Answers

Page 4

1. 14

2. $2\sqrt{2}$

3. $22\sqrt{2}$

4. 1

5. $3\sqrt{2}-3\sqrt{5}$

6. $^{-}17$

7. $52+6\sqrt{35}$

8. $\frac{2}{3}$

Page 5

9. $\frac{2\sqrt{15}}{5}$

10. $\frac{^{-}\sqrt{15}-5\sqrt{3}+3\sqrt{5}+15}{20}$

11. $\frac{2\sqrt{6}+3\sqrt{2}}{6}$

12. $\frac{4\sqrt{14}-3\sqrt{10}}{10}$

13. $\frac{11+6\sqrt{2}}{7}$

14. $^{-}8-10\sqrt{2}$

15. $\frac{^{-}3\sqrt{14}+9\sqrt{2}+5\sqrt{7}-15}{4}$

16. $\frac{12\sqrt{10}+4\sqrt{15}-6\sqrt{2}-2\sqrt{3}}{15}$

17. $\frac{a\sqrt{6}+3\sqrt{2}}{6}$

18. $\frac{2\sqrt{5}a+\sqrt{5b}}{5}$

19. $\frac{6+3\sqrt{a}}{4-a}$

20. $\frac{4\sqrt{b}+2b}{4b-b^2}$

Page 6

21. $\frac{a\sqrt{b}-b\sqrt{a}}{a-b}$

22. $\frac{^{-}2\sqrt{a}}{4-a}=\frac{2\sqrt{a}}{a-4}$

23. $\frac{1+6\sqrt{a}+9a}{1-9a}$

24. $\frac{2}{3x-1}$

25. $34+9\sqrt{2}$

26. $\frac{4}{1-a}$

Page 6 cont...

27. a) $\frac{a\sqrt{a}+a-2}{a-1}$

b) $\frac{a\sqrt{a}-2\sqrt{a}+a}{a-1}$

Page 9

28. $(x+10(x-2)=0$
$x=^{-}10, 2$

29. $(2x+1)(3x-4)=0$
$x=\ ^{-}\frac{1}{2},\ \frac{4}{3}\left(1\frac{1}{3}\right)$

30. $(4k-7)(4k+7)=0$
$k=\frac{7}{4}\left(1\frac{3}{4}\right),\ \frac{^{-}7}{4}\left(^{-}1\frac{3}{4}\right)$

31. $(x-4)(2x+3)=0$
$x=4,\ \frac{^{-}3}{2}\left(^{-}1\frac{1}{2}\right)$

32. $6x(3-4x)=0$
$x=0,\ \frac{3}{4}$

33. $(5x-4)(2x-3)=0$
$x=\frac{4}{5},\frac{3}{2}\left(1\frac{1}{2}\right)$

34. $(4x-1)(3x+5)=0$
$x=\frac{1}{4},\frac{^{-}5}{3}\left(^{-}1\frac{2}{3}\right)$

35. $(k-6)(9k+2)=0$
$k=6,\frac{^{-}2}{9}$

36. $(x-7)(x+1)=0$
$x=7, ^{-}1$

37. $2(1-2k)(1+2k)=0$
$k=\frac{1}{2},\frac{^{-}1}{2}$

38. $(3x-1)(x-5)=0$
$x=5,\frac{1}{3}$

39. $3(a-3)(a+3)=0$
$a=3, ^{-}3$

40. $(h-3)(3h+5)=0$
$h=3,\frac{^{-}5}{3}\left(^{-}1\frac{2}{3}\right)$

41. $(x-\frac{1}{2})(x+\frac{1}{2})=0$
$x=\frac{1}{2},\frac{^{-}1}{2}$

42. $(x-a)(x-a)=0$
$x=a$

43. $(x-a)(x+2a)=0$
$x=a, ^{-}2a$

44. $(x-a)(x-3a)=0$
$x=a, 3a$

45. $(x-2a)(x+2a)=0$
$x=2a, ^{-}2a$

Page 11

46. $(x+2)^2-5=0$
$x=^{-}2\pm\sqrt{5}$

47. $(x-3)^2-18=0$
$x=3\pm3\sqrt{2}$

48. $(x+4)^2-22=0$
$x=^{-}4\pm\sqrt{22}$

49. $(x+2.5)^2-5.25=0$
$x=\frac{^{-}5}{2}\pm\sqrt{\frac{21}{4}}$

50. $(x+2)^2-7=0$
$x=^{-}2\pm\sqrt{7}$

51. $(x-4)^2-19=0$
$x=4\pm\sqrt{19}$

52. $(x-1)^2-11=0$
$x=1\pm\sqrt{11}$

53. $(x-5)^2-12=0$
$x=5\pm2\sqrt{3}$

54. $(x+3)^2-9+k=0$
$x=\ ^{-}3\pm\sqrt{9-k}$

55. $(x-5)^2-25+k=0$
$x=\ 5\pm\sqrt{25-k}$

56. $(x-k)^2-k^2+5=0$
$x=\ k\pm\sqrt{k^2-5}$

57. $(x+2k)^2-4k^2+1=0$
$x=\ ^{-}2k\pm\sqrt{4k^2-1}$

Page 12

58. $6(x-1)^2-24=0$
$x=^{-}1, 3$

59. $5(x-3)^2-35=0$
$x=3\pm\sqrt{7}$

60. $4(x-2)^2-24=0$
$x=2\pm\sqrt{6}$

61. $3(x+2)^2-6=0$
$x=^{-}2\pm\sqrt{2}$

62. $3(m+4)^2-3=0$
$m=^{-}3, ^{-}5$

63. $3(x+2)^2-14=0$
$x=\ ^{-}2\pm\sqrt{\frac{14}{3}}$

64. $3(x-1)^2-4=0$
$x=1\pm\sqrt{\frac{4}{3}}$ or $1\pm\frac{2}{\sqrt{3}}$

Page 12 cont...

65. $2(k+2)^2 - 1 = 0$
$k = -2 \pm \sqrt{\frac{1}{2}}$ or $-2 \pm \frac{1}{\sqrt{2}}$

66. $x = -1 \pm \sqrt{1 + \frac{k}{2}}$

67. $x = 2 \pm \sqrt{4 - \frac{k}{2}}$

68. $x = -1 \pm \sqrt{1 + \frac{6}{k}}$

69. $x = \frac{-1 \pm \sqrt{7}}{k}$

Page 14

70. $x = -0.146, -6.854$

71. $x = 1.854, -4.854$

72. $x = 5, -6$

73. $x = -1, 2.5$

Page 15

74. $x = 1.143, 0.180$

75. $x = 3.886, -0.886$

76. $x = -3 \pm \sqrt{10}$

77. $x = 3 \pm \sqrt{6}$

78. $x = \frac{-5 \pm \sqrt{29}}{2}$

79. $x = 2 \pm \sqrt{14}$

80. $x = -4 \pm \sqrt{16 + k}$

81. $x = \frac{-2 \pm \sqrt{13}}{k}$

82. $x = \frac{(k+2) \pm k}{2}$ or $k + 1, 1$

83. $x = 5k \pm \sqrt{26k}$

Page 17

84. $\Delta = 89$. Roots are **unequal**, **real** and **irrational**.

85. $\Delta = 25$. Roots are **unequal**, **real** and **rational**.

86. $\Delta = 0$. Roots are **equal** and **real**.

87. $\Delta = -23$. Roots are **unequal** and **complex**.

88. $4 - 12c \geq 0$ so $c \leq \frac{1}{3}$. Includes equal as equal roots are real.

89. $4 + 16d < 0$ so $d < -\frac{1}{4}$.

Page 17 cont...

90. $e^2 - 144 = 0$ so $e = \pm 12$.

91. $f^2 - 8 < 0$ so $-\sqrt{8} < f < \sqrt{8}$.

92. $9k^2 - 32k < 0$ so
$k(9k - 32) < 0$
$0 < k < \frac{32}{9}$

93. $9k^2 - 60k + 96 < 0$ so
$(3k - 8)(k - 4) < 0$
$\frac{8}{3} < k < 4$

Page 19

94. $p(-1) = -3$

95. $p(2) = 15$

96. $p(-1) = 3$

97. $p(-0.5) = -3.4375$

98. $p(-2) = -35$

99. $p(\frac{1}{3}) = 3.691$

100. $p(3) = 27 + 63 - 18 - 72 = 0$

101. $k = 22$

Page 20

102. $p(-1.5) = 2(-1.5)^3 + 9(-1.5)^2 - 1.5 - 12 = 0$
hence $(2x + 3)$ is a factor.

103. $k = 7$

104. $k = 12.5$

105. $k = 3, -6$

106. $k = 19$

107. $q = 2$

108. $p(2a) = 16a^3 - 4a^3 - 6a^3 - 6a^3$
$p(2a) = 0$ hence a factor

109. $m = 2, n = 5$

110. $a = 3, b = -7$

111. $a = 1, b = -8$

Page 23

112. $(x + 1)(2x - 1)(2x + 1)$
$x = -1, 0.5, -0.5$

113. $(x - 4)(x - 2)(x + 1)$
$x = 4, 2, -1$

114. $(x + 1)(2x + 1)(3x - 2)$
$x = 0.667, -1, -0.5$

115. $(x - 1)(2x - 5)(2x + 3)$
$x = -1.5, 2.5, 1$

Page 24

116. $(x + 3)(x - 3)(4x - 1)$
$x = -3, 3, 0.25$

117. $(x - 2)(x + 4)(2x - 1)$
$x = 2, -4, 0.5$

118. $(x - 4)(3x - 1)(2x + 3)$
$x = 4, 0.333, -1.5$

119. $(2x + 1)(2x - 1)(x - 1)$
$x = -0.5, 0.5, 1$

120. $(3x - 1)(x - 2)(5x - 1)$
$x = 0.333, 2, 0.2$

121. $(x + 4)(x + 2)(x - 6)$
$x = -4, -2, 6$

122. $(3x - 1)(4x - 3)(2x - 3)$
$x = 0.333, 0.75, 1.5$

123. $(x - 3)(2x - 5)(3x - 2)$
$x = 3, 2.5, 0.667$

124. $(2x - 1)^2(2 - x)$
$x = 0.5, 2$

125. $(-3x - 2)^3$
$x = -0.667$

Page 25

126. a) $p(-3) = -275$
b) $p(2) = 48 - 52 + 4 = 0$
c) $x = \frac{2}{3}, \frac{-1}{2}, 2$

127. a) $p(-1) = 3$
b) $(x + 2)$

128. a) $p(-1) = -1 + 4 - 8 + 5 = 0$
b) $p(x) = (x + 1)(x^2 + 3x + 5)$

129. a) $(6 - k)$
b) $k = 6$
c) $(x + 2)(2x + 1)(x - 3)$

130. $p(x) = (x + 3)(x - 4)(x + 1)$

131. $a = 13, b = 8$

Page 28

132. $x = 1$

133. $x = 3$

134. $x = 2$

135. $x = 8$

136. $x = 6, 5$

137. $x = 16$

138. $x = 9$

139. $x = 0.333$

Page 29

140. $x = 0.589$

141. $x = 12.685$

142. $x = 1.804$

143. $x = 2.303$

144. $x = \dfrac{(4-t)^2}{16}$

145. $x = \dfrac{(t+16)^2}{64}$

146. $x = \dfrac{^{-}q^2}{q^2-9}, q \neq \pm 3$

147. $x = \dfrac{k^2}{k^2-16}, k \neq \pm 4$

Page 34

148. $6 + 3i$

149. $5 - 3i$

150. $5 + 3i$

151. $22 + 31i$

152. $1 + 3i$

153. $10 - 10i$

154. $23 + 2i$

155. $^{-}5 + 14i$

156. $11 + 13i$

157. $^{-}1 - 9i$

158. $21 - i$

159. $^{-}14 + 44i$

160. ^{-}i

161. 2

162. $^{-}5 + 12i$

163. $^{-}2 - 2i$

164. $10 + 6i$

165. $^{-}46 - 9i$

Page 35

166. 17

167. $2 + 6i$

168. $\dfrac{3+2i}{13}$

169. $\dfrac{^{-}36-52i}{25}$

170. $\dfrac{4+3i}{5}$

171. $\dfrac{4-\sqrt{5}i}{21}$

Page 35 cont...

172. $\dfrac{^{-}4+i}{17}$

173. $\dfrac{7-3i}{3}$

174. $\dfrac{^{-}1+2\sqrt{2}i}{3}$

175. $\dfrac{12+5i}{2}$

176. $\dfrac{9+i}{5}$

177. $\dfrac{3-i}{2}$

178. $\dfrac{8-14i}{5}$

179. $\dfrac{58+4i}{169}$

Page 37

180. $z = 3 \pm \sqrt{2}i$

181. $z = \dfrac{3 \pm i}{2}$

182. $z = \dfrac{5 \pm \sqrt{83}i}{6}$

183. $z = \dfrac{^{-}2 \pm \sqrt{11}i}{3}$

184. $z = \dfrac{1 \pm \sqrt{55}i}{14}$

185. $z = \dfrac{5 \pm \sqrt{7}i}{4}$

Page 38

186. $z = 2 \pm \sqrt{2}i$

187. $z = 5 \pm 2i,$

188. $z = \dfrac{4 \pm \sqrt{92}i}{6}$

189. $z = \dfrac{^{-}3 \pm \sqrt{31}i}{4}$

190. $z = \dfrac{4 \pm \sqrt{104}i}{10}$

191. $z = \dfrac{1 \pm \sqrt{3}i}{2}$

192. $z = \dfrac{6 \pm \sqrt{60}i}{6}$

193. $z = \dfrac{1 \pm \sqrt{23}i}{4}$

Page 38 cont...

194. $z = a \pm 2ai$

195. $z = ^{-}a \pm 3ai$

196. $z^2 - 8z + 17 = 0$

197. $z^2 - 2z + 5 = 0$

198. $1 - \sqrt{3}i$, $k = 4$

199. $^{-}2 + \sqrt{2}i$, $k = 6$

Page 40

200. $x = ^{-}1,$
$x = 0.75 + 1.561i,$
$x = 0.75 - 1.561i$

201. $x = 2,$
$x = 1 + 1.732i,$
$x = 1 - 1.732i$

Page 41

202. $x = ^{-}1$, $x = ^{-}2 + 1.414i,$
$x = ^{-}2 - 1.414i$

203. $x = 2$, $x = ^{-}0.5 + 0.866i,$
$x = ^{-}0.5 - 0.866i$

204. a) $p(2) = 8 + 20 - 2 - 26 = 0$
b) $x = 2,$
$x = ^{-}3.5 + 0.866i,$
$x = ^{-}3.5 - 0.866i$

205. $x = 2$, $x = ^{-}3 + 1.732i,$
$x = ^{-}3 - 1.732i$

206. $A = 8$ and $x = 2$, $x = 3 + 2i,$
$x = 3 - 2i$

207. $A = 2$, $x = ^{-}4$, $x = 1 + 3i,$
$x = 1 - 3i$

208. $x = 1$, $x = ^{-}1 + 2i,$
$x = ^{-}1 - 2i$

209. $x = 2,$
$x = ^{-}2 + i,$
$x = ^{-}2 - i$

210. $z = ^{-}2,$
$z = ^{-}3 + i,$
$z = ^{-}3 - i$

211. $x = 1,$
$x = 2.5 + 1.658i,$
$x = 2.5 - 1.658i$

Page 42

212. a) $2 + 3i$
b) $z^2 - 4z + 13$
c) $A = 7, z = 3$

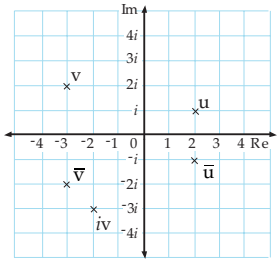
213. a) $p(^-1) = ^-1 + 4 - 8 + 5 = 0$
b) $(x + 1)(x^2 + 3x + 5)$
c) $x = ^-1$,
$x = ^-1.5 + 1.658i$,
$x = ^-1.5 - 1.658i$

214. a) $^-k^2 + 5k + 6$
b) $k = 6, ^-1$
c) $k = 6$
d) $x = ^-0.375 + 2.09i$
$x = ^-0.375 - 2.09i$
$x = 1$

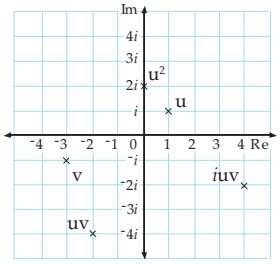
215. a) $1 - i$
b) $(z - (1 - i))(z - (1 + i))$
$= z^2 - 2z + 2$
$= (z + 2)(z^2 - 2z + 2)$
$= z^3 - 2z + 4$
so $a = ^-2, b = 4$

Page 44

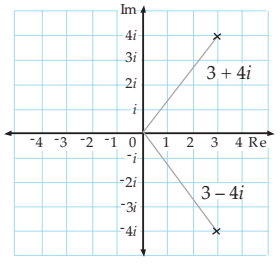
216. a) $u = 2 + i$, $v = ^-3 + 2i$
b) $\overline{u} = 2 - i$, $\overline{v} = ^-3 - 2i$
c) $iv = i(^-3 + 2i) = ^-2 - 3i$



217. a) $uv = ^-2 - 4i$
b) $iuv = 4 - 2i$
c) $u^2 = 2i$

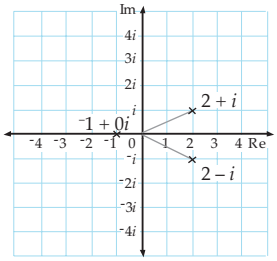


218.



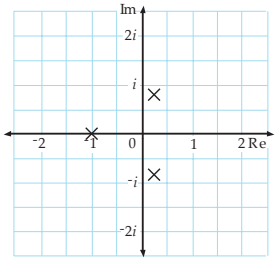
Page 44 cont...

219.



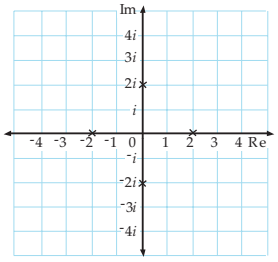
Page 45

220. $z^3 + 1 = 0$
$(z + 1)(z^2 - z + 1) = 0$
$z = ^-1, \frac{1}{2} \pm \frac{\sqrt{3}}{2}i$



All are 1 unit from the centre and are symmetrically spaced every $\frac{2\pi}{3}$ radians (120°).

221. $z^4 - 16 = 0$
$(z^2 + 4)(z^2 - 4) = 0$
$(z + 2i)(z - 2i)(z - 1)(z + 2) = 0$
$z = 2, ^-2, 2i, ^-2i$



All are 2 units from the centre and are symmetrically spaced every quarter turn. The roots sum to 0.

222. $z = 3, ^-1 \pm i$

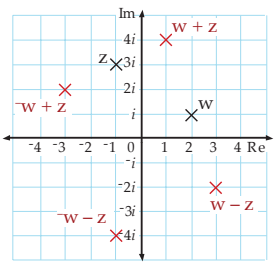
i) $(z - 3)(z + 1 - i)(z + 1 + i) = 0$
$z^3 - z^2 - 4z - 6 = 0$

ii) Sum = 1

iii) Product = 6

223. $w = 2 + i$, $z = ^-1 + 3i$
$w + z = 1 + 4i$
$w - z = 3 - 2i$
$^-w + z = ^-3 + 2i$
$^-w - z = ^-1 - 4i$

Page 45 Q223 cont...



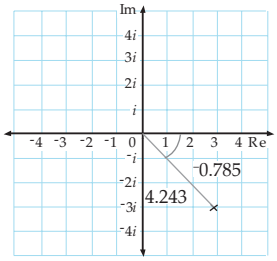
The resulting shape is a parallelogram.

Page 48

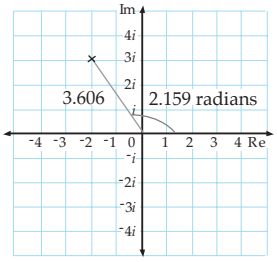
224. a) $u = 2.236 \text{ cis } 0.464$
$v = 3.606 \text{ cis } 2.554$
b) $\overline{u} = 2.236 \text{ cis } ^-0.464$
$\overline{v} = 3.606 \text{ cis } ^-2.554$
c) $uv = 8.062 \text{ cis } 3.017$

225. a) $2 \text{ cis } 0.5236$
b) $4 \text{ cis } ^-1.047$
c) $4 \text{ cis } ^-1.571$
d) $3 \text{ cis } 3.142$
e) $\sqrt{2}k \text{ cis } 0.7854$
f) $\sqrt{5}k \text{ cis } ^-0.4636$
g) $k \text{ cis } 1.571$

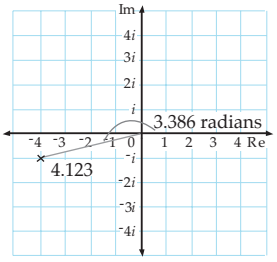
226. $z = 4.243 \text{ cis } ^-0.785$



227. $z = 3.606 \text{ cis } 2.159$

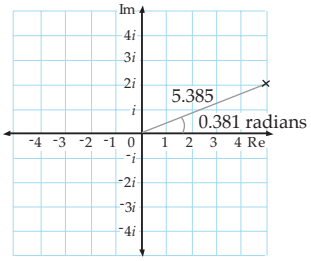


228. $z = 4.123 \text{ cis } ^-2.897$ or
$z = 4.123 \text{ cis } 3.386$



Page 48 cont...

229. $z = 5.385 \text{ cis } 0.381$



Page 49

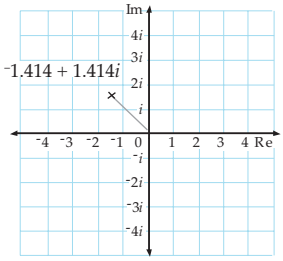
230 a) $1 + i$ b) $^{-}3$

c) $^{-}1.732 + i$ d) $3.464 - 2i$

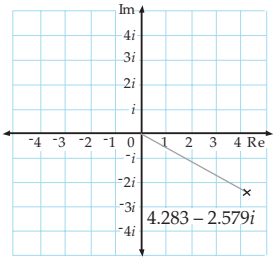
e) $0.707 - 1.225i$

f) $^{-}4.807 - 3.591i$

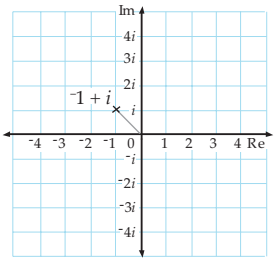
231. $z = ^{-}1.414 + 1.414i$



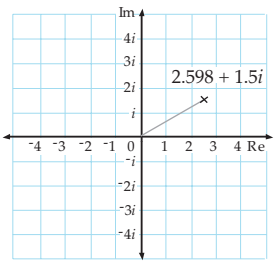
232. $z = 4.283 - 2.579i$



233. $z = ^{-}1 + i$

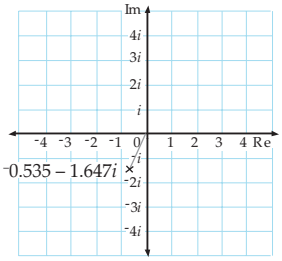


234. $z = 2.598 + 1.500i$

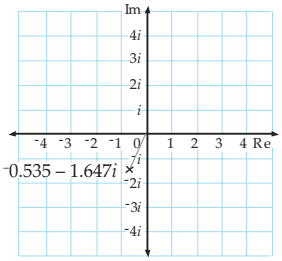


Page 49 cont...

235. $z = ^{-}1.848 - 0.765i$



236. $z = ^{-}0.535 - 1.647i$



Page 52

237. a) 108 cis 1.12

b) 324 cis $^{-}1.16$

c) 3 cis $^{-}2.283$

d) 2 cis 0.84

238. a) 7.616 cis 1.976

b) 9.849 cis $^{-}1.153$

c) 75 cis 0.823

d) 0.7733 cis 3.128

e) 441.7 cis $^{-}0.356$

239. a) 16 cis 1.15

b) 0.25 cis $^{-}0.575$

c) 0.0625 cis $^{-}1.15$

d) 64 cis 1.725

240. a) 0.2 cis 0.6435

b) 0.04 cis 1.287

c) 25 cis $^{-}1.287$

d) 125 cis $^{-}1.9305$

241. a) 28.285 cis $^{-}0.142$

b) 800 cis $^{-}0.284$

c) 0.884 cis $^{-}1.712$

d) 4.419 cis $^{-}2.639$

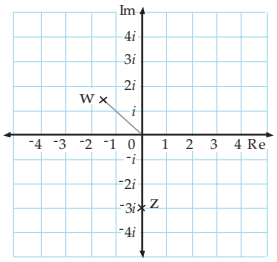
242. a) $6 + 10i$

b) 11.66 cis 1.030

c) 20.58 cis $^{-}1.030$

Page 53

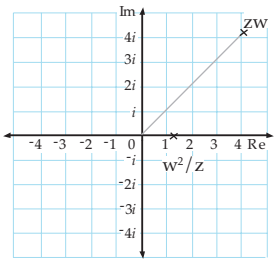
243. a)



b) 6 cis 0.785

c) 1.333 cis 0 (2π)

d)



244. a) $\dfrac{1}{z_{comb.}} = 0.28 + 0.04i$

$z_{comb.} = 3.5 - 0.5i$

b) $z_{comb.} = 3.54 \text{ cis } ^{-}0.142$

c) I = 36.77 cis 0.2869 (amps)

245. a) V = 161.8 cis 1.700

b) Z = 17.68 cis $^{-}1.429$ or
Z = 17.68 cis 4.854

246. a) z = 1 cis 2.094

b) $z^3 = 1$ cis 2π
$z^3 = 1 + 0i$ in rect. form,
hence $z^3 - 1 = 1 - 1 = 0$

247. I = 15.18 cis $^{-}0.9653$ amps

Page 55

248. a) 8 cis 180° = $^{-}8$

b) 625 cis 120° = $^{-}312.5 + 541i$

c) 5.196 cis π = $^{-}5.196$

d) 64 cis $^{-}4\pi = 64$

249. a) $^{-}597 - 122i$

b) $64 - 110.85i$

c) $^{-}240.1 - 218.1i$

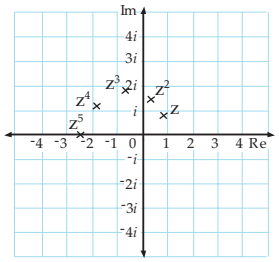
d) $5.657 + 5.657i$

Page 55 cont...

250. $z^2 = 4 \text{ cis } 2.094$
$= -2 + 3.464i$
$z^3 = 8 \text{ cis } 3.142$
$= -8 + 0i$

251. $z^2 = 20 \text{ cis } -2.214$
$= -12 - 16i$
$z^3 = 89.44 \text{ cis } 2.96$
$= -88 + 16i$

252. $z^2 = 1.44 \text{ cis } 1.256$
$z^3 = 1.73 \text{ cis } 1.885$
$z^4 = 2.07 \text{ cis } 2.513$
$z^5 = 2.49 \text{ cis } \pi$



Values appear to spiral out.

253. $$\frac{(\sqrt{3}-i)^9}{(1+i)^7} = \frac{\left(2\text{cis}\frac{-\pi}{6}\right)^9}{\left(\sqrt{2}\text{cis}\frac{\pi}{4}\right)^7}$$

$$= \frac{2^9\text{cis}\frac{-3\pi}{2}}{2^{7/2}\text{cis}\frac{7\pi}{4}}$$

$$= 2^{11/2}\text{cis}\frac{-13\pi}{4}$$

$= -32(1 - i)$

Page 57

254. $3 \text{ cis } 0.7854 = 2.12 + 2.12i$
$3 \text{ cis } 2.880 = -2.90 + 0.776i$
$3 \text{ cis } 4.974 = 0.776 - 2.90i$

255. $1.732 \text{ cis } 0.262 = 1.673 + 0.448i$
$1.732 \text{ cis } 1.833 = -0.448 + 1.673i$
$1.732 \text{ cis } 3.403 = -1.673 - 0.448i$
$1.732 \text{ cis } 4.974 = 0.448 - 1.673i$

256. $1.523 \text{ cis } -0.2975$
$1.523 \text{ cis } 1.273$
$1.523 \text{ cis } 2.844$
$1.523 \text{ cis } 4.415 (-1.868)$

Page 57 cont...

257. $1.378 \text{ cis } 0.1470$
$1.378 \text{ cis } 1.718$
$1.378 \text{ cis } 3.289 (-2.994)$
$1.378 \text{ cis } 4.859 (-1.424)$

258. $1.800 \text{ cis } 0.3435$
$1.800 \text{ cis } 2.438$
$1.800 \text{ cis } 4.532 (-1.751)$

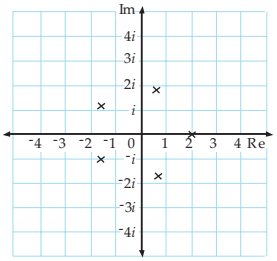
259. $1 \text{ cis } 0 = 1$
$1 \text{ cis } 2.094 = -0.5 + 0.886i$
$1 \text{ cis } 4.189 = -0.5 - 0.886i$

260. $1.414 \text{ cis } 0.349$
$1.414 \text{ cis } 2.443$
$1.414 \text{ cis } 4.538 (-1.745)$

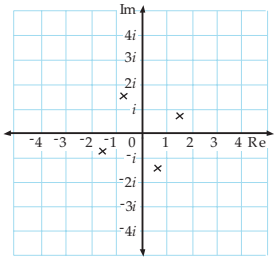
261. $1.378 \text{ cis } -0.2457$
$1.378 \text{ cis } 1.325$
$1.378 \text{ cis } 2.896$
$1.378 \text{ cis } 4.467 (-1.816)$

Page 58

262. $2 + 0i$
$0.618 + 1.902i$
$-1.618 + 1.176i$
$-1.618 - 1.176i$
$0.618 - 1.902i$



263. $1.643 \text{ cis } 0.4623$
$1.47 + 0.733i$
$1.643 \text{ cis } 2.033$
$-0.733 + 1.47i$
$1.643 \text{ cis } 3.604$
$-1.47 - 0.733i$
$1.643 \text{ cis } 5.175$
$0.733 - 1.47i$

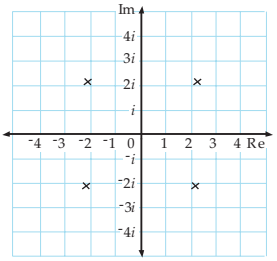


Page 58 cont...

264. $1.378 \text{ cis } -0.5398$
$1.378 \text{ cis } 1.031$
$1.378 \text{ cis } 2.602$
$1.378 \text{ cis } 4.173 (-2.110)$

265. $(2 - i)^3 =$
$2^3 + 3(2)^2(-i) + (3)(2)(-i)^2 + (-i)^3$
$= 2 - 11i$
$z_1 = 2.24 \text{ cis } -0.464$
$= 2 - i$
$z_2 = 2.24 \text{ cis } 1.63$
$= -0.134 + 2.23i$
$z_3 = 2.24 \text{ cis } -2.56$
$= -1.87 - 1.23i$

266. $2.121 + 2.121i$
$-2.121 + 2.121i$
$-2.121 - 2.121i$
$2.121 - 2.121i$



267. $1.495 \text{ cis } 1.339$
$(0.343 + 1.455i)$
$1.495 \text{ cis } 2.910$
$(-1.455 + 0.343i)$
$1.495 \text{ cis } 4.481$
$(-0.343 - 1.455i)$
$1.495 \text{ cis } 6.051 (-0.2318)$
$(1.455 - 0.343i)$

Page 59

268. $1.395 + 1.061i$
$-1.617 + 0.6774i$
$0.2217 - 1.739i$

269. $-1.414 - 1.414i$
$-1.414 + 1.414i$
$1.414 - 1.414i$
$1.414 + 1.414i$

270. $1 \text{ cis } 0.2618$
$1 \text{ cis } 2.3562$
$1 \text{ cis } 4.4506 (-1.8326)$

271. $1.414 \text{ cis } 0.7854$
$1.414 \text{ cis } 2.3562$
$1.414 \text{ cis } 3.9270 (-2.3562)$
$1.414 \text{ cis } 5.4978 (-0.7854)$

Page 59 cont...

272. 2 cis 0.1309, 2 cis 1.7017
2 cis 3.2725 ($^-$3.010)
2 cis 4.8433 ($^-$1.440)

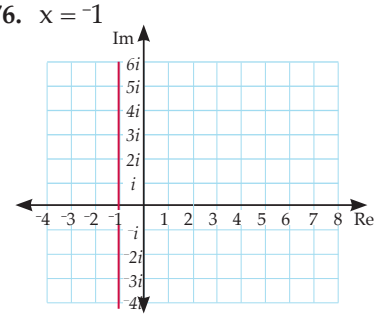
273. $z_1 = \sqrt{k}\text{cis}\frac{\pi}{8}$
$z_2 = \sqrt{k}\text{cis}\frac{5\pi}{8}$
$z_3 = \sqrt{k}\text{cis}\frac{9\pi}{8}$
$z_4 = \sqrt{k}\text{cis}\frac{13\pi}{8}$

Page 62

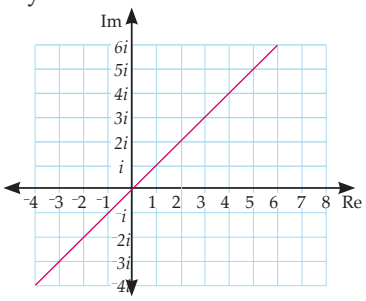
274. $(x-1)^2 + (y-2)^2 = 9$
Circle centre $1 + 2i$, radius 3

275. $x^2 + (y+3)^2 = 16$
Circle centre $0 - 3i$, radius 4

276. $x = ^-1$



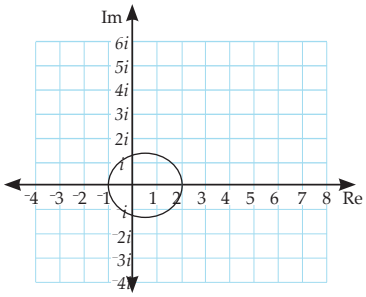
277. $y = x$



Page 63

278. $\frac{4(x-\frac{1}{2})^2}{9} + \frac{y^2}{2} = 1$

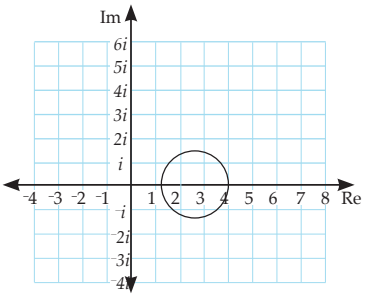
Ellipse centre $\frac{1}{2} + 0i$.
Major axis length = 3
Minor axis length = $2\sqrt{2}$



Page 63 cont...

279. $(x-\frac{8}{3})^2 + y^2 = \frac{16}{9}$

Circle centre $\frac{8}{3} + 0i$, radius $\frac{4}{3}$



280. $y^2 - \frac{x^2}{8} = 1$

Hyperbola centre (0, 0).
Vertices (0, 1) and (0, $^-$1)

Page 64 Excellence Questions

281. a) $(a + bi)^3 = a^3 - 3ab^2 + i(3a^2b - b^3)$
so $3a^2b - b^3 = 0$
so $3a^2 - b^2 = 0$ or $b = 0$
b) If $b \neq 0$ then $3a^2 = b^2$
$(a + bi)^3 = a^3 - 3ab^2$
$= a^3 - 9a^3$
$= ^-8a^3$
so $k = ^-8a^3$

282. $(z - (1 - i))(z - (1 + i))$
$= z^2 - 2z + 2$
$(z - k)(z^2 - 2z + 2)$
$= z^3 - 2z^2 - kz^2 + 2z + 2zk - 2k$
Equating
$2 + 2k = 6$ so $k = 2$
$b = ^-2k$ so $b = ^-4$
$a = 2 + k$ so $a = 4$.

283. $a^2 + 2abi - b^2 = 48 + 14i$
equating real parts
$a^2 - b^2 = 48$
and imaginary parts
$2ab = 14$ gives
$a = \pm 7$, $b = \pm 1$
or $a = \pm i$, $b = \mp 7i$
That is
(a, b) = (7, 1), ($^-$7, $^-$1), (i, ^-7i)
or (^-i, $7i$)

Page 65

284. $z_1.z_2 = (a_1 + b_1i)(a_2 + b_2i)$
$= a_1a_2 - b_1b_2 + i(a_1b_2 + a_2b_1)$
$\overline{z_1.z_2} = a_1a_2 - b_1b_2 - i(a_1b_2 + a_2b_1)$
$\overline{z_1}.\overline{z_2} = (a_1 - b_1i)(a_2 - b_2i)$
$= a_1a_2 - b_1b_2 - ia_1b_2 - ia_2b_1$
$= a_1a_2 - b_1b_2 - i(a_1b_2 + a_2b_1)$
$= z_1.z_2$

285. $k = \frac{z^2 - 4z + 5}{z - 2}$
$0 = z^2 - (4 + k)z + 5 + 2k$
Complex solutions when
$b^2 - 4ac < 0$
$k^2 + 8k + 16 - 20 - 8k < 0$
$k^2 - 4 < 0$
$(k + 2)(k - 2) < 0$
$^-2 < k < 2$

286. $\frac{x - yi + 2x + 2yi}{x^2 + y^2} = 1 + i$
$\frac{3x + yi}{x^2 + y^2} = 1 + i$
Equating
$\frac{3x}{x^2 + y^2} = 1$ and $\frac{y}{x^2 + y^2} = 1$
gives $y = 3x$ and substitution back gives $x = 0.3$ and $y = 0.9$
Note: $x = 0$ and $y = 0$ is NOT a solution.

Page 66

287. Finding the differences between vertices so we can work out the length of each side.

$\text{Line}_1 = v - u$
$= 6 - 3i$
$|L_1| = \sqrt{45}$
$L_2 = v - w$
$= 2 + 4i$
$|L_2| = \sqrt{20}$
$L_3 = w - u$
$= 4 - 7i$
$|L_3| = \sqrt{65}$
As $|L_3|^2 = |L_1|^2 + |L_2|^2$ triangle u, v and w must be right angled. Area 15 units^2.

Page 66 cont...

288. $z = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

If $\Delta \geq 0$ then real roots
If $\Delta < 0$ then complex roots as $b^2 - 4ac$ is negative (and $4ac - b^2$ is positive). Therefore:

$$z = \frac{-b \pm \sqrt{(^-1)}\sqrt{4ac - b^2}}{2a}$$

$$z = \frac{-b \pm i\sqrt{4ac - b^2}}{2a}$$

As the imaginary component is ± the two roots are conjugates of each other.

289. If $z = u + iv$ is a root then the conjugate $\overline{z} = u - iv$ is a root.
$(z - (u + iv))(z - (u - iv))$
$= z^2 - 2uz + (u + iv)(u - iv)$
$= z^2 - 2uz + u^2 + v^2$
Therefore $a = 1$, $b = {}^-2u$ and $c = u^2 + v^2$.

Page 67

290. $v = a + ib$, $w = c + id$
$z = ac - bd + i(ad + bc)$
$|z|^2 = (ac - bd)^2 + (ad + bc)^2$
$= (ac)^2 - 2abcd + (bd)^2 + (ad)^2 + 2abcd + (bc)^2$
$= (ac)^2 + (bd)^2 + (ad)^2 + (bc)^2$
$= (ac)^2 + (ad)^2 + (bc)^2 + (bd)^2$
$= a^2(c^2 + d^2) + b^2(c^2 + d^2)$
$= (a^2 + b^2)(c^2 + d^2)$
As a, b, c and d are non zero integers $(a^2 + b^2)$ and $(c^2 + d^2)$ are whole numbers so $|z|^2$ is a product of whole numbers and therefore NOT prime.

291. $z^{-1} = \dfrac{1}{a + ib}$

$$= \frac{a - ib}{a^2 + b^2}$$

$$|z^{-1}| = \sqrt{\frac{a^2}{(a^2 + b^2)^2} + \frac{b^2}{(a^2 + b^2)^2}}$$

$$|z^{-1}| = \frac{1}{\sqrt{(a^2 + b^2)}}$$

$$|z| = \sqrt{(a^2 + b^2)}$$

$$|z|^{-1} = \frac{1}{\sqrt{(a^2 + b^2)}} = |z^{-1}|$$

Page 67 cont...

292. $|x - 1 + i(y + 1)| = 2$

$(x - 1)^2 + (y + 1)^2 = 2^2$

Circle centre $(1, {}^-i)$ radius 2.
point on circle (many different answers but 2 from centre).
$(3, {}^-i)$.

Pages 68 – 74

Practice Assessment – Complex Numbers

In the external examinations NZQA uses a different approach to marking based on understanding (u), relational thinking (r) and abstract thinking (t). They then allocate marks to these concepts and add them up to decide upon the overall grade. This approach is not as easy for students to self mark as the NuLake approach but the results should be broadly similar.

Question One

(a) $(p + 2i)^3$
$= p^3 + 3p^2.2i + 3p(2i)^2 + (2i)^3$
$= p^3 + 6p^2i - 12p - 8i$
$= p^3 - 12p + i(6p^2 - 8)$ **A**
Working must be shown.

(b) $54 + 9A + 3B - 105 = 0$
or $3A + B - 17 = 0$
and
$^-250 + 25A - 5B - 105 = 0$
or $5A - B - 71 = 0$
$A = 11$, $B = {}^-16$ **A**
Working must be shown.

(c) $z^2 = 1 \text{ cis } \pi/2$
$z = 1 \text{ cis } \pi/4,\ 1 \text{ cis } 5\pi/4$ **A**
$\mathbf{z = \dfrac{1}{\sqrt{2}} + \dfrac{i}{\sqrt{2}},\ \dfrac{^-1}{\sqrt{2}} - \dfrac{i}{\sqrt{2}}}$ **M**
Or decimal equivalent.

(d) $\left|\dfrac{(2+i)^3}{2-i+1}\right| = \left|\dfrac{2+11i}{3-i}\right|$ **A**

$\left|\dfrac{(2+11i)(3+i)}{(3-i)(3+i)}\right| = \left|\dfrac{^-5+35i}{10}\right|$

$= \dfrac{5\sqrt{2}}{2}$ **M**
Working must be shown.

Question One cont...

(e) $z^n + \dfrac{1}{z^n} = (\text{cis } \theta)^n + (\text{cis } \theta)^{-n}$
$= \cos n\theta + i\sin n\theta + \cos({}^-n\theta) + i\sin({}^-n\theta)$ **M**
$= \cos n\theta + i\sin n\theta + \cos n\theta - i\sin n\theta$
$= 2 \cos n\theta$
$= \text{RHS}$ **E**
Working must be shown.

Question Two

(a) $64 \text{ cis } {}^-\pi = 64 \text{ cis } \pi$ **A**

(b) $2z + 1 = (z + 3)^2$
$z^2 + 6z + 9 = 2z + 1$
$z^2 + 4z + 8 = 0$
$z = \dfrac{^-4 \pm \sqrt{4^2 - 32}}{2}$
$z = {}^-2 \pm 2i$ **A**
Working must be shown.

(c) Conjugate $2 + i$ is also a root.
So polynomial is:
$(mz + n)(z - (2 - i))(z - (2 + i))$
$= (mz + n)(z^2 - 4z + 5)$ **A**
$m = 2$ and $n = {}^-1$ by inspection.
third factor is $(2z + 1)$ so
$z = (2 - i)$, $(2 + i)$ and 0.5
and $a = {}^-9$ and $b = 14$ **M**
Working must be shown.

(d) $z = (4.5 + 2.5981i)^{1/3}$
$= \sqrt{3}\text{cis}\left(\dfrac{\pi + 12k\pi}{18}\right)$ **A**
$= \sqrt{3}\text{cis } \dfrac{\pi}{18}, \sqrt{3}\text{cis } \dfrac{13\pi}{18}, \sqrt{3}\text{cis } \dfrac{25\pi}{18}$ **M**
Working must be shown.

(e) $|x + iy + 1| = |x + iy - 3i|$
$|x + 1 + iy| = |x + i(y - 3)|$
$(x + 1)^2 + y^2 = x^2 + (y - 3)^2$
$x^2 + 2x + 1 + y^2 = x^2 - 6y + 9 + y^2$
$3y + x - 4 = 0$ **M**
The equation represents the line $y = \dfrac{^-x + 4}{3}$ (the perpendicular bisector of points $({}^-1, 0i)$ and $(0, 3i)$). **E**
Working must be shown.
Geometrical interpretation required as well as solution.

Question Three

(a) $z^2 = 3 - 4i$

$$\frac{5}{3-4i}\frac{3+4i}{3+4i} - \frac{6}{2-i}\frac{2+i}{2+i} + 2$$

$0.2 - 0.4i$ **A**

Working must be shown

(b) $z_1 = 2\text{cis}(\pi/6)$, $z_2 = 2\text{cis}(7\pi/6)$

$z_1 = \sqrt{3} + i$, $z_2 = -\sqrt{3} - i$ **A**

(c) $z^4 = n \text{ cis } 0$

$$z = n^{1/4}\text{cis}\frac{(0+2k\pi)}{4}$$

$z_1 = \sqrt[4]{n} \text{ cis } 0 = \sqrt[4]{n}$ **A**

$z_2 = \sqrt[4]{n} \text{ cis } \frac{\pi}{2} = \sqrt[4]{n}\, i$

$z_3 = \sqrt[4]{n} \text{ cis } \pi = -\sqrt[4]{n}$

$z_4 = \sqrt[4]{n} \text{ cis } \frac{3\pi}{2} = -\sqrt[4]{n}\, i$ **M**

Working must be shown.

(d) $z^3 - 4z^2 + 14z - 20 = 0$
Use Factor Theorem to find $(z - 2)$

$(z - 2)(z^2 - 2z + 10) = 0$ **A**

$z = 2, 1 + 3i, 1 - 3i$ **M**

(e) $$w = \frac{(x-1)+yi}{(x+1)+yi} \times \frac{(x+1)-yi}{(x+1)-yi}$$

$$w = \frac{x^2+y^2-1+2yi}{x^2+2x+1+y^2}$$ **M**

since $x^2 + y^2 = 1$

$$w = \frac{yi}{x+1}$$

hence w is purely imaginary.
E

Sufficiency. For each question award yourself a score out of 8 using this table. Add the three scores for a score out of 24 and compare to the cut scores. All answers must show working where appropriate.

Quest.	N0	N1	N2	A3	A4	M5	M6	E7	E8
ONE	None correct	1A with error	1A correct	2A correct.	3A correct.	1M + 3A	2M correct.	1E almost	1E all correct
TWO	None correct	1A with error	1A correct	2A correct.	3A correct.	1M + 3A	2M correct.	1E almost	1E all correct
THREE	None correct	1A with error	1A correct	2A correct.	3A correct.	1M + 3A	2M correct.	1E almost	1E all correct
Cut Scores									

Not Achieved	Achievement	Achievement with Merit	Achievement with Excellence	
0 – 8	9 – 14	15 – 20	21 – 24	