x) = 2x^3 - 0.75x^2$
52. $f'(x) = 1.2x^5 + 0.9x^2 - 1.5$
53. $f'(x) = 1.5x^2 + 0.6x - 0.8$
54. $f'(x) = 6x^4 + 7x - 1.4$
55. $f'(x) = \frac{3x}{2} - \frac{1}{5} - \frac{x^2}{2}$
56. $f'(x) = \frac{10x^4}{3} - 3x^3 - \frac{6x^2}{5} + 8x - 2$
57. $f(x) = x^2 - 2x - 15$
 $f'(x) = 2x - 2$
58. $f(x) = x^3 - 2x^2 + 5x - 10$
 $f'(x) = 3x^2 - 4x + 5$
59. $f(x) = 3x^4 + 5x^3$
 $f'(x) = 12x^3 + 15x^2$
60. $f(x) = x^4 - 2x^2 - 35$
 $f'(x) = 4x^3 - 4x$

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61. $f'(x) = 27x^2 + 2$

62. $f''(x) = 126x^5 - 30x$

63. $\frac{dy}{dx} = 5 - 15x^4$

64. $\frac{dy}{dx} = 2x$

65. $f''(x) = 60x^3$

66. $\frac{d^2y}{dx^2} = 24x - 12$

67. $\frac{dy}{dx} = x - \frac{9}{4}x^2$

68. $\frac{dy}{dx} = -6x^4 + 5x$

69. $f''(x) = \frac{9x}{2} - \frac{4}{3}$

70. $\frac{d^2y}{dx^2} = 192x^2 - 48$

71. $f''(x) = \frac{-48x^2}{5} + \frac{18x}{5} + \frac{8}{3}$

72. $\frac{d^2y}{dx^2} = 3 - \frac{8x}{3}$

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73. $f'(0) = 2$

74. $f'(-2) = -2$

75. $f'(-3) = 7$

76. $f'(-4) = -4.25$

77. $f'(-1) = 2$

78. $f'(4) = 41$

79. $y = x + 1$

80. $y = 2x - 6$

81. $y = -5$

82. $y = -x$

83. $y = -3x - 7$

84. $y = 7x - 4$

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85. Gradient = 8

86. Gradient = -42

87. (-1, 4)

88. (0.5, 1.25)

89. (0, -2), (2, 0)

90. (-3, -1.5), (0, 3)

91. a) $f'(x) = 0.5x - 1.5$

b) $f'(1) = -1$ and Grad. = -1
 $f'(5) = 1$ and Grad. = 1

c) $y = 6 - x$ for (1, 5)
 $y = x$ for (5, 5)

d) Intersection (3, 3)

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92. $f'(x) = 2x + 8$ min. at (-4, -1)

93. $f'(x) = 2x - 2$ min. at (1, -4)

94. $f'(x) = 2x + 1$ min. at (-0.5, 2.75)

95. $f'(x) = 2x$
minimum at (0, -5)

96. $\frac{dy}{dx} = 6 - 2x$
maximum at (3, 1)

97. $f'(x) = 8x - 12$
minimum at (1.5, 0)

98. $\frac{dy}{dx} = 6x - 8$
minimum at (1.333, 5.667) (4 sf)

99. $f'(x) = -4x - 16$
maximum at (-4, 47)

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100. $\frac{dy}{dx} = x^2 - 4x - 12$
minimum (6, -72)
maximum (-2, 13.33) (4 sf)

101. $f'(x) = 3x^2 - 6x - 9$
minimum (3, -17)
maximum (-1, 15)

102. $\frac{dy}{dx} = -3x^2 + 6x + 9$
minimum (-1, -2)
maximum (3, 30)

103. $f'(x) = 3x^2 - 16x + 5$
minimum (5, -52)
maximum (0.333, -1.185) (4 sf)

104. $\frac{dy}{dx} = 6x^2 + 18x + 12$
minimum (-1, -10)
maximum (-2, -9)

105. $f'(x) = -3x^2 - 2x + 8$
minimum (-2, -12)
maximum (1.333, 6.519)

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106. Turning point (-1, 2)
Increasing $x > -1$
Decreasing $x < -1$

107. Turning point (-3, 13)
Increasing $x < -3$
Decreasing $x > -3$

108. Turning points (-3, -5) and
(-1, $-6\frac{1}{3}$).
Increasing $x < -3$ or $x > -1$
Decreasing $-3 < x < -1$

109. Turning points (-2, $5\frac{1}{3}$) and
(2, $-5\frac{1}{3}$).
Increasing $x < -2$ or $x > 2$
Decreasing $-2 < x < 2$

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110. Turning points (-4, 51.67)
and (5, -69.83)
Increasing $x < -4$ or
Increasing $x > 5$
Decreasing $-4 < x < 5$

111. Turning points (-0.5, 2.8)
and (3, -11.5)
Increasing $x < -0.5$ or $x > 3$
Decreasing $-0.5 < x < 3$

112. Turning point (2, -1)
Increasing $x > 2$
Decreasing $x < 2$

113. Turning point (1, 2)
Increasing $x < 1$
Decreasing $x > 1$

114. Turning points (0, 0)
and ($\frac{1}{2}$, -0.0417)
Increasing $x < 0$ or $x > \frac{1}{2}$
Decreasing $0 < x < \frac{1}{2}$

115. Turning points (-2, -4)
and (0, 0)
Increasing $-2 < x < 0$
Decreasing $x < -2$ or $x > 0$

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116. Prod. = $x(31 - x)$
 $x = 15.5$
Max prod. = 240.25

117. Area = $w(500 - w)$
Width = 250 m
Max area = 62 500 metres²

118. Area = $w(1000 - 2w)$
Width = 250 m
Height = 500 m
Max area = 125 000 metres²

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119. Rail: $2x + 2y = 18$
Area = $-x^2 + 5x + 54$
 $x = 2.5$ m, $y = 6.5$ m
Area = 60.25 m²

120. Area = $0.5 \times x \times y$
 $x + y = 40$
Area = $20x - 0.5x^2$
Max Area = 200 cm²

121. SA = $300 = 4x^2 + 6xh$
Vol = $2x^2h$
width (x) = 5 cm
length = 10 cm
height = $6\frac{2}{3}$ cm

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122. Link equation

$$\begin{aligned} 3y + 2x &= 600 \\ x &= 300 - 1.5y \\ \text{Area} &= 2xy \\ &= 2y(300 - 1.5y) \\ &= 600y - 3y^2 \\ (\text{Area})' &= 600 - 6y \\ y &= 100 \text{ m for max} \\ \text{length} &= 100 \text{ m} \\ \text{width (x)} &= 150 \text{ m} \end{aligned}$$

123. $r^2 + h^2 = 22^2$

$$\begin{aligned} V &= \frac{1}{3}\pi r^2 h \\ &= \frac{1}{3}\pi(22^2 - h^2)h \\ \text{radius} &= 18.0 \text{ cm} \\ \text{height} &= 12.7 \text{ cm} \end{aligned}$$

124. Link equation

$$\begin{aligned} y &= 9 - x^2 \\ \text{Area} &= 2xy \\ &= 2x(9 - x^2) \\ &= 18x - 2x^3 \\ (\text{Area})' &= 18 - 6x^2 \\ x &= \pm\sqrt{3} \\ \text{width (2x)} &= 3.464 \text{ (4 sf)} \\ \text{height (y)} &= 6 \\ \text{Area} &= 20.78 \text{ units}^2 \end{aligned}$$

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125. Link equation

$$\begin{aligned} \pi x + 4x + 2y &= 5 \\ y &= 0.5(5 - x(\pi + 4)) \\ \text{Area} &= 2xy \\ &= x(5 - x(\pi + 4)) \\ &= 5x - x^2\pi - 4x^2 \\ (\text{Area})' &= 5 - 2x\pi - 8x \\ x &= 0.35 \text{ m for max} \\ \text{height} &= 1.25 \text{ m} \\ \text{width (2x)} &= 0.70 \text{ m} \end{aligned}$$

126. $y_2 - y_1 = D$

$$\begin{aligned} D &= (x + 2)(6 - x) - ((x - 1)^2 - 9) \\ D &= -x^2 + 4x + 12 - x^2 + 2x + 8 \\ D &= -2x^2 + 6x + 20 \\ D' &= -4x + 6 \\ D' &= 0 \text{ for max / min} \\ x &= 1.5 \\ D &= 24.5 \text{ units} \end{aligned}$$

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127. Corner x units long

$$\begin{aligned} V &= x(600 - 2x)(400 - 2x) \\ V &= 240\,000x - 2000x^2 + 4x^3 \\ V' &= 240\,000 - 4000x + 12x^2 \\ V' &= 0 \text{ for max / min} \\ x &= 78.47, 254.85 \\ \text{Max } x &= 78.5 \text{ mm (3 sf) gives} \\ \text{Vol.} &= 8\,450\,000 \text{ mm}^3 \text{ (3 sf)} \end{aligned}$$

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128. a) $v(t) = 60 - 10t$ m/s

$$\begin{aligned} \text{b) } a(t) &= -10 \text{ m/s}^2 \\ \text{c) } a(4) &= -10 \text{ m/s}^2 \end{aligned}$$

129. a) $s(0) = 0$ m

$$\begin{aligned} s(6) &= -144 \text{ m} \\ s(12) &= 0 \text{ m} \end{aligned}$$

b) $v(0) = -48$ m/s

$$\begin{aligned} v(6) &= 0 \text{ m/s} \\ v(12) &= 48 \text{ m/s} \end{aligned}$$

130. a) $s(0) = 9$ m

$$\begin{aligned} \text{b) } s(2) &= 1 \text{ m} \\ s(3) &= 0 \text{ m} \\ s(6) &= 9 \text{ m} \end{aligned}$$

c) $v(3) = 0$ m/s

$$\begin{aligned} v(6) &= 6 \text{ m/s} \\ \text{d) } a(1) &= 2 \text{ m/s}^2 \end{aligned}$$

131. a) $h(t) = 0$

$$t = -10 \text{ and } 50 \text{ seconds}$$

b) $v(5) = 30$ m/s

c) $v(t) = 0$

$$t = 20 \text{ seconds}$$

d) $h(20) = 900$ metres

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132. a) $V'(t) = 12 - 1.6t$

$$V'(5) = 4 \text{ m}^3/\text{h}$$

b) $12 - 1.6t = 6$

$$t = 3.75 \text{ hours}$$

133. a) $v(t) = -360 + 12t$

$$v(15) = -180 \text{ m/s}$$

$$v(30) = 0 \text{ m/s}$$

$$v(40) = 120 \text{ m/s}$$

b) min. when $v(t) = 0$ so $t = 30$ s

$$s(30) = 14\,600 \text{ m}$$

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134. a) $s = 4$ m

b) $t = 1$ second (and $t = 4$).

c) 7 m/s

d) Acceleration = 2 m/s^2 for $t = 1, 2$ and 3 .
Constant acceleration.

135. a) $s = 30$ m

b) $v = 3$ m/s

c) $a = 0 \text{ m/s}^2$

d) $t = 1$ second

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136. $V' = 4\pi r^2$ and at $r = 15$ mm

$$V' = 2800 \text{ mm}^3 \text{ per mm increase (2 sf)}$$

137. $H' = 2 - d$ and at $d = 3.5$ m

$$H' = -1.5 \text{ m per horizontal m}$$

138. $V = 6w^3$

$$V' = 18w^2 \text{ and at } w = 0.25 \text{ m}$$

$$V' = 1.1 \text{ m}^3 \text{ drop per m decrease (2 sf)}$$

139. $\text{Vol.} = \frac{\pi d^3}{48\,000}$

$$V' = \frac{\pi d^2}{16\,000} \text{ and at } d = 75 \text{ m}$$

$$V' = 1.1 \text{ m}^3 \text{ increase per m travelled (2 sf)}$$

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140. $f(x) = x^3 - 6x^2 + 10x + C$

141. $f(x) = 3x^4 - 2x^3 - 5x + C$

142. $f(x) = 4x^5 - 4x^3 + C$

143. $f(x) = \frac{1}{3}x^3 + \frac{3}{2}x^2 + 2x + C$

144. $f(x) = 3x^3 + 3x^2 + x + C$

145. $f(x) = \frac{1}{6}x^3 - \frac{3}{8}x^2 + 5x + C$

146. $f(x) = \frac{2x^5}{15} + \frac{x^3}{5} - \frac{x^2}{8} + x + C$

147. $f(x) = \frac{x^4}{4} - \frac{5x^3}{3} + \frac{7x^2}{2} - 3x + C$

148. $f(x) = \frac{x^5}{25} - \frac{x^3}{6} - \frac{x^2}{6} - 8x + C$

149. $f(x) = \frac{3x^5}{20} + \frac{5x^3}{9} + \frac{7x^2}{4} - 4x + C$

150. $f(x) = 0.5x^5 - 0.8x^4 + 0.5x^3 - 6x + C$

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151. $f(x) = 0.214x^7 - 1.53x^6 + 3x + C$

152. $f(x) = 2x^7 - x^6 + 9x + C$

153. $f(x) = \frac{-3x^4}{10} - \frac{4x^3}{9} - \frac{2x^2}{5} + C$

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154. $f(x) = 6x^2 - 5x - 3$

155. $f(x) = 4x - 2x^2 + 16$

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156. $y = 3x - \frac{x^2}{2} - 4$

157. $f(x) = x - x^3 - 1$

158. $f(x) = x^3 - x^2 + x + 2$

159. $f(x) = x - \frac{x^3}{3} + \frac{x^2}{2}$

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160. $f(x) = x^3 - \frac{9x^2}{2} - 18x + 20$

161. $y = 2x + 3x^2 - \frac{x^3}{3} + 2\frac{2}{3}$

162. $f(x) = x^4 - 6x^3 + 12x^2 - 10x + 3$

163. $f(x) = x^4 - 5x^3 + 5x^2 + 5x - 6$

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164. $0 = 4 - 2x$ for Max/Min.
Max. at $x = 2$

$f(x) = 4x - x^2 + C$
through $(2, 7)$

$f(x) = 4x - x^2 + 3$

165. $0 = 4x + 12$ for Max/Min.
Min. at $x = -3$.

$f(x) = 2x^2 + 12x + C$
through $(-3, 0)$

$f(x) = 2x^2 + 12x + 18$

166. $f'(x) = k(x-1)(x-3)$

$f'(0) = 3$ gives $k = 1$

$f'(x) = x^2 - 4x + 3$

$f(x) = \frac{x^3}{3} - 2x^2 + 3x$

167. $f'(x) = k(x+2)(x-3)$

$f'(0) = 12$ gives $k = -2$

$f'(x) = -2(x^2 - x - 6)$

$f'(x) = 12 + 2x - 2x^2$

$f(x) = 12x + x^2 - \frac{2x^3}{3} + C$

through $(0, 2)$

$f(x) = 12x + x^2 - \frac{2x^3}{3} + 2$

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168. a) $3t^2 - 12 = 0$, $t = 2$ seconds ($t = -2$)

b) $s(t) = t^3 - 12t + 2$

c) $a(t) = 6t$. After 3 seconds
 $a(3) = 18 \text{ m/s}^2$

169. a) $v(t) = t^2 + t + 6$

b) $s(t) = \frac{1}{3}t^3 + \frac{1}{2}t^2 + 6t + 5$
 $s(2) = 21.67 \text{ m}$ (4 sf)

170. a) $a(t) = 140 - 14t$ When $t = 0$, $a = 140 \text{ m/s}^2$

b) $s(t) = 70t^2 - \frac{7}{3}t^3 + C$
as $s(t) = 0$ when $t = 0$

$s(t) = 70t^2 - \frac{7}{3}t^3$

$s(1) = 67.67 \text{ m}$ (4 sf)

c) $v(t) = 0$ when $t = 0$ and 20 seconds.

$s(20) = 9333 \text{ m}$ (4 sf)

171. a) When $v = 0$, $t = 3$

$s(t) = 30t - 5t^2 + 2$

$s(3) = 47 \text{ m}$

b) $s(0) = 2$, $s(2) = 42$
therefore 40 m in the first 2 seconds.

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172. a) 15 cm/s

b) $a(t) = 6t - 18$
 $t = 3$ seconds

c) $s(t) = t^3 - 9t^2 + 15t + 150$

d) $3t^2 - 18t + 15 = 0$
 $t = 1$ and $t = 5$ seconds

e) $s(5) = 125 \text{ cm}$

f) $s(1) = 157 \text{ cm}$

173. a) 34 m/s

b) $a = -6.8 \text{ m/s}^2$

c) $v(t) = 0$ at $t = 5$ seconds

d) $h(t) = 34t - 3.4t^2$

e) $s(5) = 85 \text{ m}$

f) $s(4) = 81.6 \text{ m}$

g) $s(t) = 0$ so
 $t = 10$ seconds

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174. a) $v(t) = 2t^2 - t + 3$
 b) $v(3) = 18 \text{ m/s}$
 c) $s(t) = \frac{2}{3}t^3 - \frac{1}{2}t^2 + 3t + 4$
 d) $s(6) = 148 \text{ m}$
 e) Dist. = $s(3) - s(2)$
 $= 13.17 \text{ m (4 sf)}$
 f) $s(0) = 4, s(3) = 26.5 \text{ m}$
 velocity = 7.5 m/s

175. a) $a(t) = 2t - 14$
 $a(3) = -8 \text{ m/s}$
 b) $s(t) = \frac{1}{3}t^3 - 7t^2 + 40t + 20$
 c) $v(t) = 0$ at
 $t = 4, 10$
 $s(10) = 53.3 \text{ m}$

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176. a) $a(t) = 0$
 $t = 40 \text{ s}$
 b) $v(t) = 8t - 0.1t^2 + 0$ (from plane)
 $v(40) = 160 \text{ m/s}$
 c) $s(t) = 4t^2 - 0.0333t^3 + 0$ (from plane)
 $s(40) = 4270 \text{ m (3 sf)}$
177. a) $f'(x) = 2ax + b$ at $x = 2$ $f'(2) = 0$
 $0 = 4a + b$
 $b = -4a$
 $f(x) = ax^2 - 4ax - 3$
 $3 = 4a - 8a - 3$
 $a = -1.5$ and $b = 6$
 b) For the parabola to have a maximum 'a'
 must be negative.
 $3 = 4a - 8a + c$ for $a > 0$
 $3 - c > 0$
 $c < 3$

178. a) $f'(x) = 6(x - 2)(x - 5)$
 At turning points $f'(x) = 0$ giving
 $x = 2, 5$
 b) $f(x) = 2x^3 - 21x^2 + 60x + C$
 through $(5, 26)$
 $26 = 2 \times 125 - 21 \times 25 + 60 \times 5 + C$
 $C = 1$
 $f(x) = 2x^3 - 21x^2 + 60x + 1$
 At $x = 2$
 Maximum value $(2, 53)$

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179. a) $a(t) = -12.5 \text{ m/s}^2$
 $v(t) = -12.5t + C$ and $(2, 10)$
 $10 = -25 + C$
 $v(t) = -12.5t + 35$
 b) $s(t) = -6.25t^2 + 35t + 0$ as distance from $t = 0$.
 $v(t) = 0$ at $t = 2.8 \text{ s}$
 Max. distance therefore 49 m .

180. a) $V = x(200 - 2x)(300 - 2x)$
 $V = 4x^3 - 1000x^2 + 60\,000x$
 b) $V' = 12x^2 - 2000x + 60\,000$
 which is equal to 0 when
 $x = 39.2$ and 127.4 mm
 Discard 127.4 as it is over half the 200 mm of
 one side.
 $V(\text{max}) = 1\,060\,000 \text{ mm}^3$ (3 sf)

181. a) $C'(v) = -0.25 + 0.003\,333v$
 Set equal to 0 for max / min
 $v = 75 \text{ km/h}$
 b) $C(v) = 2.625 \text{ litres/100 km}$ at minimum
 Petrol = 5.17×2.625
 $= 13.57 \text{ litres}$
 Cost = $\$29.58$

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182. a) Diameter = 26, w and h on diagram.
 b) $26^2 = w^2 + h^2$
 $S = 10wh^2$
 $S = 10w(26^2 - w^2)$
 $S' = 6760 - 30w^2$
 $S' = 0$ for max gives
 $w = 15.0 \text{ cm}$ and $h = 21.2 \text{ cm (3 sf)}$
183. a) Area = $0.5 \times \text{base} \times \text{height}$
 Area = $0.5xy$
 Area = $6x + 2x^2 - 0.5x^3$
 b) $A' = 6 + 4x - 1.5x^2$
 which is equal to 0 when
 $x = 3.737$ (ignore negative answer).
 Area = 24.3 units^2

184. a) $f'(x) = x^3 - 6x^2 + 8x$
 $f'(x) = 0$ when
 $x = 0, 2$ or 4
 b)

x	-1	0	1	2	3	4	5
$f'(x)$	-ve	0	+ve	0	-ve	0	+ve
grad.	↘	min	↗	max	↘	min	↗

Increasing $0 < x < 2$ or $x > 4$.

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Practice External Assessment Task

Question One

- a) i) $f'(x) = 3x^2 - 4$
 $f'(2) = 8$ **A**
- ii) $\frac{dy}{dx} = 3 - 6x - 3x^2$
 $m = 6$ **(A)**
 $y = 6x + 5$ **M**
- b) i) $h'(x) = -x^2 + 3x - 2$
 Turning points $h'(x) = 0$
 $x = 1, 2$
 Minimum point (1, 1.667)
 Maximum point (2, 1.833) **(M)**
 Difference = 0.1667 m
 = 16.7 cm (3 sf)
 which is less than the required 20 cm. **E**
- ii) Starts 2.5 m above the ground.
 Hollow is across 1 m and up 1.667 m. **(A)**
 Lip is across 2 m and up 1.883 m.
 End is across 3 m and 1 m above the ground. **M**
- c) $f'(x) = 3x^2 - 6x - 6$
 $3 = 3x^2 - 6x - 6$
 $x = -1, 3$
 (-1, 10) and (3, -10) **M**
- d) i) $C'(s) = \frac{2s}{3} - 12$
 $s = 18$ km/h **A**
- ii) $C_{\min} = \$117$ per h
 $t = \frac{54}{18}$
 = 3 hours
 Cost = \$351 **E**

Question Two

- a) $f'(x) = 6x^3 - 3x^2 + x$
 $f'(2) = 38$ **A**
- b) $f'(x) = 6x^4 - 15x^2 - 7$
 $f'(-2) = 29$ **A**
- c) $f(x) = 1.5x^4 - 2x^3 - 4x^2 + 5x + C$
 $f(x) = 1.5x^4 - 2x^3 - 4x^2 + 5x - 2$ **M**
- d) $f'(x) = 4x^3 - 12x^2 - 4x + 12$
 $0 = x^3 - 3x^2 - x + 3$
 $x = -1, 1, 3$
 Min. at (-1, -7) and (3, -7) and max. at (1, 9)
 Demonstrate max and min with Calc. **E**
- e) i) $w'(t) = 30t^2 - 570t + 2100$
 $w'(8) = -540$ kg/day **M**

Question Two cont...

- ii) $w'(t) = 30t^2 - 570t + 2100$
 $0 = t^2 - 19t + 70$
 $t = 5, 14$ days
 Min. (14, 1100) **(M)**
 Demonstrate this is a minimum. **E**

Question Three

- a) i) $f'(x) = 3x^2 + 3x - 6$
 $f'(1) = 0$ **(A)**
 $x = -2$
 (-2, 12) is a max. and (1, -1.5) is a min. **M**
- ii) Function is increasing before the min. and after the max. points so $x < -2$ or $x > 1$ **M**
- b) i) $f(x) = x^3 - 4.5x^2 - 12x + C$
 $f(x) = x^3 - 4.5x^2 - 12x + 2$ **A**
- ii) $f'(x) = 3x^2 - 9x - 12$
 $0 = x^2 - 3x - 4$
 $x = -1, 4$
 Min. (4, -54) **M**
- c) i) $a(t) = 6t - 24$
 $a(2) = -12$ m/s² **A**
- ii) Flying back towards the radar station when velocity is negative, i.e. $2 < t < 6$ seconds. **E**
- iii) $s(t) = t^3 - 12t^2 + 36t + C$
 through (5, 10) gives $C = 5$ m. **(M)**
 Furthest point at $t = 2$ where $s = 37$ m **E**

Judgement

The grade in brackets is an alternative if the full grade is not earned. In each question the student needs at least 2 **A** for an **Achievement**, 2 **M** for a **Merit** and 1 **E** plus 1 **M** for **Excellence**.

- E** The final grade is found by combining the results of the three questions.

Achievement

- A** Requires two question **Achievements** or better.

Merit

- A** Requires two question **Merits** or better.

Excellence

- M** Requires two question **Excellences**.

NZQA External Examination

In the external examinations NZQA uses a different approach to marking based on understanding (u), relational thinking (r) and abstract thinking (t). They then allocate marks to these concepts (maximum of 8 for a question) and add them up to decide upon the overall grade. This approach is not as easy for students to self mark as the NuLake approach, but the results should be broadly similar.