Answers

Note: Undef. means Undefined.

Page 7

1. 9

2. 1

3. No limit as approaching 5 from above and below gives different results.

4. 0

5. $^{-}2$

6. 7

7. 8

8. 5

9. 2x

10. 4x

11. $3a^2$

12. 2x + 5

Page 8

13. a) 1.5
b) Undef.
c) 2
d) 5
e) 4

14. a) 0
b) 2
c) Undef.
d) $^{-}1$
e) $^{-}2$
f) 2
g) 1

15. a) 0 b) 1
c) $^{-}2$ d) 0
e) x = 0.5, 1, 2

16. a) 1 b) ∞
c) Undef. d) 1

Page 9

17. a) 2 b) 0
c) $^{-}3$ d) Undef.

18. a) 3 b) $^{-}2$
c) Undef. d) 3
e) 1 f) 2

19. a) Undef. b) 1
c) Undef. d) 1
e) 3 f) Undef.

Page 11

20. a) 0
b) $^{-}1$, 1 and 2
c) $x < {}^{-}1$ and $x \geq 3$
d) $\{x: x \neq 1, 2, x \in R\}$
e) Undef.

Page 11 cont...

21. a) $\{x: x \neq {}^{-}1, 0, x \in R\}$
b) $^{-}1$, 0, 1 and 2
c) x = 1
d) $\{x: x \geq 2, x \in R\}$
e) Approximately 2.8
f) 0, 1, 2

22. a) f(2.5) = 1.75, f(4.5) = 4
b) $\{x: x \neq 2, 3, x \in R\}$
c) x = 4
d) 2, 3, 4.5
e) 5.5

23. a) $\{x: x \neq 2, x \in R\}$
b) 0.5
c) $^{-}1$, 2, 4
d) $^{-}1$, 2, 4, 6
e) 6

Page 12

24. a) 3
b) f(3) = 4
c) $^{-}1 < x < 1$
d) $x = {}^{-}1, 3$
e) $x = {}^{-}1, 1, 3$
f) f′(2) = 1

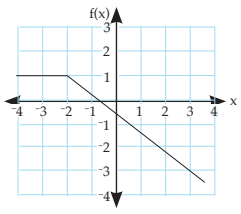
25. a) x > 5 (possibly $5 < x < 6$)
b) $f({}^{-}2) = 2$
c) $f'({}^{-}2) = 1$
d) Maximum at (1, 4)
e) Inflection (5, 4)
f) x = 3

26. a) Maximum at ($^{-}1$, 3) and (4, 2.5)
b) f′(2) = 4 approx.
c) $f'({}^{-}1) = 0$
d) x = 6
e) x = 1.5, 6
f) $^{-}3 \leq x \leq 6, x \neq 1.5$

27. a) Minimum at (1, $^{-}4$)
b) $x = {}^{-}0.5$ approx.
c) $x = {}^{-}2.25$, 1.5, 2.4, 5.3 all approximate.
d) $^{-}0.5 < x < 2$ or $4 < x < 6$
e) $x = {}^{-}0.5$, 2 and 4
f) $x = {}^{-}2$, 1 and 4

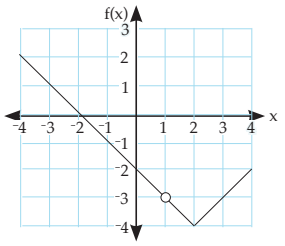
Page 13 (Other answers possible)

28.

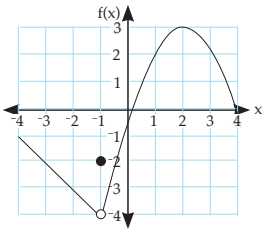


Page 13 cont...

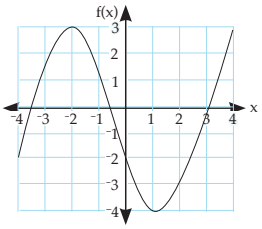
29.



30.



31.



Page 15 (Needs multiple steps)

32. $f'(x) = \lim_{h \to 0} \frac{4x + 4h - 4x}{h}$
$f'(x) = 4$

33. $f'(x) = \lim_{h \to 0} \frac{4x + 4h + 5 - 4x - 5}{h}$
$f'(x) = 4$

34. $f'(x) = \lim_{h \to 0} \frac{{}^{-}8x - 8h + 8x}{h}$
$f'(x) = {}^{-}8$

35. $f'(x) = \lim_{h \to 0} \frac{7x + 7h - 7x}{h}$
$f'(x) = 7$

36. $f'(x) = \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - x^2}{h}$
$f'(x) = 2x$

37. $f'(x) = \lim_{h \to 0} \frac{x^2 + 2xh + h^2 + 5 - x^2 - 5}{h}$
$f'(x) = 2x$

38. $f'(x) = \lim_{h \to 0} \frac{2x^2 + 4xh + 2h^2 - 2x^2}{h}$
$f'(x) = 4x$

39. $f'(x) = \lim_{h \to 0} \frac{2xh + h^2}{h}$
$f'(x) = 2x$

Page 16 (Needs multiple steps)

40. $f'(x) = \lim_{h \to 0} \frac{2xh + h^2 + h}{h}$
$f'(x) = 2x + 1$

41. $f'(x) = \lim_{h \to 0} \frac{{}^{-}2xh - h^2}{h}$
$f'(x) = {}^{-}2x$

Page 16 cont...

42. $f'(x)=\lim_{h\to0}\frac{2xh+h^2+5h}{h}$
$f'(x) = 2x + 5$

43. $f'(x)=\lim_{h\to0}\frac{3x^2+6xh+3h^2-3x^2}{h}$
$f'(x) = 6x$

44. $f'(x)=\lim_{h\to0}\frac{ax^2+2axh+ah^2-ax^2}{h}$
$f'(x) = 2ax$

45. $f'(x)=\lim_{h\to0}\frac{x^3+3x^2h+3xh^2+h^3-x^3}{h}$
$f'(x) = 3x^2$

46. $f'(x)=\lim_{h\to0}\frac{3x^2h+3xh^2+h^3+h}{h}$
$f'(x) = 3x^2 + 1$

47. $f'(x)=\lim_{h\to0}\frac{2axh+ah^2+bh}{h}$
$f'(x) = 2ax + b$

Page 18

48. $f'(x) = 12x^3 + 18x^2 - 14x - 2$

49. $\frac{dy}{dx} = 20x^3 - 6x + 6$

50. $f'(x) = \frac{8}{3}x^3 + \frac{3}{2}x - 1$
$f'(3) = 75.5$

51. $f'(x) = \frac{16}{5}x^3 - \frac{12}{5}x^2 + 6x$
$f'(^-1) = ^-11.6$

52. $f'(x) = \frac{7}{2}x^6 + 10x^5 - 2x$
$f'(2) = 540$

53. $\frac{dy}{dx} = 2.5x^4 - 9.6x^3 - 10.5x^2 + 3.2x$

54. $f'(x) = 4x + 5$

55. $\frac{dy}{dx} = 3x^2 - 2x - 6$

56. $f'(x) = 18x^2 + 20x$

57. $f'(x) = 60x^4 - 32x^3 + 12x^2$

58. $f'(x) = 2ax + b$

59. $\frac{dy}{dx} = 3a^2x^2 - 2abx$

60. $f'(x) = 28x^6 + 2 - \frac{1}{x^2}$

61. $f'(x) = \frac{4}{3}x^2 - \frac{2}{x^2} + 2x$

62. $f'(x) = 6x^3 - \frac{5}{x^2} - \frac{4}{x^3} + 1$
$f'(^-1) = ^-6$

63. $\frac{dy}{dx} = \frac{^-3}{x^2} - \frac{2}{x^3} + \frac{27}{x^4}$

Page 19

64. $f'(x) = 4 + \frac{6}{x^2} + \frac{10}{x^3}$

65. $\frac{dy}{dx} = 5 - \frac{2}{x^2} - \frac{6}{x^3}$

66. $f'(x) = \frac{^-2a}{x^3} + \frac{b}{x^2}$

67. $\frac{dy}{dx} = 6x + 2 + \frac{1}{2\sqrt{x}}$

68. $f'(x) = 4 + \frac{5}{x^2} + \frac{1}{2\sqrt{x}}$

69. $f'(x) = \frac{^-5}{3\sqrt[3]{x^4}}$

70. $\frac{dy}{dx} = \frac{^-1}{x^2} + \frac{1}{\sqrt{x^3}} - \frac{1}{\sqrt[3]{x^4}}$

71. $f'(x) = \frac{^-3}{2\sqrt{x^3}} + \frac{2}{5\sqrt[5]{x^6}} - \frac{5}{4\sqrt[4]{x^5}}$

72. $f'(x) = \frac{^-a}{2\sqrt{x^3}} + \frac{b}{3\sqrt[3]{x^4}}$

Page 22

73. $\frac{dy}{dx} = 15(5x - 1)^2$

74. $\frac{dy}{dx} = 4(4x + 3)(2x^2 + 3x - 5)^3$

75. $\frac{dy}{dx} = ^-10x(4 - x^2)^4$

76. $\frac{dy}{dx} = 6ax(ax^2 + b)^2$

77. $g'(x) = 12(2x - 1)^5$

78. $f'(x) = x(x^2 - 12)^{-1/2}$

79. $\frac{dy}{dx} = ^-0.5(24x - 5)(12x^2 - 5x + 8)^{-3/2}$

80. $\frac{dy}{dx} = ^-x(9 - x^2)^{-1/2}$

81. $\frac{dy}{dx} = ax(ax^2 - b)^{-1/2}$

82. $k'(x) = (2x + 8)^{-1/2}$

83. $k'(x) = ^-2x(x^2 - 2)^{-2}$

84. $h'(x) = ^-24(2x + 1)^{-5}$

85. $\frac{dy}{dx} = 6x(6x^2 - 5)^{-1/2}$

86. $k'(x) = (3x - 4)(3x^2 - 8x + 2)^{-1/2}$

87. $f'(x) = ^-4ax(ax^2 - b)^{-2}$

88. $f'(x) = 3(x + \frac{2}{x})^2(1 - \frac{2}{x^2})$

89. $\frac{dy}{dx} = \frac{1}{5}(10x - 2)(5x^2 - 2x)^{^-4/5}$

90. $\frac{dy}{dx} = ^-3(6x^2 - x)(4x^3 - x^2)^{^-3}$

Page 23

91. $\frac{dy}{dx} = \frac{a}{b^2}$

92. $\frac{dP}{dr} = \frac{3}{2}(r - 2)^{1/2}$

93. $\frac{dz}{dt} = ^-3(t - 1)^{-4}$

94. $\frac{dy}{dx} = \frac{3b}{x^2}(a + \frac{b}{x})^{-4}$

95. $\frac{dA}{db} = 2b(1 - b^2)^{-2}$

96. $\frac{dk}{dx} = (3 - x)^{-5/4}$

97. $\frac{dy}{dx} = \frac{3}{4}(x - 1)^5$

98. $\frac{dy}{dx} = \frac{2a}{b^2}(x + 1)$

Page 25

99. $f'(x) = 4e^{4x}$

100. $y' = ^-2e^{-2x}$

101. $g'(x) = 6e^{6x - 1}$

102. $y' = ^-3e^{8 - 3x}$

103. $k'(x) = 6e^{3x}$

104. $y' = 12e^{3x - 1}$

105. $f'(x) = 20e^{-4x + 3}$

106. $m'(x) = ^-6e^{-8x}$

Page 26

107. $f'(x) = \frac{^-3e^{2x+1}}{4}$

108. $h'(x) = \frac{^-8e^{5-4x}}{7}$

109. $y' = 12e^{4x} + 10e^{-5x + 3}$

110. $k'(x) = 10e^{-2x + 1} - 7e^{4 - x}$

111. $y' = 42xe^{3x^2-4} - 16xe^{1-4x^2}$

112. $f'(x) = ^-168x^2e^{7x^3-1} - 6e^{1+2x}$

113. $g'(x) = \frac{e^{\sqrt{x}}}{2\sqrt{x}}$

114. $m'(x) = \frac{2e^{\sqrt{x}}}{\sqrt{x}}$

115. $f'(x) = \frac{^-e^{1/x}}{x^2}$

116. $p'(x) = \frac{3e^{1/x}}{x^2}$

Page 26 cont...

117. $\frac{dy}{dx} = \frac{-5e^{1-x}}{4}$

118. $f'(x) = ae^{ax+b}$

119. $\frac{dy}{dx} = 3(2x+e^{3x})^2(2+3e^{3x})$

120. $\frac{dy}{dx} = \frac{4e^{-2x}}{\sqrt{1-4e^{-2x}}}$

121. $v'(x) = \frac{3e^{\sqrt{2x-3}}}{\sqrt{2x-3}}$

122. $\frac{dy}{dx} = \frac{-6a}{e^{2x}}$

123. $f'(x) = 2(a + be^{bx})(ax + e^{bx})$

124. $\frac{dy}{dx} = \frac{ae^{ax}}{\sqrt{1+2e^{ax}}}$

Page 28

125. $f'(x) = \frac{7}{7x+2}$

126. $f'(x) = \frac{1}{x}$

127. $f'(x) = \frac{-5}{9-5x}$

128. $m'(x) = \frac{6x}{3x^2-1}$

129. $\frac{dy}{dx} = \frac{-8x}{1-4x^2}$

130. $f'(x) = \frac{5}{x}$

131. $g'(x) = \frac{12x^2}{4x^3-3}$

132. $g'(x) = \frac{a}{ax+b}$

133. $\frac{dy}{dx} = \frac{2x+5}{x^2+5x+5}$

Page 29

134. $\frac{dy}{dx} = 3e^{3x} + \frac{1}{x}$

135. $f'(x) = \frac{2ax-b}{ax^2-bx-c}$

136. $p'(x) = \frac{6x-9}{x^2-3x}$

137. $k'(x) = \frac{16x+4}{2x^2+x}$

138. $q'(x) = \frac{-6x^2}{x^3-1}$

Page 29 cont...

139. $k'(x) = \frac{5}{1-x}$

140. $q'(x) = \frac{5-20x}{2x^2-x+1}$

141. $k'(x) = \frac{2x-3}{1-x^2+3x}$

142. $f'(x) = \frac{4x}{x^2-1}$

143. $f'(x) = \frac{6}{1-x}$

144. $f'(x) = \frac{48x}{2x^2+1}$

145. $\frac{dy}{dx} = \frac{108x}{3x^2-4}$

146. $f'(x) = \frac{1}{2x}$

147. $f'(x) = \frac{1}{x+1}$

148. $f'(x) = \frac{-3}{2x-5}$

149. $\frac{dy}{dx} = \frac{-1}{x}$

150. $\frac{dy}{dx} = \frac{1}{2(x-3)}$

151. $g'(x) = \frac{-1}{x(3x+1)}$

152. $\frac{dy}{dx} = \frac{9(\ln(3x+2))^2}{3x+2}$

153. $\frac{dy}{dx} = \frac{1}{4x\sqrt[4]{(\ln(x))^3}}$

Page 34

154. $\frac{dy}{dx} = 3\sec^2 3x$

155. $f'(x) = 5\cos 5x$

156. $\frac{dy}{dx} = -12\sin 4x$

157. $f'(x) = -6\cos 3x$

158. $\frac{dy}{dx} = 10\sec 2x\tan 2x$

159. $f'(x) = -6\cos(2x+1)$

160. $\frac{dy}{dx} = -6\sec^2(1-3x)$

161. $f'(x) = 4\sin(4x-1)$

162. $f'(x) = \frac{-3}{4}\sin\frac{1}{4}x$

Page 34 cont...

163. $f'(x) = -5\operatorname{cosec}^2(5x-2)$

164. $f'(x) = -4\operatorname{cosec}^2 x\cot x$

165. $\frac{dy}{dx} = \frac{-2\cos\left(\frac{2}{x}\right)}{x^2}$

166. $\frac{dy}{dx} = -\tan x$

167. $\frac{dy}{dx} = \frac{3\cos\sqrt{x}}{\sqrt{x}}$

168. $\frac{dy}{dx} = 2\sec 2x\tan 2x - 12\operatorname{cosec}(3x+1)\cot(3x+1)$

169. $\frac{dy}{dx} = 4\sec^2 4x - 3\cos 3x$

170. $\frac{dy}{dx} = -a\sin(ax+b)$

171. $\frac{dy}{dx} = 6ax\cos(ax^2-b)$

Page 37

172. $f'(x) = (6x+5)e^{2x}$

173. $q'(x) = (-9x^2-9x+5)e^{-3x}$

174. $n'(x) = (12x^3+6x)e^{2x^2}$

175. $q'(x) = 2x(15x^2-8)e^{5-3x^2}$

176. $f'(x) = (12x^2+8x-27)e^{3x}$

177. $\frac{dy}{dx} = 3e^{2x}(4x^2-14x-9)$

178. $u'(x) = (12x^2+8x)e^{3x-1}$

179. $p'(x) = (6x^2+6x-2)e^{2x+1}$

180. $p'(x) = 2\ln\sqrt{x} + \frac{2x+1}{2x}$

$p'(x) = \ln(x) + \frac{2x+1}{2x}$

181. $n'(x) = 3\ln 5x^2 + \frac{6x-2}{x}$

182. $v'(x) = 2x\ln\sqrt{x-2} + \frac{x^2-3}{2x-4}$

183. $n'(x) = 4\ln\left(\frac{1}{x^2}\right) - 8 - \frac{2}{x}$

184. $f'(x) = e^x\ln(x^2+1) + \frac{2xe^x}{x^2+1}$

185. $f'(x) = 2e^{2x-1}\ln(\sqrt{x}) + \frac{e^{2x-1}}{2x}$

$= e^{2x-1}\ln(x) + \frac{e^{2x-1}}{2x}$

Page 38

186. $\frac{dy}{dx} = 2x\cos x - x^2\sin x$

187. $\frac{dy}{dx} = 3\tan 3x + (9x-6)\sec^2 3x$

188. $\frac{dy}{dx} = 2x\cot(5x-1) - 5(x^2-1)\operatorname{cosec}^2(5x-1)$

189. $\frac{dy}{dx} = 3x^2\operatorname{cosec} 2x - 2x^3\operatorname{cosec} 2x\cot 2x$

190. $h'(x) = 4\cos^2 x - 4\sin^2 x$

191. $f'(x) = 24\sin x\cos^2 x - 12\sin^3 x$

192. $h'(x) = \sec x\tan^2 x + \sec^3 x$

193. $k'(x) = \sin x(\sec^2 x + 1)$

194. $f'(x) = -24\sin 4x\cos 4x$

195. $q'(x) = 3x^4\cos x + 12x^3\sin x$

196. $q'(x) = (12x^3 + 6x^2 + 4x + 1)e^{3x^2+2}$

197. $q'(x) = 6x\ln(3x-1) + \frac{9x^2}{3x-1}$

198. $\frac{dy}{dx} = (a + 2a^2x^2 + 2abx)e^{ax^2+b}$

199. $f'(x) = ae^{ax+b}\ln(ax+b) + \frac{ae^{ax+b}}{ax+b}$

Page 41

200. $g'(x) = \frac{6x^2-6x-1}{(2x-1)^2}$

201. $g'(x) = \frac{30x^2-16x-17}{(2x^2+8x-1)^2}$

202. $\frac{dy}{dx} = \frac{2x-3x^2}{2\sqrt{x}(x^2-2x)^2}$

203. $\frac{dy}{dx} = \frac{-(4x-5)}{2\sqrt{x}(4x+5)^2}$

204. $g'(x) = \frac{12x+1-15x^2}{3x^{2/3}(3x^2-6x+1)^2}$

205. $h'(x) = \frac{x^{1/3}+3}{6\sqrt{x}(x^{1/3}+1)^2}$

Page 42

206. $\frac{dy}{dx} = \frac{60x^2+64x+8}{e^{3x}(5x^2+2x)^2}$

207. $q'(x) = \frac{8x^2+8x-2}{e^x(4x^2-1)^{3/2}}$

208. $\frac{dy}{dx} = \frac{(24x^3-18x)e^{x^2+1}}{(4x^2-1)^{3/2}}$

209. $f'(x) = \frac{(2x^3-8x)e^{x^2}}{(x^2-3)^2}$

Page 42 cont...

210. $\frac{dy}{dx} = \frac{(48x^2-12x-3)e^{4x}}{x^2(4x^2+1)^2}$

211. $\frac{dy}{dx} = \frac{1-2\ln 2x}{x^3}$

212. $\frac{dy}{dx} = \frac{2ae^{ax}(ax^2+b-2x)}{(ax^2+b)^2}$

213. $\frac{dy}{dx} = \frac{(x-b)-2x\ln(x+b)}{(x^2-b^2)^2}$

214. $\frac{dy}{dx} = \frac{-1}{1+\sin x}$

215. $f'(x) = 8x\cos 4x - 4(4x^2-1)\sin 4x$

Page 43

216. $m'(t) = \frac{2\cos 2t(1-t^2)+2t\sin 2t}{(1-t^2)^2}$

217. $f'(x) = \frac{a(1+\sin x-x\cos x)}{(1+\sin x)^2}$

Page 44

218. $f'(x) = \frac{-8}{x^3} - 10x$

219. $g'(x) = 15x^2 + 7 - \frac{3}{x^2}$

220. $\frac{dy}{dx} = 4x^3$

221. $h'(r) = 4\pi r + \frac{3}{\pi}$

222. $\frac{dy}{dx} = -10e^{2x}(1-e^{2x})^4$

223. $\frac{dy}{dx} = \frac{1}{x} + 2e^{2x}$

224. $m'(x) = \frac{\sec^2 x}{2\sqrt{\tan x}}$

225. $p'(x) = \frac{-11}{(2x-3)^2}$

226. $k'(x) = \frac{2x}{x^2+2}$

227. $\frac{dy}{dx} = \frac{\sec^2 x}{\tan x} = \operatorname{cosec} x\sec x$

228. $f'(x) = \frac{6}{5}(3x+5)^{-3/5}$

229. $\frac{dy}{dx} = \frac{-4}{(3x-1)^2}$

230. $k'(x) = \frac{-8}{x^3} + \frac{2}{x^2} - \frac{9}{x^4}$

231. $\frac{dy}{dx} = 4(6x-2)(3x^2-2x+1)^3$

Page 45

232. $f'(x) = (10x^2+5)e^{x^2}$

233. $g'(x) = \frac{e^{2x}}{2\sqrt{x}} + 2\sqrt{x}e^{2x}$

$g'(x) = \frac{e^{2x}(1+4x)}{2\sqrt{x}}$

234. $\frac{dy}{dx} = 10(2x^2+x-3)(4x^2+2x-1)$

235. $g'(x) = 3e^{3x}\sin x + e^{3x}\cos x$

$g'(x) = e^{3x}(3\sin x + \cos x)$

236. $h'(x) = 3e^{3x}\ln(2x) + \frac{e^{3x}}{x}$

237. $j'(x) = \frac{(x^2+6)\sec x\tan x - 2x\sec x}{(x^2+6)^2}$

238. $k'(x) = \frac{12x^2(1+2\ln x)-8x^2}{(1+2\ln x)^2}$

$k'(x) = \frac{4x^2(1+6\ln x)}{(1+2\ln x)^2}$

239. $\frac{dy}{dx} = \frac{2(1-x^2)\cos x + 4x\sin x}{(1-x^2)^2}$

240. $\frac{dy}{dx} = \frac{2\cos^2 x + 2\sin x + 2\sin^2 x}{\cos^2 x}$

$\frac{dy}{dx} = \frac{2}{1-\sin x}$

or $\frac{dy}{dx} = 2 + 2\sec x\tan x + 2\tan^2 x$ if using the product rule.

241. $h(x)' = \frac{-(\cos x+1)}{\sin^2 x}$

242. $g'(x) = \frac{2(2x-1)^{-1/2}e^{3x} - 6e^{3x}(2x-1)^{1/2}}{(2e^{3x})^2}$

$g'(x) = \frac{(2-3x)e^{-3x}}{\sqrt{2x-1}}$

243. $k'(x) = 12(3x+1)^3(x-2)^{1/2} + \frac{1}{2}(3x+1)^4(x-2)^{-1/2}$

$k'(x) = \frac{(3x+1)^3(27x-47)}{2(x-2)^{0.5}}$

Page 47

244. $\frac{dy}{dx} = 2x - \frac{1}{x}$

Setting $2x - \frac{1}{x} = 1$

$x = 1, x = {}^{-}0.5$

Coordinates (1,1)

245. $\frac{dy}{dx} = \frac{{}^{-}3}{x^2} + \frac{1}{3}$

Setting $\frac{{}^{-}3}{x^2} + \frac{1}{3} = {}^{-}1$

$x = {}^{-}1.5, x = 1.5$

246. $\frac{dy}{dx} = \frac{{}^{-}1}{(x+1)^2}$

When $x = {}^{-}3, \frac{dy}{dx} = \frac{{}^{-}1}{4}$

Gradient of normal $= 4$

247. $6y - x - 31 = 0$

248. $y = 5x$

249. $y = {}^{-}0.3296x + 6.397$

Page 48

250. $y = 2x + 8$

251. $\frac{dy}{dx} = {}^{-}2x + 2$. At $x = 2$

$\frac{dy}{dx} = {}^{-}2$ Grad. of the line is ${}^{-}2$.

252. $4y - 65x = {}^{-}68$

253. $y = {}^{-}3$

254. $y = 3x + 12$

255. $y = {}^{-}3x + 15$

256. $\frac{dy}{dx} = \frac{{}^{-}k}{(x-1)^2} - 2$

$1 = \frac{{}^{-}k}{4} - 2$

$k = {}^{-}12$

257. $\frac{dy}{dx} = \frac{k}{kx-1} + 3$

$5 = \frac{k}{2k-1} + 3$

$k = \frac{2}{3}$

258. $y = \frac{{}^{-}1}{4}x + \frac{15}{4}$

259. $y = 4x + 1.369$

or $y - 2\sqrt{3} = 4(x - \frac{\pi}{6})$

Page 52

260. Minimum at (3, ${}^{-}1$)

261. Minimum at (1.5, ${}^{-}20.25$)

262. Maximum at (1.5, 12.25)

263. a) Minimum at (${}^{-}3$, ${}^{-}32$)
Maximum (1, 0)

b) Decreasing:
$x < {}^{-}3$ or $x > 1$

264. Maximum at (${}^{-}2$, 16.33)
Minimum at (4, ${}^{-}19.67$)
Increasing: $x < {}^{-}2$ and $x > 4$

265. Maximum at (5, 98)
Minimum at (${}^{-}1$, ${}^{-}10$)
Increasing: ${}^{-}1 < x < 5$

Page 53

266. Maximum at (0, 4)
Minimum at (2, 0)

267. Maximum at (${}^{-}2$, ${}^{-}4$)
Minimum at (2, 4)

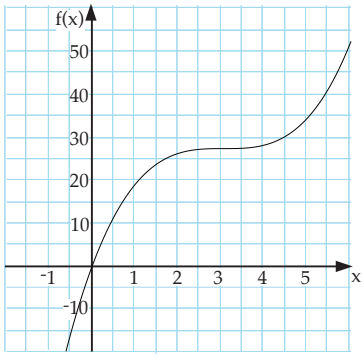
268. $\frac{dy}{dx} = 3x^2 - 18x + 27$
$= 3(x-3)^2$

Stationary point (3, 27)

$\frac{d^2y}{dx^2} = 6x - 18$

$\frac{d^2y}{dx^2} = 0$ when $x = 3$,

so (3, 27) is a point of inflection.



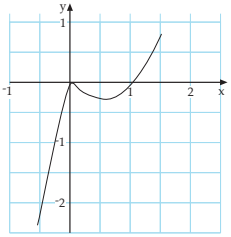
269. a) $x = 0, 1$

b) $f'(x) = 5x^{2/3} - 4x^{1/3}$

c) Maximum at (0, 0)
Minimum at (0.512, ${}^{-}0.25$)

d) Increasing $x < 0$ or $x > 0.512$

e)

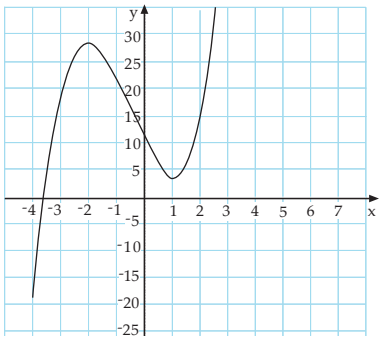


Page 57

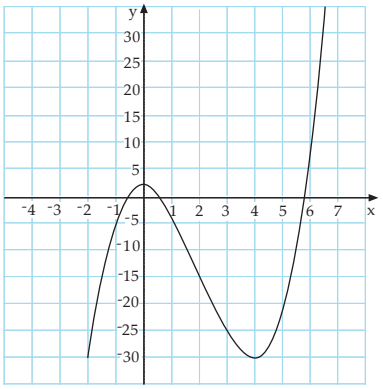
270. Maximum (${}^{-}2$, 41)
Minimum at (0.5, 9.75)
Inflection (${}^{-}0.75$, 25.375)

271. Maximum (${}^{-}2$, 53)
Minimum at (3, ${}^{-}72$)
Inflection (0.5, ${}^{-}9.5$)

272. Maximum (${}^{-}2$, 27)
Minimum at (1, 0)
Inflection (${}^{-}0.5$, 13.5)
Concave down $x < {}^{-}0.5$



273. Maximum (0, 2)
Minimum at (4, ${}^{-}30$)
Inflection (2, ${}^{-}14$)
Concave up $x > 2$



Page 58

274. $f'(x) = \frac{1}{x} - 0.25x$

Stationary points $x = 2$ and ${}^{-}2$, but ignore ${}^{-}2$ as x must be positive for ln to exist.
Maximum at (2, 3.193)
No point of inflection as $f''(x) = 0$ has no real solutions.

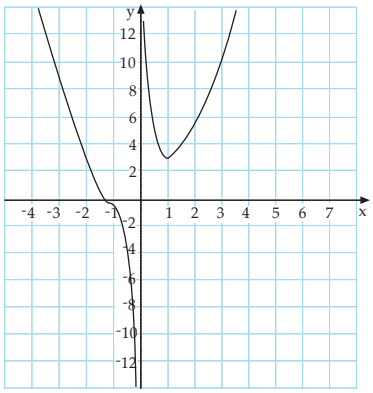
275. $f'(x) = e^x - 4$

$f'(x) = 0$ at $x = 1.386$.
Minimum (1.386, 0.455).
Concave up for all x (no point of inflection).

Page 58 cont...

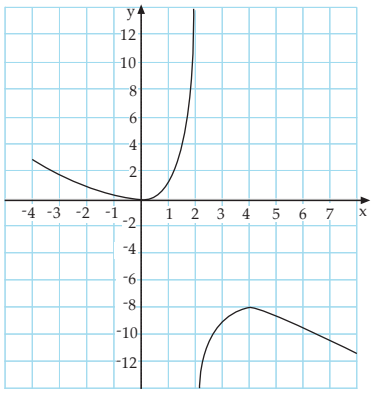
276. $f'(x) = 2x - \frac{2}{x^2}$

Minimum (1, 3)
Inflection ($^-$1.26, 0)
Concave up when $x < ^-1.26$ and $x > 0$.



277. $f'(x) = \frac{4x - x^2}{(2-x)^2}$

Minimum (0, 0)
Maximum at (4, $^-$8)
Decreasing $x < 0$ or $x > 4$



Page 59

278. A = 4, B = 3

279. A = 2, B = 3, C = $^-$1

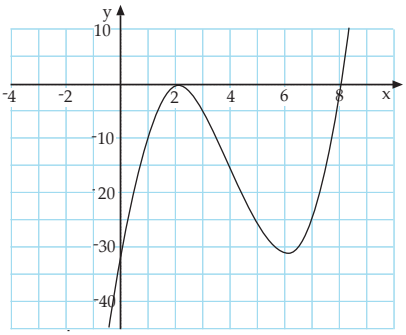
280. k = 3 and $k = 1\frac{1}{3}$

281. $A = \frac{2}{3}$, $B = ^-1\frac{1}{2}$, C = $^-$2

Page 63

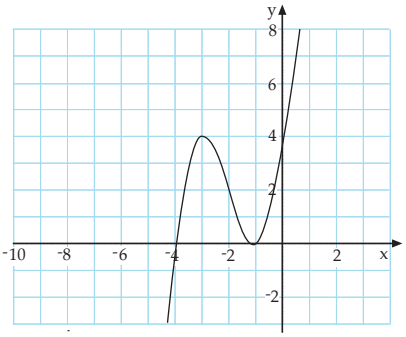
282. $\frac{dy}{dx} = 3x^2 - 24x + 36$

Intercepts (0, $^-$32), (2, 0), (8, 0)
Maximum (2, 0)
Minimum (6, $^-$32)
Inflection (4, $^-$16)



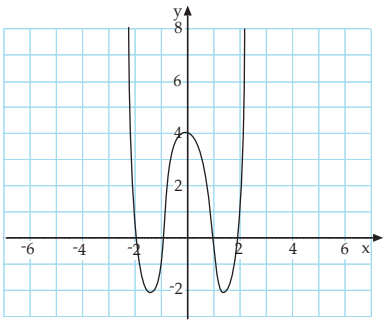
283. $\frac{dy}{dx} = 3x^2 + 12x + 9$

Intercepts (0, 4), ($^-$4, 0), ($^-$1, 0)
Maximum ($^-$3, 4)
Minimum ($^-$1, 0)
Inflection ($^-$2, 2)



284. $\frac{dy}{dx} = 4x^3 - 10x$

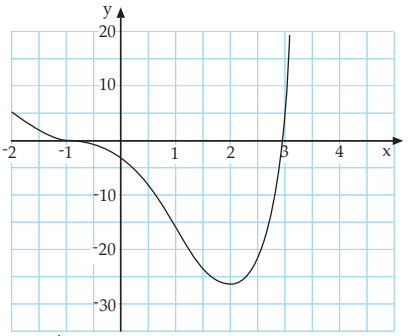
Intercepts (0, 4), ($^-$2, 0), ($^-$1, 0), (1, 0) and (2, 0)
Maximum (0, 4)
Minimum ($^-$1.58, $^-$2.25) and (1.58, $^-$2.25)
Inflection ($^-$0.913, 0.527), (0.913, 0.527)



Page 64

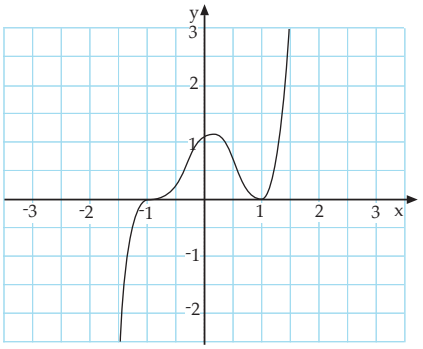
285. $\frac{dy}{dx} = 4x^3 - 12x - 8$

Intercepts (0, $^-$3), ($^-$1, 0), (3, 0)
No maximum
Minimum (2, $^-$27)
Points of inflection ($^-$1, 0) and (1, $^-$16)



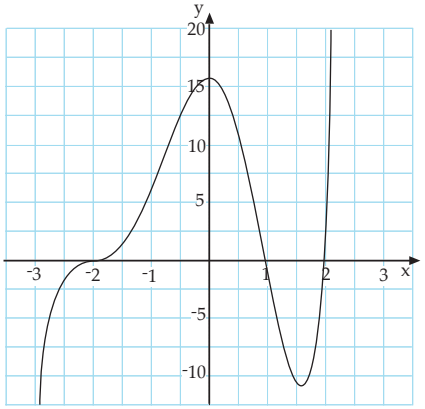
286. $\frac{dy}{dx} = 5x^4 + 4x^3 - 6x^2 - 4x + 1$

Intercepts (0, 1), ($^-$1, 0), (1, 0)
Maximum (0.2, 1.106)
Minimum (1, 0)
Points of inflection ($^-$1, 0), ($^-$0.29, 0.60) and (0.69, 0.46)



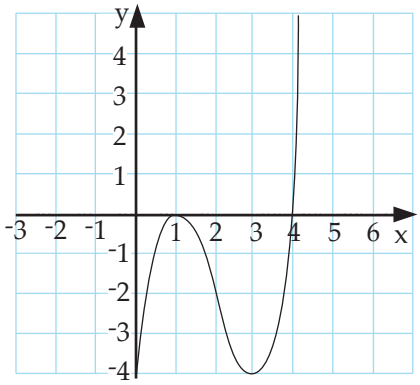
287. $\frac{dy}{dx} = 5x^4 + 12x^3 - 12x^2 - 32x$

Intercepts (0, 16), ($^-$2, 0), (1, 0) and (2, 0)
Maximum (0, 16)
Minimum (1.6, $^-$11.197)
Points of inflection ($^-$2, 0), ($^-$0.8, 8.71) and (1, 0)



Page 65

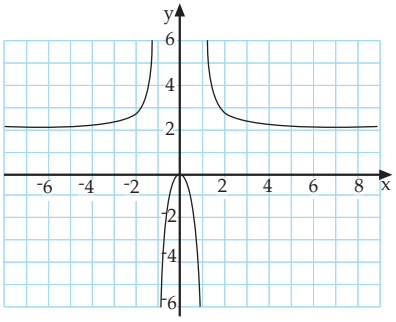
288. a) Maximum (1, 0)
Minimum (3, ⁻4)
b) Inflection (2, ⁻2)
c)



d) $x > 2$

289. a) A = (⁻0.645, 0)
B = (⁻0.5, 0.25)
C = (0, 0)
D = (0.5, ⁻0.25)
E = (0.645, 0)
b) At the points of inflection
$x = 0, \pm 0.3536$
Gradient when $x = 0.3536$ is ⁻0.938 (3 sf)

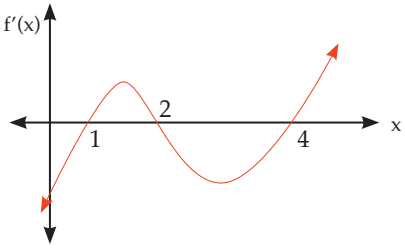
290. a) Maximum point (0, 0)
b)



c) limit = 2

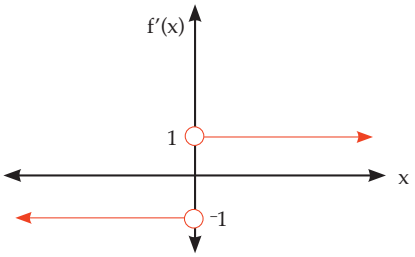
Page 68

291.

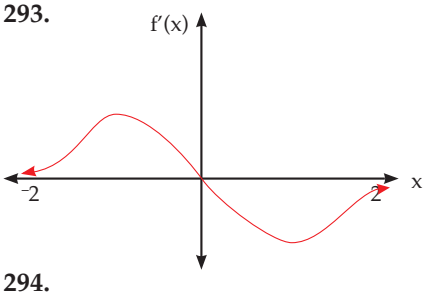


Page 68 cont...

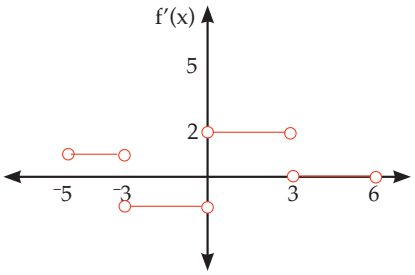
292.



293.

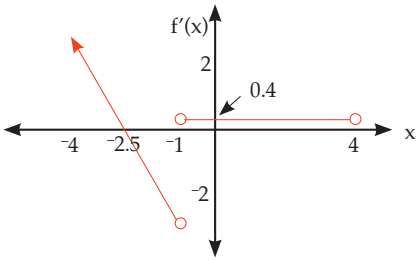


294.

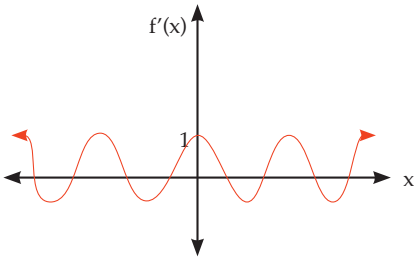


Page 69

295.



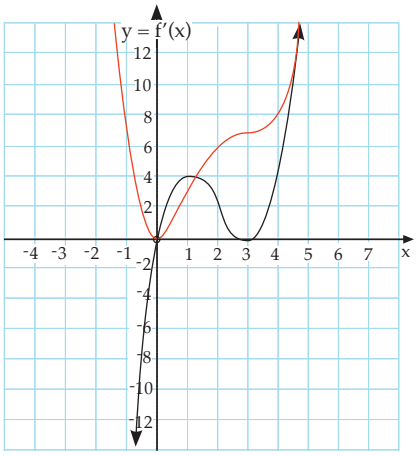
296.



297. a) $x = 0$
b) $x = 1$ and 3
c) Stationary point of inflection because at $x = 3$, $y = f'(x)$ is both an intercept and a stationary point.

Page 69 cont...

297. d)



Page 71

298. $\frac{dy}{dx} = \frac{3}{10t}$

299. $\frac{dy}{dx} = \frac{t}{2}$

300. $\frac{dy}{dx} = \frac{-2}{5}\tan t$

301. $\frac{dy}{dx} = \frac{\cos t}{e^t}$

302. $\frac{dy}{dx} = 4t\sqrt{t}$

303. $\frac{dy}{dx} = 2t^2(1-t)$

304. $\frac{dy}{dx} = \frac{3t^2-1}{2t-3}$

305. $\frac{dy}{dx} = \frac{2t^2+1}{t^2-1}$

306. $\frac{d^2y}{dx^2} = \frac{3}{4t}$

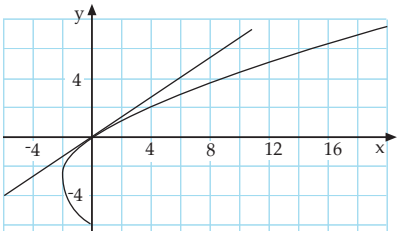
307. $\frac{d^2y}{dx^2} = \frac{-5}{16\cos^3 t}$

Page 72

308. $\frac{d^2y}{dx^2} = -3\sec^3 t$

309. $\frac{d^2y}{dx^2} = \frac{-(t+1)}{t^2e^{2t}}$

310. $\frac{dy}{dx} = \frac{2}{2t+3}$
At $t = 0$ the tangent is $3y - 2x = 0$



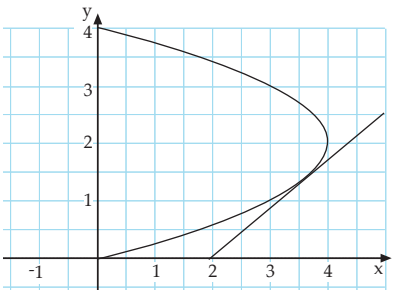
Page 72 cont...

311. $\frac{dy}{dx} = \frac{-1}{2t}$

At t = −0.6 the tangent is

$y = \frac{1}{30}(25x - 49)$

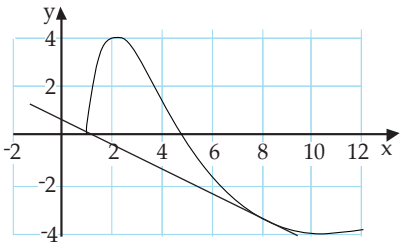
$= 0.833x - 1.6333$



Page 73

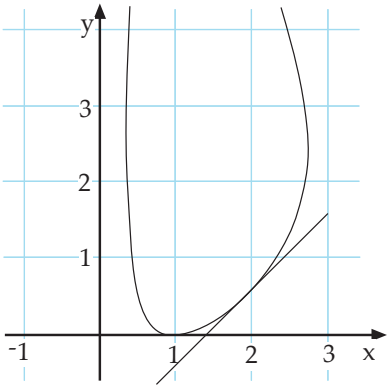
312. $\frac{dy}{dx} = \frac{4\cos t}{0.5e^{t/2}}$

$y = -0.524x + 0.790$



313. $\frac{dy}{dx} = \frac{2t}{\cos t \times e^{\sin t}}$

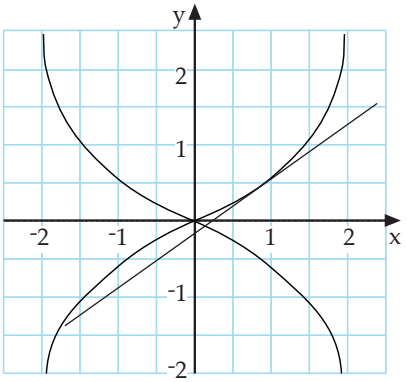
$y = 1.061x - 1.537$



Page 73 cont...

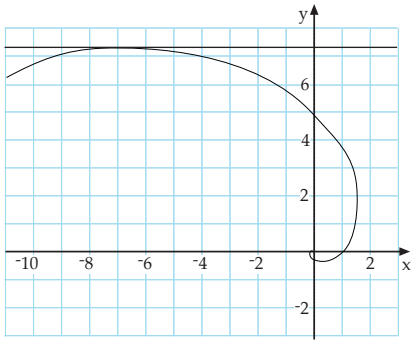
314. $\frac{dy}{dx} = \frac{1}{2\cos^3 t}$

$y = 0.740x - 0.163$



315. $\frac{dy}{dx} = \frac{\cos t + \sin t}{\cos t - \sin t}$

$y = 7.461$



Page 76

316. $\frac{dy}{dx} = \frac{1}{y}$

$= \frac{1}{3}$

317. $\frac{dy}{dx} = \frac{-y}{x + 2y}$

$= \frac{-1}{3}$

318. $\frac{dy}{dx} = \frac{4x}{3y}$

319. $\frac{dy}{dx} = \frac{-x - 2y}{2x + 3y}$

320. $\frac{dy}{dx} = \frac{3x^2 - y^2}{2xy - 2y}$

321. $\frac{dy}{dx} = \frac{-3x^2 - 2y}{3y^2 + 2x}$

322. $\frac{dy}{dx} = \frac{-2x}{e^y}$

323. $\frac{dy}{dx} = 3x^2y + 4y$

Page 76 cont...

324. $\frac{dy}{dx} = \frac{-2xy^2}{1 + x^2y}$

325. $\frac{dy}{dx} = \frac{3y^2 - 8xy}{4x^2 - 6xy}$

326. $\frac{dy}{dx} = \frac{-\sqrt{y}}{\sqrt{x}}$

327. $\frac{dy}{dx} = \frac{-3x^2 + 6xy^3 - 3y^2}{-9x^2y^2 + 6xy - 2y}$

Page 79

328. $\frac{dh}{dt} = \frac{1}{\pi(0.45)^2} \times 0.123$

$= 0.193 \text{ m/min}$

329. $\frac{dh}{dt} = \frac{1}{4\pi(3.25)^2} \times 1.45$

$= 0.0109 \text{ m/s}$

330. a) 0.94 cm/s (2 sf)

b) 3.3 cm^2/s (2 sf)

Page 80

331. $\frac{dh}{dt} = \frac{9}{4\pi h^2} \times \frac{dV}{dt}$

$= 0.895 \text{ cm/m}$

332. a) 9.425 mm^2/s

b) 1.696 mm^2/s

333. $\frac{5.26}{x + y} = \frac{1.71}{y}$

$y = \frac{1.71x}{3.55}$

$\frac{dy}{dt} = 2.2 \text{ m/s}$

Page 81

334. $x = 1000 \cot \theta$

$\frac{dx}{dt} = \frac{-1000}{(\sin x)^2} \times \frac{d\theta}{dt}$

$= -43.91 \text{ m/s}$

335. $\frac{dx}{dt} = \frac{s}{\sqrt{s^2 - 4.2^2}} \times \frac{ds}{dt}$

$= 1050 \text{ km/h}$

Page 81 cont...

336. $\frac{dA}{dt} = 0.00263\ m^2/s$

337. r = 9.35 cm, $\frac{dV}{dt} = 125\ cm^2/s$

$$\frac{dSA}{dt} = \frac{dSA}{dr} \times \frac{dr}{dV} \times \frac{dV}{dt}$$

$$= 8\pi r \times \frac{1}{4\pi r^2} \times \frac{dV}{dt}$$

$= 26.7\ cm^2/s$

Page 85

338. $Cost = 150w^2 + \frac{2400}{w}$

$Cost' = 300w - \frac{2400}{w^2}$

Minimum cost when w = 2, h = 2.5 m giving Cost = $1800

339. $SA = \frac{500000}{r} + 2\pi r^2$

$SA' = \frac{-500000}{r^2} + 4\pi r$

Minimum cost when r = 34.1 cm, h = 68.3 cm

340. a) Maximum height v = 0.
t = 2 s, h(2) = 27 m

b) h(t) = 0 when t = 5 s
v(t) = 12 – 6t
v(5) = −18 m/s

Page 86

341. a) $S = 150h^2 - h^3$
$S' = 300h - 3h^2$
h = 100 mm, w = 50 mm

b) $S = 112^2w - w^3$
$S' = 112^2 - 3w^2$
h = 91.4 mm,
w = 64.7 mm

342. $Area = -y^2 + 3y + 25$
$Area' = -2y + 3$
y = 1.5 m, x = 3.5 m
Area of deck = 27.25 m^2

343. a) $v = -3t^2 + 30t - 60$
$0 = -3t^2 + 30t - 60$
t = 2.764, 7.236 seconds
Minimum h = 27.6 m
Maximum h = 72.4 m

b) h = 0, t = 10 seconds
v = −60 m/s
a = −30 m/s^2

Page 87

344. $P = x(450 - 9.5x) - (2500 + 15x + 0.25x^2)$
$P' = 435 - 19.5x$
x = 22.3
= 22 puppies per year

345. a) $C = 9.125x + \frac{5000}{x} + 250$

b) $C' = \frac{-5000}{x^2} + 9.125$
x = 23 rats
C''(23) > 0 to show min.

Page 88

346. $V = \frac{1}{3}\pi r^2 h$ and $R^2 = r^2 + h^2$

$V = \frac{1}{3}\pi R^2 h - \frac{1}{3}\pi h^3$

$V' = \frac{1}{3}\pi R^2 - \pi h^2$

h = 0.5774R, r = 0.8165R
$2\pi r = R(2\pi - A)$
A = 1.153 radians
$V_{max} = 0.404R^3$

347. $Save = \frac{x}{6} + \frac{32}{1.8} - \frac{\sqrt{x^2 + 32^2}}{1.8}$

$Save' = \frac{1}{6} - \frac{x}{1.8(x^2 + 32^2)^{0.5}}$

x = 10.06 m

Page 89

348. Since $\frac{3}{x-7} = \frac{y-3}{7}$

then $y = \frac{21}{(x-7)} + 3$

$A = 0.5xy - 21$

so $A = \frac{3x^2}{2(x-7)} - 21$

and $A' = \frac{12x(x-7) - 6x^2}{(2(x-7))^2}$

so x = 14, y = 6

and $m = \frac{-3}{7}$

3x + 7y = 42

Page 89 cont...

349. h = u + v

$h = \frac{v^2}{v - f}$

$h' = \frac{2v(v-f) - v^2}{(v-f)^2}$

$v^2 - 2vf = 0$

v = 2f

Substituting in h

h = 4f

Pages 90 – 95

Practice External Assessment – Apply differentiation methods in solving problems.

In the external examinations NZQA uses a different approach to marking based on understanding (u), relational thinking (r) and abstract thinking (t). They then allocate marks to these concepts and add them up to decide upon the overall grade. This approach is not as easy for students to self mark as the NuLake approach but the results should be broadly similar.

Question One

a) $\frac{dy}{dx} = 6x^2 \tan 3x + 3(2x^3 + 3)\sec^2 3x$ **A**

b) $f'(x) = 4\sin(2x + \pi)\cos(2x + \pi)$

$f'(\frac{\pi}{8}) = 2$

$y - \frac{1}{2} = 2\left(x - \frac{\pi}{8}\right)$ **A**

c) (i) 1. $x = 0, 4$

2. $x = -4, 0, 4$

3. $x = -6, 2$

4. $-8 < x < -4$ **At least 2 answers correct A**

(ii) Does not exist. **At least 4 answers correct M**

d) $\frac{dy}{dx} = 2e^x \cos 2x + e^x \sin 2x$ **A**

$\tan 2x = -2$

$x = 1.017$ (4 sf) **M**

e) $\frac{dy}{dx} = 8e^{-x} - 8xe^{-x}$ **A**

Turning point $x = 1$ **M**

$\frac{d^2y}{dx^2} = -8e^{-x} - 8e^{-x} + 8xe^{-x}$

At $x = 1$, this is negative so maximum point.

Required proof E

Question Two

a) $f'(x) = \frac{1}{3}(4x + x^2)^{-2/3}(4 + 2x)$ **A**

b) $f'(x) = 3x^2 - 8x + 3$

$f'(3) = 6$

$m_N = \frac{-1}{6}$

Equation of normal $6y + x - 3 = 0$ **A**

c) Area = xy

$= x.8x\, e^{-x}$ **A**

$A' = 16x\, e^{-x} - 8x^2 e^{-x}$

Max at $x = 2$

Area = 4.331 **M**

d) $\text{Cost} = (32 - x)k + 3k\sqrt{16^2 + x^2}$ **A**

$\text{Cost}' = -k + \frac{3xk}{\sqrt{16^2 + x^2}}$

$x = 5.66$ km **M**

e) $\frac{dH}{dt} = \frac{dH}{dr} \cdot \frac{dr}{dV} \cdot \frac{dV}{dt}$

$V = \frac{4\pi R^3}{3}$. After 40 seconds, $V = 3.68\ m^3$

$R = 0.9577$ m

$\frac{dH}{dt} = 3 \times \frac{1}{4\pi R^2} \times 0.0920$ **M**

$= 0.0239$ m/s **E**

Question Three

a) $f'(x) = \frac{2e^{2x}(1 + \tan 2x) - 2e^{2x}\sec^2 2x}{(1 + \tan 2x)^2}$ **A**

b) $f(x) = e^{-(x+k)^2}$

$f'(x) = -2(x + k)e^{-(x+k)^2}$ **A**

$f''(x) = e^{-(x+k)^2}(4(x + k)^2 - 2)$ **M**

Setting $f''(x) = 0$

$4(x + k)^2 - 2$

$x = -k \pm \frac{1}{\sqrt{2}}$

Correct solution with f'(x) and f''(x) E

c) $f'(x) = 0.5(3 + x^2)^{-0.5} \times 2x$ **A**

$f'(x) = 0.5$ at $x = 1$, $f(1) = 2$, so coordinates (1, 2)

Derivative set equal to 0.5 and answer of (1, 2) found. **M**

d) $\frac{dy}{dt} = 2t$

$\frac{dx}{dt} = \frac{0.25}{(t + 2)^{0.75}}$

$\frac{dy}{dx} = 8t(t + 2)^{0.75}$ **A**

Turning points $t = 0, -2$.

Minimum $t = 0$.

Coordinates (1.189, 0)

Turning points found (t = 0 and t = −2).

Minimum identified (t = 0) and justified by use of the second derivative $\frac{d^2y}{dx^2} = 8(t + 2)^{0.5}(7t + 8)$ **M**

e) $\tan\theta = \frac{h}{x}$, $L = \frac{x + d}{\cos\theta}$

$L = h\,\text{cosec}\,\theta + d\sec\theta$

$L' = -h\,\text{cosec}\,\theta\cot\theta + d\sec\theta\tan\theta$ **M**

$L' = 0$

$\frac{h}{\sin\theta\tan\theta} = \frac{d\tan\theta}{\cos\theta}$

$\tan\theta = \sqrt[3]{\frac{h}{d}}$ **E**

Sufficiency. For each question award yourself a score out of 8 using this table. Add the three scores for a score out of 24 and compare to the cut scores. All answers must include derivatives where appropriate.

Quest.	N0	N1	N2	A3	A4	M5	M6	E7	E8
ONE	No diff's. correct.	1 diff. with error.	1 A or 1 diff. correct.	2A or 2 diff's. correct.	3A or 3 diff's. correct.	1M + 1M minor error.	2M all correct.	1E minor error.	1E all correct.
TWO	No diff's. correct.	1 diff. with error.	1 A or 1 diff. correct.	2A or 2 diff's. correct.	3A or 3 diff's. correct.	1M + 1M minor error.	2M all correct.	1E minor error.	1E all correct.
THREE	No diff's. correct.	1 diff. with error.	1 A or 1 diff. correct.	2A or 2 diff's. correct.	3A or 3 diff's. correct.	1M + 1M minor error.	2M all correct.	1E minor error.	1E all correct.
Cut Scores									
Not Achieved		**Achievement**		**Achievement with Merit**		**Achievement with Excellence**			
0 – 6		7 – 13		14 – 20		21 – 24			