Answers

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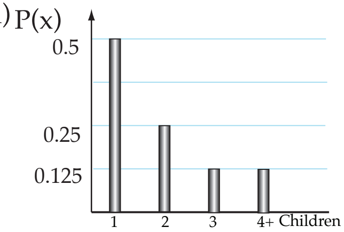
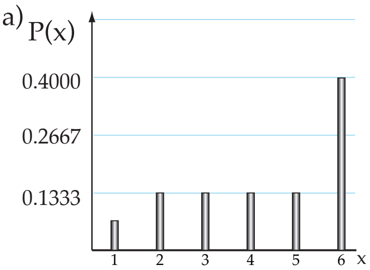
1. a)



b) P(X = 2+) = 0.5

c) P(1 brother) = 0.5 x 0.25
= 0.125

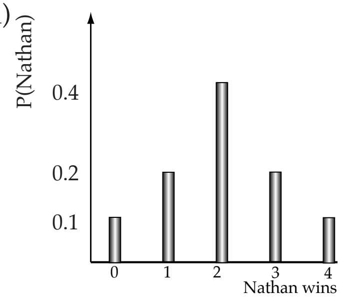
2. a)



b) P(X = 6) = 0.4

c) P(X = odd) = 0.3333

3. a)



b) P(X = 2) = 0.4

c) P(X = 3+) = 0.3

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4. a)

X	1	2	3	4
P(X = x)	0.25	0.25	0.25	0.25

b) E(X) = 2.5

5. a) E(X) = 1.59 hours
Var(X) = 0.637 (3 sf)

b) P(X < 2) = 0.47

6. a)

X	1	2	3	4	5
P(X = x)	0.45	0.30	0.12	0.10	0.03

b) P(5 days) = 0.03

c) E(X) = 1.96 days

7. a)

X	0	1	2	3
P(X = x)	p	6p	2p	p
P(X = x)	0.1	0.6	0.2	0.1

b) P(3 lambs) = 0.1

c) E(X) = 1.3 lambs
Var(X) = 0.610 (3 sf)

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8.

X	$^{-}6$	3	4	5	12
P(X = x)	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

E(X) = 2 squares
Var(X) = 40.3 (3 sf)

9.

X	0	\$5	\$10
P(X = x)	0.375	0.375	0.25

E(Winnings) = 4.375
E(Result) = $^{-}0.625$
Loss of 62 or 63 cents
Var(X) = 15.2

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10. E(2X) = 14
Var(2X) = $2^2 \times 5.83$
= 23.32

11. E(X + X) = 14
Var(X + X) = 5.83 + 5.83
= 11.66

12. a) E(M) = 4.2
Var(M) = 2.676^2
= 7.16
E(C) = 3.75
Var(C) = 2.095^2
= 4.39

b) P(some) = 1 – 0.03
= 0.97

c) E(Tot.) = 7.95
Var(Tot) = 11.55

13. a) About 10.
Distribution fairly symmetrical about 10.

b) E(C) = 9.9
Var(C) = 45.1

c) E(\$) = 123.75
Var(\$) = 7047

14. a) Kaydee E(C) = 15 (symmetrical)
Jenni E(C) = 12 Skewed distribution.

b) Kaydee E(C) = 15
Var(C) = 30.7
Jenni E(C) = 11.85
Var(C) = 18.4.

c) E(K – J) = 3.15
Var(K – J) = 49.1

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15. a) P(X = 2) = 0.2791

b) P(X < 3) = 0.8217

c) P(X ≤ 2) = 0.8217(6)

d) P(X > 2) = 0.1783(4)

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16. a) P(X = 0) = 0.0576

b) P(X = 4) = 0.1361

c) P(X ≤ 3) = 0.8059

d) P(X ≥ 6) = 0.0113

17. a) P(X = 5) = 0.0001

b) P(X = 0) = 0.4437

c) P(X > 3) = 0.0023

d) P(X < 2) = 0.8352

18. a) P(X = 5) = 0.0467

b) P(X ≥ 5) = 0.0580

c) P(X = 1) = 0.1977

d) P(X ≤ 4) = 0.9420

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19. a) P(X = 6) = 0.1762

b) P(X < 5) = 0.0196

c) P(3 ≤ X ≤ 7) = 0.5636(5)

d) P(X = 9) = 0.1342

20. a) P(X = 2) = 0.0229

b) P(X < 5) = 0.2616

c) P(X = 0) = 0.0003

d) P(X = 8) = 0.0763

21. a) P(X = 4) = 0.2322

b) P(X ≤ 3) = 0.1737

c) P(X > 4) = 0.5941

d) P(X = 0) = 0.0007

22. a) P(X ≥ 5) = 0.8552(1)

b) P(X = 6) = 0.2731

c) P(5 ≤ X ≤ 7) = 0.7121

d) P(X > 0) = 0.9999

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23. a) Fixed number of events.
Independence assumed.
Probability constant.
Only two outcomes.

b) P(X ≥ 2)
= 1 – [P(X = 0) + P(X = 1)]
= 0.4573(4)

c) P(X ≥ 2) and P(X ≥ 2)
= 0.2091

24. a) P(X = 3) = 0.2668

b) P(X > 2) = 1 – 0.3828
= 0.6172

c) $[P(X>2)]^2$ = 0.3809

25. a) P(X ≥ 5) = 0.9936

b) $[P(X \geq 5)]^5$ = 0.9686

c) Yes. Only two outcomes.
Fixed No. of trials = 10.
Independence assumed.
Constant probability = 0.8.
If a disease or fungi spreads from one bulb to another, results are not independent, for example.

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26. a) Fixed No. of trials = 5
Independence assumed
Constant probability = 0.2
Only two outcomes.

b) Possibly not independent. If they are late one day it may affect probability on following days.

c) $P(X = 1)^2 = 0.1678$

d) $P(X = 1$ in two weeks)
$= 0.2684$

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27. a) Fixed number of events.
Independence of one test to the next test.

b) $P(X \le 2) = 0.2616\ (7)$

c) $P(X > 7) = 1 - P(X \le 7)$
$= 0.3990$

28. a) Fixed No. of throws per game.
Independence assumed.
Probability constant = 0.8.
Only two outcomes.

b) $P(X = 6) \times P(X = 3)$
$= 0.02147$

c) $P(X = 9) = 0.2362$

d) No for independence as one result could affect the next and no for constant probability as Heidi will tire during the game and may not shoot as well as at the start.

29. a) $[P(X = 0)]^{10} = 0.735110$
$= 0.0461$

b) $P(X \le 1) = 0.967\,226$
$P(X \le 1)^{10} = 0.7166$

30. a) $P(X \ge 1) = 0.2262$

b) $P(X \le 1) = 0.3019$

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31. a) $P(X = 0) = 0.0183$

b) $P(X = 2) = 0.1465$

c) $P(X > 3) = 0.5665$

d) $P(4 \le X \le 6) = 0.4559\ (8)$

32. a) $P(X = 2) = 0.0107$

b) $P(X > 3) = 0.9576$

c) $P(4 \le X \le 7) = 0.4106$

d) $P(X \ge 6) = 0.8088$

33. a) $P(X = 0) = 0.0334$

b) $P(X > 4) = 0.2558$

c) $P(2 \le X \le 5) = 0.7237$

34. a) $P(X = 3) = 0.2090$

b) $P(X = 0) = 0.3012$

c) $P(X \le 5) = 0.6510(09)$

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35. a) $\lambda = 1.609$

b) 2 ways of 0 with 2 gives
$= 0.1036$

36. a) $\lambda = 2.4$ (1 dp)

b) $0.2613^5 = 0.0012$

37. a) Poisson $\lambda = 2.4$ (rejt./ shift)
$P(X \le 1) = 0.3084$
$P(X \le 1)^2 = 0.0951$

b) Constant rate of rejections.
Rejections occur randomly.
Rejections are independent.
No rejections occur simultaneously.

c) It may not be reasonable to assume the rate of car rejections is constant as there is likely to be differences between early and late shifts, also the start and end of a shift.

The Poisson distribution has a mean of 2.4 rejections and the observation mean is similar
mean = $\sum x.p = 2.44$
but the spread appears different. The Poisson
S.D. $= \sqrt{2.4} = 1.55$
while the observations appears more spread out with a
S.D. $= \sqrt{\sum x^2 p - \mu^2} = 1.8.$

The Poisson distribution has a peak close to its mean and then steadily declines from this peak. The observation is bimodal with peaks at 1 and 4. The Poisson would predict the probability of four rejections as 0.1254 while the observation has a probability of 0.18. Therefore the conclusion is the observations are poorly modelled by a Poisson distribution.

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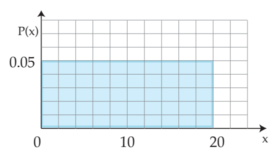
38. a)


b) $P(0 < X < 2) = 0.1$

c) $P(5 < X < 18) = 0.65$

d) $P(6.5 < X < 7.5) = 0.05$

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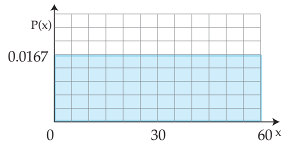
39. a)


b) $P(X < 5) = 0.0833$

c) $P(X > 55) = 0.0833$

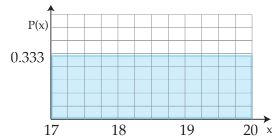
d) $P(X < 45) = 0.75$

40. a) $P(av) = 0.125$

b) $P(11 < X < 12) = 0.125$

c) $P(X = 11) = 0$
Continuous distribution

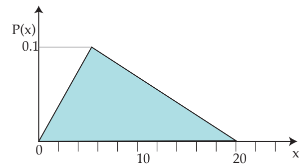
d) $P(X > 11) = 0.875$

41. a)


b) $P(1800 < X < 1815)$
$= 0.0833$ (4 dp)

c) Yes. Expect 6.67 of 20 but it is random so will vary.

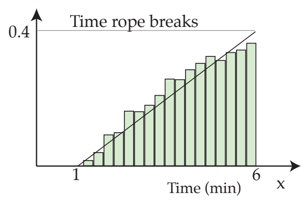
Page 31 (Rounded to 4 dp)

42. a)


b) $P(X < 5) = 0.5 \times 5 \times 0.1$
$= 0.25$

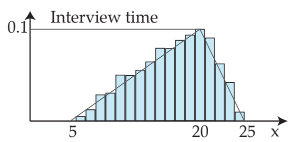
c) $f(2) = 0.04$
$P(0 < X < 2) = 0.5 \times 2 \times 0.04$
$= 0.04$

d) $A = 0.5 \times 13 \times (0.1 + 0.01333)$
$= 0.7367$
$P(5 < X < 18) = 0.7367$

43. a)


b) $A = 0.5 \times 1 \times (0.08 + 0.16)$
$= 0.12$
$P(2 < X < 3) = 0.12$

c) $A = 0.5 \times 0.25 \times (0.4 + 0.38)$
$= 0.0975$
$P(5.75 < X < 6) = 0.0975$

44. a)


b) $P(X > 20) = 0.5 \times 5 \times 0.1$
$= 0.25$

c) $P(X < 10) = 0.5 \times 5 \times 0.03333$
$= 0.0833$

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44. d) $P(10 < X < 20) = 0.6667$
$P(20 < X < 22) = 0.16$
$P(10 < X < 22) = 0.8267$

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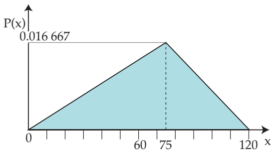
45. a)



b) Area triangle = 0.025
$P(0 < X < 15) = 0.025$

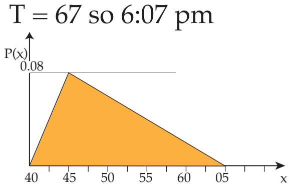
c) Area of trap. = 0.225
Area of triangle = 0.375
$P(60 < X < 120) = 0.6$

d) Area of trap. 1 = 0.225
Area of trap. 2 = 0.2083
$P(60 < X < 90) = 0.4333$

e) Probability of arriving after 6:15 is 0.375. Should be 0.5 for median.

f) Find triangle with area 0.5
$0.5 \times T \times 0.000222T = 0.5$
T = 67 so 6:07 pm

46. a)



b) $P(X > 45) = 0.5 \times 20 \times 0.08$
$= 0.8$

c) $P(X < 60) = 1 - P(X > 60)$
$= 0.95$

d) Probability of arriving after 9:45 is 0.8. Should be 0.5 for median.

e) Let T = time to 10:05
Find triangle with area 0.5
$0.5 \times T \times 0.004T = 0.5$
T = 15.8 minutes
Median = 9:49.2

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47. a) $P(0<Z<1.452) = 0.4268$ (7)
b) $P(Z>1.452) = 0.0732$ (3)
c) $P(Z<1.452) = 0.9268$ (7)
d) $P(^{-}1.452 < Z < 1.452)$
$= 0.8536$ (5)

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48. a) $P(Z < {}^{-}0.973) = 0.1652$ (3)
b) $P(^{-}0.895 < Z < 1.059)$
$= 0.3147 + 0.3552$
$= 0.6699$
c) $P(0.652 < Z < 2.074)$
$= 0.4810 - 0.2428$
$= 0.2382$
d) $P(^{-}1.953 < Z < {}^{-}1.049)$
$= 0.1217$

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49. a) $P(X > 108) = 0.0548$
b) $P(X < 95) = 0.1587$
c) $P(98 < X < 107) = 0.5746$
d) 159

50. a) $P(X > 4250) = 0.0345$
b) $P(2600 < X < 4000)$
$= 0.7950(1)$
c) Number outside range
= 205

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51. a) $P(X < 24) = 0.0913$ (2)
b) $P(X > 20) = 0.9962$
c) $P(25 < X < 30)$
$= 0.5890$ (89)
d) 61 days

52. a) $P(X > 30) = 0.0159$
b) $P(18.5 < X < 25)$
$= 0.6423$ (1)
c) $P(X < 17.5 \text{ or } X > 30)$
$= 0.0790$ (1)
d) 0.07898×650
= 51 or 52 boys

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53. a) $P(X > 5) = 0.1587$
$[P(X > 5)]^3 = 0.0040$
b) $P(X < 3) = 0.0668$
$[P(X < 3)]^3 = 0.0003$

54. a) $P(X < 3) = 0.141\ 99$
$P(Y < 3) = 0.308\ 54$
Both = 0.0438
b) $P(X < 2) = 0.037\ 07$
$P(Y < 2) = 0.040\ 06$
Either = 0.0756
c) $P(X > 5) = 0.3605$
$P(Y < 2) = 0.0400$
$P(X > 5)$ and $P(Y < 2)$
$= 0.0144$

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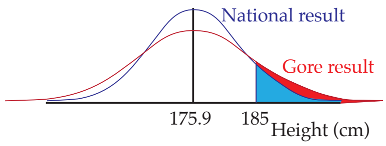
55. a) $P(X < 0.1667) = 1.6 \times 10^{-6}$
(any answer that rounds to 4 dp).
Only one call so it does not call into question the parameters.
b) $P(\text{three} > 15) = (0.2602)^3$
$= 0.0176$
c) Time to answer each call is independent of any other call and the parameters are constant throughout the day. This is unlikely as they will change at times of high demand.

56. a) $P(X > 185) = 0.2222$
b) $P(X < 165) = 0.1798$
$P(\text{three} < 165) = 0.0058$
c) Not reasonable as each of the friends may feel more comfortable with someone close to their own height therefore not random.
d) Standard deviation of 17 yo male heights in Gore must be larger (16.1) as it has more results further from the mean.



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57. 73.3 m

58. Distinction = 63 or better
Merit = 55 to 62

59. Lower = 117
Upper = 131

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60. a) Mean = 17.4 mm
b) Reject > 20.5 mm

61. a) Std. Dev. = 18.5 kg
b) $P(X > 70.0) = 0.1587$

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62. a) $P(X > 65.5) = 0.0846$
b) $P(X < 57.5) = 0.2660$
c) $P(57.5 < X < 61.5) = 0.3802$
d) 0.0846×500
$= 42.3$
= 42 or 43 sacks

63. a) $P(X < 1.45) = 0.1908$
b) $P(X < 1.45)^2 = 0.0364$
c) $P(X < 1.55)^4 = 0.1743$

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64. a) $P(X > 5.025) = 0.3951$
b) $P(X > 4.975) = 0.4250$
c) $P(4.975 < X < 5.025)$
$= 0.0299$
d) Lower quartile = 4.4135 kg
So salmon recorded as 4.40 kg and lighter.

65. a) $P(X > 30.5) = 0.0401$
b) $P(X > 28.5) = 0.0861$
2.14 times or just over twice as likely.

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66. a) $P(X < 37.5) = 0.2647$
b) $P(X > 42.5) = 0.4296$
c) Top 15% = 47.8 g so recorded as 50 g
d) $P(42.5 < X < 47.5) = 0.2670$ Number 32 mice expected.

67. a) $P(54.5 < X < 55.5) = 0.0498$
b) $P(X < 37.5) = 0.0143$ (4)
c) $P(X > 64.5) = 0.1175$ (5) Only 11.75% achieving 65 lengths.

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68. a) $P(X = 1) = 0.2707$
b) $P(X = 1 \text{ and } X = 1) = 0.0733$
c) $P(X = 0 \text{ and } X = 2)$
$= 0.1353 \times 0.2707 = 0.0366$

69. a) $P(X = 10) = 0.1969$
b) $P(X = 8) = 0.2759$
$P(X = 8 \text{ and } X = 10)$
$= 0.2759 \times 0.1969 = 0.0543$

70. P(Late once)
= P(Late) x P(≠ late)
+ P(≠ late) x P(Late)
= 0.04446

71. $P(X \geq 3) = 0.7619$
$P(X \geq 3 \text{ and } X \geq 3)$
$= 0.7619^2$
$= 0.5805$

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72. a) $P(X < 6)^2 = 0.0112$
b) $n = 5, \pi = 0.1056$
$P(X = 2) = 0.0798$

73. a) $P(X = 0) = 0.0907$
b) $n = 8, \pi = 0.3012$
$P(X = 5) = 0.0474$
c) $n = 8, \pi = 0.3012$
$P(X \geq 5) = 0.0589$

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74. $P(X \geq 1) = 1 - P(X = 0)$
$= 0.2592$
P(three successive days)
$= 0.2592^3$
$= 0.0174$

75. $n = 5, \pi = 0.1649$
$P(X \geq 3) = 0.0345$

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76. a) Binomial.
$\text{Failure}^5 = 0.30$
Failure = 0.7860
Success = 0.2140
b) $P(X = 2) = 0.2224$
So consistent with 22 results of two 'sixes'. Biased as expect success to be 0.1667 not 0.2140.
c) Mean = 1.07 sixes / throw
d) $P(X = 5) = 0.000448$
Expect to throw 2232 times.

77. a) Unlikely to be a symmetrical distribution around the mean. Some amounts will be very common.
b) P(Money > 30) = 0.22
σ = $7.12
c) Binomial, $\pi = 0.22$, $n = 4$
$P(X \geq 2) = 0.2122$
d) The income of the four friends is unlikely to be independent of each other.

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78. a) Poisson as rate proportional to interval (area), independent, distribution must be random and no simultaneous results.

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78. b) $\lambda = 1.77$ worms / m^2
Pop. = 160 000 worms.
c) Reality. Worms will depend upon the quality of soil so distribution unlikely to be random. Experiment. Potassium permanganate may not force up all worms. Selection of the 100 square plots may not be random. Other answers possible.

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79. a) Binomial. Independent, Two outcomes, fixed probability and probability must be constant. After 2500 spins the reality is likely to be reflected in the results.
$\pi = 0.1008$.
b) $P(X = 2) = 0.0274$
c) Reel 1 p = 0.096 so expect cherries = 7
Reel 2 p = 0.108 so expect cherries = 8
Reel 3 p = 0.0984 so expect cherries = 7

80. a) Poisson as rate proportional to interval, may be independent, distribution random and no simultaneous results.
b) Mean = 1.7 texts / min.
Variance = 1.8. Similar so Poisson confirmed.
c) $\lambda_{5\text{-minute}} = 8$ or 9 texts
d) Binomial trials = 5, $\pi = 0.2$
$P(X = 2) = 0.2048$

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Practice External Assessment – Probability Distributions

Q	Evidence	Achievement	Merit	Excellence
1 a)	P(Light > 4) = 0.0459 P(Regular < 4) = 0.0408 Approximately the same so no misquoting.	2 probabilities plus conclusion.		
b) i)	P(Light > 3) = 0.3004 P(Regular > 10) = 0.1743 Probability both = 0.0524	Correct approach with one error.	All probabilities correct.	
b) ii)	Binomial P(X = 3) = 0.0657, So 6.5% you would get 3 out of 20 by **chance alone** so on this **one sample** there is not enough evidence to suspect the distribution.		Binomial probability correct.	Correct conclusion.
c)	1 kg is 0.91536 standard deviation above the mean. One standard deviation = 0.180 kg.	Partial calculation.	Correct answer.	

Q	Evidence	Achievement	Merit	Excellence
2 a) i)	$P(X > 2) = 0.2390$	Correct solution.		
ii)	The Binomial distribution requires the results to be independent. If the delivery van was in an accident or something occurred the independence of damage to each item would likely not hold.	Correctly identifies independence with explanation.		
iii)	Binomial probability. Insurance income = \$19.50 / van load. Only pay out if ≥ 3 items damaged. Payout = 0.1546 x \$50 + 0.0611 x \$100 + 0.0181 x \$150 + 0.0042 x \$200 + 0.0001 x \$250 = \$17.42 Profit or return = \$19.50 – \$17.42 = \$2.08 per van load.		Correct return.	
b) i)	Mean = 1.7 and Variance = 1.8 Poisson as random, no simultaneous solution, mean approximately equals variance and results are likely to be independent.		λ (mean) correct.	Correct justification.
ii)	$\lambda = 5.1$. $P(X \geq 4) = 0.7488$	λ (mean) correct.	Correct solution.	
3 a)	Triangular distribution. Mode 8, Height = 0.1667 $P(X > 7) = 1 - P(X < 7)$ $P(X > 7) = 0.8125$	Correct solution.		
b)	i) First office $P(X > 12) = 0.167$ so second office slower. ii) Mode = 9.33 minutes.	i) Correct.	i) Correct solution and conclusion.	i) Correct and conclusion plus mode.
c)	Assuming a binomial distribution with $\pi = 0.5$ and $n = 18$ probability of getting a result from 0 to 6 answers is 0.119. Therefore if this occurs 12% of the time by chance alone we cannot conclude it was not chance. Also only one trial.		Some evidence but small errors.	Correct conclusion with evidence.
d) i)	$\lambda = 4.5$ / 15 min., $P(X > 4) = 0.4679$	Correct solution.		
ii)	$P(X = 0) = 0.16$ implies $\lambda = 1.832$ / 5 min. $\lambda = 5.5$ / 15 min., $P(X > 4) = 0.6425$	A correct λ.	Correct solution.	

Practice Assessment – Probability Distributions

In the external examinations NZQA uses a different approach to marking based on understanding (u), relational thinking (r) and abstract thinking (t). They then allocate marks to these concepts and add them up to decide upon the overall grade. This approach is not as easy for students to self mark as the NuLake approach, but the results should be broadly similar.

Sufficiency. For each question award yourself a score out of 8 using this table. Add the three scores for a score out of 24 and compare to the cut scores. All answers must include evidence / justification where appropriate.

Quest.	N0	N1	N2	A3	A4	M5	M6	E7	E8
ONE	Nil correct	Part correct	1 A	2A or equiv.	3A or equiv.	1M + 1M minor error	2M	1E Minor error	1E all correct
TWO	Nil correct	Part correct	1 A	2A or equiv.	3A or equiv.	1M + 1M minor error	2M	1E Minor error	1E all correct
THREE	Nil correct	Part correct	1 A	2A or equiv.	3A or equiv.	1M + 1M minor error	2M	1E	2E all Minor error

Cut Scores

Not Achieved	Achievement	Achievement with Merit	Achievement with Excellence	
0 – 6	7 – 13	14 – 20	21 – 24	