Answers

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1. $\frac{1}{2}x^4 + \frac{4}{3}x^3 - \frac{5}{2}x^2 + 7x + C$

2. $5x^3 - \frac{35}{2}x^2 + C$

3. $\frac{Ax^3}{3} - 3Bx + C$

4. $3x^4 + 2x^3 + 5x + 3x^{-2} + C$

$= 3x^4 + 2x^3 + 5x + \frac{3}{x^2} + C$

5. $2x - x^{-1} + x^{-3} + C$

$= 2x - \frac{1}{x} + \frac{1}{x^3} + C$

6. $6x^2 + 4x^{3/2} + 2x^{5/2} + C$

7. $\frac{2}{3}x^{3/2} - 2x^{5/2} + C$

$= \frac{2}{3}\sqrt{x^3} - 2\sqrt{x^5} + C$

8. $\frac{3}{16}x^4 - \frac{2}{15}x^3 + \frac{1}{10}x^2 - 7x + C$

9. $\frac{2}{3}x^{3/2} - x^{-1} + C$

$= \frac{2}{3}\sqrt{x^3} - \frac{1}{x} + C$

10. $9x^{4/3} + 16x^{1/2} + C$

$= 9\sqrt[3]{x^4} + 16\sqrt{x} + C$

11. $2x^{0.5} + 8x^{1.25} + C$

$= 2\sqrt{x} + 8\sqrt[4]{x^5} + C$

12. $12x^{1/2} + C = 12\sqrt{x} + C$

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13. $y = 18x^{4/3} + C = 18\sqrt[3]{x^4} + C$

14. $21x^{5/3} + C = 21\sqrt[3]{x^5} + C$

15. $\frac{4}{5}\sqrt[4]{x^5} + 3x + C$

$= 0.8x^{1.25} + 3x + C$

16. $\frac{1}{5}x^5 + \frac{1}{4}x^4 + \frac{1}{3}x^3 + C$

17. $\int x^2 - 10x + 25 \ dx$

$= \frac{1}{3}x^3 - 5x^2 + 25x + C$

18. $\int x^2 - 4 \ dx = \frac{1}{3}x^3 - 4x + C$

19. $\int 4x^2 + 4x + 1 \ dx$

$= \frac{4}{3}x^3 + 2x^2 + x + C$

20. $= B\int x^2 + 8x + 12 \ dx$

$= B(\frac{1}{3}x^3 + 4x^2 + 12x) + C$

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21. $\int 2x^2 - 6x^{-2} \ dx$

$= \frac{2}{3}x^3 + 6x^{-1} + C$

$= \frac{2x^3}{3} + \frac{6}{x} + C$

22. $\int x^{1.5} - 4x^{-0.5} \ dx$

$= \frac{1}{2.5}x^{2.5} - \frac{4}{0.5}x^{0.5} + C$

$= \frac{2}{5}\sqrt{x^5} - 8\sqrt{x} + C$

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23. $\frac{1}{7}e^{7x} + C$

24. $\frac{5}{4}e^{4x} + C$

25. $^-2e^{-3x} + C$

26. $\frac{A^2}{4}e^{4x} + C$

27. $16e^{x/2} + C$

28. $\frac{1}{3}e^{3x+4} + C$

29. $\int e^{-2x} \ dx = \frac{^-1}{2}e^{-2x} + C$

30. $\int e^{0.5x} \ dx = 2e^{0.5x} + C$

$= 2\sqrt{e^x} + C$

31. $\frac{e^{4x}}{4} + e^{2x} + x + C$

32. $\frac{^-1}{9}e^{-3x} - \frac{2}{5}e^{-5x} + C$

33. $\frac{x^2}{2A} + \frac{e^{-3x}}{3} + C$

34. $\frac{^-e^{-2x}}{2} + e^x + C$

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35. $\frac{^-1}{3}\cos 3x + C$

36. $2 \sin 2x + C$

37. $4 \tan 3x + C$

38. $\frac{3}{5}\sec 5x + C$

39. $\frac{^-4}{3}\operatorname{cosec} 3x + C$

40. $^-3\sin(4x + 3) + \frac{e^{2x+1}}{2} + C$

41. $\frac{1}{AB}\tan(Bx) + C$

42. $\frac{^-2}{3}\operatorname{cosec}(3x + 1) + C$

43. $\frac{^-3}{2}\cot(2x - 1) + C$

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44. $\frac{2}{3}\sin 3x - \frac{5}{2}\cos 2x + C$

45. $\frac{^-e^{-2x}}{2} + \frac{3}{2}\sin 2x + C$

46. $\frac{^-3}{2}\cos 4x - \frac{3}{2}\sin 2x + C$

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47. $3 \ln|x| + C$

48. $\frac{1}{4}\ln|x| + C$

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49. $\frac{2}{3}\ln|x| + C$

50. $^-B \ln|x| + C$

51. $\frac{A}{B}\ln|x| + C$

52. $A\ln|x| + \frac{e^{Bx}}{B} + C$

53. $A \ln|x + 1| + C$

54. $\frac{x^2}{2A} + A\ln|x| + C$

55. $\frac{1}{2}\ln|2x + 1| + C$

56. $\ln|6x - 5| + C$

57. $\frac{A}{4}\ln|4x - 1| + C$

58. $\frac{3}{5}\ln|5x + 2| + C$

59. $3x - 2\ln|x| + C$

60. $\frac{^-1}{4}\ln|3 - 4x| + C$

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61. $\int \frac{3}{x} + 1 \ dx = 3 \ln|x| + x + C$

62. $\int \frac{4}{x} - 1 \ dx = 4 \ln|x| - x + C$

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63. $\int 3 + \frac{5}{x} \ dx = 3x + 5 \ln|x| + C$

64. $\int 2 - \frac{7}{x} \ dx = 2x - 7 \ln|x| + C$

65. $\int 3x + 7 - \frac{4}{x} \ dx$

$= \frac{3}{2}x^2 + 7x - 4 \ln|x| + C$

66. $\frac{Ax^2}{2} + Bx - D \ln|x| + C$

67. $\frac{5}{3}x^3 - 3x^2 + 7x + 3 \ln|x| + C$

68. $\frac{2}{3}x^3 + \frac{3}{2}x^2 + \frac{1}{2}x + 3 \ln|x| + C$

69. $2x^2 + x - A \ln|x| + C$

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70. $\frac{Ax^2}{2} + B\ln|x| + \frac{D}{x} + C$

71. $\frac{1}{3}x^3 - \frac{3}{2}x^2 + 5\ln|x| + C$

72. $x^5 + 2\ln|x| - \frac{3}{x} + C$

73. $\frac{1}{5}x^5 + \frac{1}{6}x^2 - \frac{5}{3}\ln|x| + C$

74. $\ln|x| + \frac{B}{Ax} + C$

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75. $3x - 11\ln|x+1| + C$

76. $3x + 9\ln|x-2| + C$

77. $2x - \frac{7}{4}\ln|4x+5| + C$

78. $3x + 18\ln|x-6| + C$

79. $3x + \frac{1}{2}\ln|2x-1| + C$

80. $^-6x - 6\ln|1-x| + C$

81. $2x + \frac{1}{4}\ln|4x+3| + C$

82. $\frac{x}{2} + \frac{1}{2}\ln|2x-2| + C$

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83. $2x - \ln|x-1| + C$

84. $^-3x + 13\ln|x+4| + C$

85. $x - 4\ln|2x+3| + C$

86. $3x + 4\ln|3-2x| + C$

87. $4x + 8\ln|x-2| + C$

88. $\frac{x}{3} - \frac{2}{9}\ln|3x+2| + C$

89. $^-7x - 32\ln|x-5| + C$

90. $2x - \ln|2-x| + C$

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91. $\ln|6x+3| + C$

92. $3\ln|x-2| + C$

93. $2\ln|x^2+1| + C$

94. $\frac{A}{3}\ln|3x-1| + C$

95. $\ln|x^2-3x+1| + C$

96. $2\ln|x^2+x-2| + C$

97. $2\ln|x^2-1| + C$

98. $\ln|e^{2x}-5| + C$

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99. $\ln|\tan 2x| + C$

100. $\ln|\cos 3x| + C$

101. $2\ln|\operatorname{cosec} 5x| + C$

102. $2\ln|7-\sin 4x| + C$

103. $\ln|\ln|x|| + C$

104. $\ln|e^{3x}+7| + C$

105. $2\ln|e^{x^2}-3| + C$

106. $^-\ln\left|\cos\left(x-\frac{\pi}{4}\right)\right| + C$

107. $\ln|\cos x + \sin x| + C$

108. $\frac{1}{2}\ln|e^{2x}+2x| + C$

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109. $\int 4(\sin 6x + \sin 4x)\,dx$
$= \frac{^-2}{3}\cos 6x - \cos 4x + C$

110. $\int 4(\sin 4x - \sin 2x)\,dx$
$= ^-\cos 4x + 2\cos 2x + C$

111. $6\sin 2x - 2\sin 6x + C$

112. $^-0.4\cos 10x - 2\cos 2x + C$

113. $\frac{5}{8}\sin 8x + \frac{5}{2}\sin 2x + C$

114. $\frac{^-3}{22}\cos 11x + \frac{3}{2}\cos x + C$

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115. $4\tan x - 4x + C$

116. $\frac{1}{4}\sin 2x + \frac{1}{2}x + C$

117. $\sin 2x + 2x + C$

118. $^-20\cot x - 20x + C$

119. $\frac{1}{2}\sin 2x + C$ OR
$\sin x\cos x + C$

120. $2x - \sin 2x + C$ OR
$^-2\sin x\cos x + 2x + C$

121. $6x + 3\sin\left(2x - \frac{\pi}{3}\right) + C$ Other forms of this answer possible.

122. $^-4\cos\left(2x + \frac{2\pi}{5}\right) + C$ Other forms of this answer possible

123. $2x^2 + 6x - \frac{3}{2}\sin 4x + C$

124. $6x - 4\sin 2x + \frac{1}{2}\sin 4x + C$

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125. $(2x+3)^6 + C$

126. $\frac{^-6}{(x-2)^5} + C$

127. $3(x-6)^4 + C$

128. $\frac{^-8}{(x+2)^3} + C$ OR
$^-8(x+2)^{-3} + C$

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129. $5(x+3) - 9\ln|x+3| + C$

130. $\frac{2}{5}(x+2)^{5/2} - \frac{4}{3}(x+2)^{3/2} + C$

131. $\frac{1}{2}(x^2+4)^6 + C$

132. $8\sqrt{x+2} + C$

133. $\frac{1}{7}(x+5)^7 - \frac{5}{3}(x+5)^6 + 5(x+5)^5 + C$

134. $\int (2u+5)u^5\,du$
$= \int 2u^6 + 5u^5\,du$
$= \frac{2}{7}(x-2)^7 + \frac{5}{6}(x-2)^6 + C$

135. $\ln|x-3| - \frac{3}{x-3} + C$

136. $\frac{^-1}{2(x^2+4x+5)^2} + C$

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137. $25(x+2)^{6/5} + C$

138. $\frac{1}{8}(x^2+5)^4 + C$

139. $\ln|\ln|x|| + C$

140. $\ln|e^x - 2| + C$

141. $\frac{1}{3}(2x-1)^{3/2} + (2x-1)^{1/2} + C$
$= \frac{1}{3}\sqrt{(2x-1)^3} + \sqrt{2x-1} + C$

142. $\frac{1}{3}\sqrt{(2x-1)^3} + \sqrt{2x-1} + C$
$= \frac{2}{3}(x+1)\sqrt{2x-1} + C$

143. $3e^{x^2} + C$

144. $3e^{x^2} + C$

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145. $\left[2x^3 - x^2 + x\right]_1^2 = 12$

146. $\left[\frac{1}{3}x^3 - 9x\right]_{-2}^{-1} = -6.667$ (4 sf)

147. $\left[\frac{2}{3}x^{1.5} + 4x^{0.5}\right]_1^4 = 8.667$ (4 sf)

148. $\left[8\ln|x| + \frac{2}{3}x^{1.5}\right]_1^5 = 19.66$ (4 sf)

149. $\left[2\ln|x| + 3x\right]_{0.112}^{1.245} = 8.216$ (4 sf)

150. $\left[\frac{1}{6}e^{3x} + x\right]_{-2}^{-1} = 1.008$ (4 sf)

151. $\left[\ln|e^x + 1|\right]_k^4 = 3.891$, $k = -2$

152. $\left[\frac{1}{2}\tan 2x\right]_0^k = 0.5$, $k = 0.3927$

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153. $\left[5\sec 2x\right]_{0.12}^{0.71} = 28.14$ (4 sf)

154. $\left[\frac{-11}{2}\ln|2x+3| + 3x\right]_{-1}^{5}$
$= 3.893$ (4 sf)

155. $\left[\frac{1}{2}\ln|2x^2 + 2x|\right]_1^3$
$= 0.8959$ (4 sf)

156. $\left[-2\cot(0.5x)\right]_{\pi/2}^{3\pi/2} = 4$

157. $\left[\frac{-1}{2}\ln|1-2x|\right]_{-3}^{0.25} = 1.320$ (4 sf)

158. $\left[\sqrt{2x}\right]_k^6 = 3.017$, $k = 0.1$

159. $\left[-e^{-x} + e^x\right]_{1.41}^{2.73} = 11.42$ (4 sf)

160. $\left[\frac{2}{3}(x-3)^{1.5}\right]_4^k = 6.787$, $k = 8$

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161. a) $\int_a^c f(x)\ dx = -K + K$
$= 0$

b) Area $= 2K$

c) Area $= 5 \times (c - a) + 0$
Area $= 5 \times (c - a)$

162. a) $\text{Area}_{a \text{ to } c} = 3.6$

b) $\int_a^c h(x)\ dx = 3.2 + -0.4$
$= 2.8$

c) $\int_a^c j(x)\ dx = 2.8 + 2(c - a)$

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163. $\left[\frac{1}{4}x^4 - \frac{8}{3}x^3 + 10x^2 - 14x\right]_2^5$
Area = 8.25 units^2

164. $\left[6x^2 - x^3\right]_1^3$
Area = 22 units^2

165. $\left[-\cos x\right]_{\pi/4}^{3\pi/4}$
Area = 1.414 units^2 (4 sf)

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166. $\left[\frac{1}{3}x^3 - 3x^2 + 8x\right]_2^4$
Area $= 1\frac{1}{3}$ units^2

167. $\left|\left[\frac{x^3}{3} - 3x^2 + 8x\right]_3^4\right| +$
$\left[\frac{x^3}{3} - 3x^2 + 8x\right]_4^5$
Area $= \frac{2}{3} + \frac{4}{3} = 2$ units^2

168. $\left[3\sin(2x)\right]_{\pi/4}^{3\pi/4}$
Area = 6 units^2

169. $\left[0.25e^{2x} - \frac{5}{2}x^2 - 5x\right]_{-0.5}^{1}$
Area = 7.620 units^2 (4 sf)

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170. $\left|\left[-2\cos(2x) - 8\sin x\right]_{-\pi/2}^{\pi/2}\right|$
Area = 16 units^2

171. $\left[2x^2 - \frac{1}{3}x^3\right]_0^4$
Area $= \frac{32}{3}\left(10\frac{2}{3}\right)$ units^2

172. $\left|\left[\frac{1}{4}x^4 - 2x^3 + x^2 + 3x\right]_2^4\right|$
Area = 34 units^2

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173. $\left[20(2x-1)^{1/2}\right]_5^k = 40$, $k = 13$

174. $\left[\ln|x^3 - 1|\right]_2^k = 4.29$, $k = 8$

175. $\left[\frac{e^{0.5x}}{5}\right]_0^k = 14.7413$, $k = 8.627$

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176. $\left[\frac{1}{4}x^4 - \frac{1}{3}x^3 - x^2\right]_{-1}^{0} +$
$\left|\left[\frac{1}{4}x^4 - \frac{1}{3}x^3 - x^2\right]_0^2\right|$
Area $= 3.083\left(3\frac{1}{12}\right)$ units^2

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177. $\int_{-3}^{0} x^2 + 2.5x + 3\ dx$
$= \left[\frac{1}{3}x^3 + \frac{5}{4}x^2 + 3x\right]_{-3}^{0}$
Area = 6.75 units^2

178. $\int_{-1}^{1} 2.5 - 2.5x^2\ dx$
$= \left[2.5x - \frac{5}{6}x^3\right]_{-1}^{1}$
Area = 3.333 units^2 (4 sf)

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179. $\left[\frac{1}{4}x^4 - \frac{10}{3}x^3 + \frac{29}{2}x^2 - 20x\right]_1^4$
Area = 11.25 units^2

180. $\left[\frac{1}{4}x^4 - 3x^3 + 12x^2 - 16x\right]_1^4$
Area = 6.75 units^2

181. $\left|\left[4x - \frac{19}{2}\ln|2x+3|\right]_0^{0.875}\right| +$
$\left[4x - \frac{19}{2}\ln|2x+3|\right]_{0.875}^{8}$
Area = 16.20 units^2 (4 sf)

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182. $\int_{-1}^{0} f(x) - g(x)dx + \int_{0}^{4} g(x) - f(x)dx$
Integral =
$\left[\frac{1}{4}x^4 - \frac{4}{3}x^3 + \frac{1}{2}x^2 - 6x + 12\ln(x+2)\right]$
Area = 24.38 units^2 (4 sf)

183. $\int_{-5\pi/6}^{-\pi/6} g(x) - f(x)dx + \int_{-\pi/6}^{\pi/2} f(x) - g(x)dx$
$I = \left[\frac{-1}{2}\sin(2x) + \cos x\right]$
Area = 3.8971 units^2 (4 sf)

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184. $2\left[\frac{(x^2-4)^6}{12}\right]_{-2}^{0}$
Area = 682.7 units^2 (4 sf)

185. $2\left[-2\cos(x^2)\right]_0^{1.772}$
Area = 8.000 units^2 (4 sf)

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186. a) $t = 2 \quad s = -3$ m
$t = 4 \quad s = 9$ m
$t = 8 \quad s = 105$ m
b) 18 m/s
c) -12 m/s
d) 6 m/s^2
e) $t = 2$ seconds

187. a) $t = \pm 1.225$ s (4 sf)
b) $s = \frac{4}{3}t^3 - 6t + 2$
c) 24 m/s^2
d) $s(2) = 0.6667$ m (4 sf)
e) 6 m/s

188. a) $a = -27$ m/s^2
b) Slowing down.
c) $v = t^3 - 6t^2 - 15t + 100$ m/s
d) $t = 5$ s (ignore $t = -4$)
e) $s = \frac{1}{4}t^4 - 2t^3 - \frac{15}{2}t^2 + 100t$
$s(5) = 218.75$ m

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189. a) -1 m
b) $\frac{t-3}{t+3} = \frac{t+3-6}{t+3}$
$s = 1 - \frac{6}{t+3}$
$v = \frac{6}{(t+3)^2}$
$a = \frac{-12}{(t+3)^3}$
c) No, because $v \neq 0$ as 6 and $(t + 3)^2$ must be positive.

190. a) -1 m/s
b) $t = 0.5$ secs
c) $s = 2t - 3\ln|t + 1|$
d) 1.841 m (4 sf)
e) $a = \frac{3}{(t+1)^2}$

191. a) $v = 81$ m/s
b) $v = 9$ m/s
c) $a = -36$ m/s^2
d) $t = 1.5$ s
e) $s = 12t^3 - 54t^2 + 81t - 30$
f) $s(1.5) = 10.5$ metres

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192. a) 49 m/s
b) $t = 5$ secs
c) $s = 49t - 4.9t^2$
d) $t = 10$ secs
e) -9.8 m/s^2
Constant deceleration (due to gravity).

193. a) $v = -9$ m/s.
Moving backwards.
b) $a = 5$ m/s^2
c) $s = 3t + \frac{5}{2}t^2 - \frac{2}{3}t^3$
d) $s = 10.67$ m (4 sf)
e) $t = 3$ and -0.5 secs

194. a) $a(0) = 5$ m/s^2
$a(2) = 1.363$ m/s^2 (4 sf)
b) $v = 5t + 4\cos t + 1$
c) $v = 9.335$ m/s (4 sf)
d) $s = 2.5t^2 + 4\sin t + t + 3$ m
e) $s = 18.64$ m

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195. $A = 8$ units^2
196. $A = 1.577$ units^2 (4 sf)
197. $A = 1.000$ units^2 (4 sf)
198. $A = 51.71$ units^2 (4 sf)

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199. $A = 4.761$ units^2 (4 sf)
200. $A = 0.7837$ units^2 (4 sf)
201. $A = 1.334$ units^2 (4 sf)
202. $A = 0.8972$ units^2 (4 sf)

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203. $A = 14.88$ units^2 (4 sf)
204. $A = 6.83$ units^2 (4 sf)
205. $A = 0.6287$ units^2 (4 sf)
206. $A = 1.486$ units^2 (4 sf)

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207. Estimate = 21.457 (4 sf)
208. $A = 5.780$ units^2 (4 sf)
209. Estimate = 3.544 (4 sf)
210. Estimate = 4.420 (4 sf)

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211. $V = 3.69395k^2$
$k = 1.802$ (4 sf)

212. Int. = 24.355
Mean = 4.060 (4 sf)
Height of a rectangle with the same width and area as the function.

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213. $f(x) = x^4 - 3x^2 + 2x + C$
214. $y = 3x^4 - 4x^3 + 6x^2 - 12x + C$
215. $f(x) = \frac{1}{2}x^4 + 8x + \frac{2}{x} + C$
216. $f(x) = \frac{x^6}{2} - \frac{2x^5}{5} - \frac{1}{x} + C$
217. $y = \frac{-1}{2}\cos(2x) + 8\sin(x) + C$
218. $y = 2e^{2x} + \frac{1}{3}\cos 6x + 3\ln|x| + C$
219. $y = \frac{5x^4}{4} - \frac{4\sqrt{x^3}}{3} - \frac{1}{2x^2} + C$
220. $y = 4\tan x + \frac{1}{x^3} + \frac{2\sqrt{x^3}}{3} + C$
221. $f(x) = 2\ln|x| + 3x + C$
222. $f(x) = 4e^{0.5x} + 4x + C$

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223. $f(x) = 12x + 3x^2 - 3x^3 - 12$
224. $y = \frac{2}{3}x^3 - \frac{1}{2}x^2 - x + 4\frac{5}{6}$
225. $f(x) = \frac{4}{3}x^3 - 16x - 20$
226. $y = \frac{-1}{2}\cos(2x) + 1$

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227. $f(x) = 3\ln|x| - 2x + 4$
228. $y = \frac{x^4}{4} + 3x^3 + \frac{27x^2}{2} + \frac{3x}{4} - 4$
229. $y = 4x - \frac{1}{2}\cos 2x$
230. $f(x) = 3\ln|x| + \frac{2\sqrt{x^3}}{3} - 1$
231. $f(x) = 2\ln|x^2 + 2x + 1| + 5$
232. $f(x) = \frac{e^{4x}}{4} + 5x + 4$
233. $f(x) = 2x + 4\ln|3x - 5| - 12$
234. $y = 3\ln|3x + 1| + x^2 - x - 3$

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235. a) $f'(x) = 5x^4 + 6x^{-2} + 4x + C$
$f(x) = x^5 - 6x^{-1} + 2x^2 + Cx + K$
b) $f(x) = x^5 - 6x^{-1} + 2x^2 + x + 13$

236. a) $f'(x) = 9x^2 - 6x^{-1} + C$
$f(x) = 3x^3 - 6\ln|x| + Cx + K$
b) $f(x) = 3x^3 - 6\ln|x| + 2x - 6$

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237. a) $f'(x) = 3x^2 + 4x^{-3} + 6x + C$

$f(x) = x^3 - 2x^{-2} + 3x^2 + Cx + K$

b) $f(x) = x^3 - 2x^{-2} + 3x^2 + 2x + 1$

238. a) $f'(x) = {}^-8\cos(4x) + 8x + C$

$f(x) = {}^-2\sin(4x) + 4x^2 + Cx + K$

b) $f(x) = {}^-2\sin(4x) + 4x^2 + 1$

239. $f'(x) = \frac{1}{2}e^{2x} + (2 - \frac{1}{2}e^2)$

$f(x) = \frac{1}{4}e^{2x} + (2 - \frac{e^2}{2})x + \frac{7}{4}$

240. $\frac{d^2y}{dx^2} = {}^-2x\cos x - 4\sin x$

Substituting in $y + \frac{d^2y}{dx^2}$

$2x\cos x + ({}^-2x\cos x - 4\sin x)$

$= {}^-4\sin x$

241. $\frac{dy}{dx} = (2x + 1)\,e^{2x}$

$\frac{d^2y}{dx^2} = (4x + 4)e^{2x}$

Substituting

$(4x + 4)e^{2x} - 4(2x + 1)e^{2x} + 4(xe^{2x}) = 0$

242. $f'(x) = 5x^4 - 20x^3 + 30x^2 - 20x + 2$

$f(x) = x^5 - 5x^4 + 10x^3 - 10x^2 + 2x + 7$

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243. a) $\frac{dy}{dx} = 3(3 + x)^4 + C$

$y = \frac{3}{5}(3 + x)^5 + Cx + K$

b) $y = \frac{3}{5}(3 + x)^5 - 240x + 5$

244. a) $f''(x) = 12 - 12\cos(4x)$

$f'(x) = 12x - 3\sin(4x) + C$

$f(x) = 6x^2 + \frac{3}{4}\cos(4x) + Cx + K$

b) $f(x) = 6x^2 + \frac{3}{4}\cos(4x) + 3x + \frac{5}{4}$

245. $f(x) = \ln|\cos x| + x + 1$

246. $\frac{dy}{dx} = 2e^{4x} + 8xe^{4x}$

$\frac{d^2y}{dx^2} = 8e^{4x} + 8e^{4x} + 32xe^{4x}$

$\frac{d^2y}{dx^2} = 4(2e^{4x} + 8xe^{4x} + 2e^{4x})$

$\frac{d^2y}{dx^2} = 4\left(\frac{dy}{dx} + \frac{2xe^{4x}}{x}\right)$

$\frac{d^2y}{dx^2} = 4\left(\frac{dy}{dx} + \frac{y}{x}\right)$

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247. $\frac{y^3}{3} = \frac{x^4}{4} + C$

248. $\ln|y| = 2x^2 + C$

This can also be rewritten as $y = ke^{2x^2}$

249. $y^2 + y = \frac{x^2}{2} + C$

250. $\frac{{}^-1}{y} = x^3 + C$

251. $\frac{y^2}{2} - \frac{y^3}{3} = \frac{x^2}{2} + C$

$3y^2 - 2y^3 = 3x^2 + K$

252. $\ln|y| = \frac{x^2}{2} + x + C$ which can be rewritten as

$y = k\,e^{0.5x^2 + x}$

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253. $\ln|y| = {}^-2\cos(0.5x)$

$y = e^{-2\cos(0.5x)}$

254. $\sin y = 6\sin 2x - 6$

255. $\ln|y| = x^3 + 15x + \ln|3|$

$y = 3e^{(x^2 + 15x)}$

256. $e^y = 6e^{2x} - 5$

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257. $y = {}^-\ln|55 - 2x^3|$

258. $\ln|2y - 1| = 2x^2 + 2x^3 + C$

$\ln|2y - 1| = 2x^2 + 2x^3 - 70.90$

259. $\frac{1}{2}\ln|1 - y^2| = \frac{1}{2}\ln\,k|1 - x^2|$

$1 - y^2 = \frac{8}{3}(1 - x^2)$

260. $\frac{y^3}{3} = (1 + x^2)^3 + 1$

$\frac{y^3}{3} = x^6 + 3x^4 + 3x^2 + 2$

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261. 7.0 years

262. 11.2 years

263. \$244 000 (3 sf)

264. 6.9% (1 dp)

265. 6.9 year (7 years)

266. 89.4 years (90 years)

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267. a) 16.2% per year

b) 9.9 years (10 years)

268. a) $\frac{dP}{dt} = kP,$

$\int\frac{1}{P}\,dP = \int k\,dt$

$\ln P = kt + c$

$P = e^{kt + c}$

$P = e^c \times e^{kt}$

$P = P_0e^{kt}$

b) $P = 725\,000\,e^{kt}$

$1\,055\,000 = 725\,000\,e^{5k}$

$k = 0.0750$

$P = 725\,000\,e^{10 \times 0.075}$

$P = 1\,535\,000$

c)

$4\,000\,000 = 725\,000\,e^{0.075\,t}$

$e^{0.075\,t} = 5.5172$

$t = 23$ years (0 dp)

269. a) \$169 000 (3 sf)

b) 23 years (0 dp)

270. Dropping at 13.9% per year

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271. a) $P = P_0e^{kt}$ t in days

$k = 0.0909$

$P = 12\,e^{0.0909\,t}$

b) $t = 7.3$ weeks (51 days)

272. a) $T = 14°$ C

b) $t = 274$ minutes

273. $r = {}^-0.1438$ (4 sf)

$t = 22.65$ minutes

274. $r = 0.026\,66$

$t = 45.2$ years

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275. $P(10) = 190$ (0 dp)

276. a) $P(10) = \$7770$ (3 sf)

b) $t = 8$ years 8 months

277. $T - T_a = Ce^{kt}$

$T = T_a + Ce^{kt}$

$18 = {}^-27 + C \quad (t = 0)$

$C = 45$

$^-15 = {}^-27 + 45e^{10k} \quad (t = 10)$

$k = {}^-0.1322$

$T(5.5) = {}^-27 + 45e^{-0.1322 \times 5.5}$

$T = {}^-5.25°C$

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278. k = starting No. mites

$k - 124 = k\,e^{-0.004\,32 \times 15}$

$k(1 - e^{-0.0648}) = 124$

$k = 1976$

$= 1980$ (3 sf)

Remain = 1852

$= 1850$ (3 sf)

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279. a) $\int \frac{1}{(500-x)}\,dx = \int k\,dt$

$-\log_e |500 - x| = kt + c_1$

$\log_e |500 - x| = -kt + c_2$

$500 - x = e^{-kt + c_2}$

$x = 500 - C\,e^{-kt}$

Substitute t = 0 and x = 0

$x = 500 - 500\,e^{-kt}$

b) t = 60 x = 250

k = 0.011 55

$x = 500(1 - e^{-0.011\,55t})$

c) $495 = 500(1 - e^{-0.011\,55t})$

t = 399 s (3 sf)

or t = 397 s if k = 0.0116 (3 sf)

280. a) $\frac{dP}{dt} = -6.52(t - 5)^2 - 1216$

$\frac{dP}{dt} = -6.52t^2 + 65.2t - 1379$

$P = -2.17t^3 + 32.6t^2 - 1379t + 15200$

b) at t = 10, P = 2497

c) $\frac{dP}{dt} = -0.187P$

$P = P_o e^{-0.187t}$

$P = 152\,00\,e^{-0.187t}$

at t = 10, P = 2343

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Practice External Assessment – Integration

In the external examinations NZQA uses a different approach to marking based on understanding (u), relational thinking (r) and abstract thinking (t). They then allocate marks to these concepts and add them up to decide upon the overall grade. This approach is not as easy for students to self mark as the NuLake approach but the results should be broadly similar.

Question One

(a) $I = \int \frac{x^2 - 8x + 16}{x}\,dx$

$I = \int x - 8 + \frac{16}{x}\,dx$

$I = \frac{1}{2}x^2 - 8x + 16\ln x + C$ **A**

(b) Simpson's rule

$I = \frac{0.5}{3}\left[3 + 4(1.3919) + 2(.6111)\right]$

$= 1.632$ (3 dp) **A**

All working must be shown.

(c) $\left[\ln(x-4)^2\right]_5^k = 3.219$

$k^2 - 8k - 9 = 0$

$(k - 9)(k + 1) = 0$

$k = 9$ only **M**

(d) i) $s = \frac{-30}{(t+3)^2} + C$ When t = 0, s = 0

$C = \frac{30}{9}$ $s = \frac{-30}{(t+3)^2} + \frac{30}{9}$

When t = 2, s = 2.133 m **A**

ii) s tends to $\frac{30}{9}$ and v to 0. **M**

(e) $\frac{dV}{dt} = -kV$

$\frac{1}{V}\frac{dV}{dt} = -k$

$\int \frac{1}{V}\,dV = \int -k\,dt$

$\ln v = -kt + C$

t = 0, V = 1000 so C = ln 1000

$\ln V = \ln 1000 - kt$

$\ln(\frac{V}{1000}) = -kt$

$V = 1000e^{-kt}$

$V(30) = 850$ so $850 = 1000e^{-30k}$

$k = 0.005417$

$200 = 1000e^{-0.005417t}$

$t = 297.1$ hours **E**

Question Two

(a) $\frac{2x^{3/2}}{3} - \frac{1}{x} + C$ **A**

(b) $38000 = 65000e^{-4i}$

$i = 0.1342\ (13.4\%)$ **A**

(c) $\int \frac{1}{y}dy = \int 2(x^2+2)dx$

$\ln y = \frac{2x^3}{3} + 4x + C$ **A**

(0,2) $\ln y = \frac{2x^3}{3} + 4x + \ln 2$

$y = 2e^{\frac{2x^3}{3}+4x}$ **M**

(d) Intersections at x = 0.5, 2.5 and 6.5 units.

$A_1 = \left[\frac{-3}{\pi}\cos\left(\frac{\pi x}{3}\right) - 0.5x\right]_{0.5}^{2.5}$

$A_2 = -\left[\frac{-3}{\pi}\cos\left(\frac{\pi x}{3}\right) - 0.5x\right]_{2.5}^{6.5}$

Area $= 4.3080$ (4 dp) **M**

(e) If $\alpha > 0$, $y = C(x-k)(x+k)$ as intercept $(0, \alpha)$

$\alpha = C(-k)(k)$

$C = \frac{\alpha}{-k^2}$ so $y = \frac{\alpha}{-k^2}(x^2 - k^2)$

$8 = 2\int_0^k \frac{a}{-k^2}(x^2 - k^2)\,dx$

$8 = \frac{-2\alpha}{k^2}\left[\frac{x^3}{3} - k^2x\right]_0^k$

$\alpha = \frac{6}{k}$ **E**

Question Three

(a) $\frac{1}{4}\cot(4x) + 5x + C$ **A**

(b) $\int_P^R f(x)dx = 4.7$ **A**

(c) $v(t) = -9.8t + t^{1.5} + 0.12t^2 + C$

$v(0) = -4.5$ so $C = -4.5$

$v(t) = -9.8t + t^{1.5} + 0.12t^2 - 4.5$ **A**

$s(t) = -4.9t^2 + 0.4t^{2.5} + 0.04t^3 - 4.5t + K$

$s(0) = 45$ so $K = 45$

Distance at 2 seconds = 19 m. **M**

(d) f(x) – g(x) is positive from x = 1 to 2.9009 while g(x) – f(x) is positive from x = 2.9009 to 3.9389.

$A = \int_1^{2.9009} f(x) - g(x)\,dx - \int_{2.9009}^{3.9389} g(x) - f(x)\,dx$

$= \left[\frac{1}{4}x^4 - \frac{8}{3}x^3 + \frac{19}{2}x - 10x - 10\ln[x+1] - \frac{3}{x}\right]_1^{2.9009}$

$- \left[\frac{1}{4}x^4 - \frac{8}{3}x^3 + \frac{19}{2}x - 10x - 10\ln[x+1] - \frac{3}{x}\right]_{2.9009}^{3.9389}$

$= 2.15565$ **M**

(e) $v(t) = 400 \sin kt$ so $(v(t))^2 = 400^2 \sin^2 kt$

$I = \int 400^2 \sin^2 kt\ dt$

$= 400^2 \int 0.5 - 0.5\cos\ 2kt\ dt$

$= \frac{400^2}{2}\left(t - \frac{\sin 2kt}{2k}\right)_0^{2\pi/k}$

$= \frac{2\pi 400^2}{2k}$

$RMS = \sqrt{\frac{\int_0^T v^2(t)dt}{T}} = \sqrt{\frac{\frac{\pi 400^2}{k}}{\frac{2\pi}{k}}}$

$RMS = \frac{400}{\sqrt{2}}$ **E**

Sufficiency. For each question award yourself a score out of 8 using this table. Add the three scores for a score out of 24 and compare to the cut scores. All answers must include integrals where appropriate.

Quest.	N0	N1	N2	A3	A4	M5	M6	E7	E8
ONE	No integ's. correct.	1 Integ. with error.	1 A or 1 integ. correct.	2A or 2 integ's. correct.	3A or 3 integ's. correct.	1M + 1M minor error.	2M incld 2 integ's.	1E Equation for V.	1E all correct.
TWO	No integ's. correct.	1 Integ. with error.	1 A or 1 integ. correct.	2A or 2 integ's. correct.	3A or 3 integ's. correct.	1M + 1M minor error.	2M all correct.	1E integ. correct.	1E integ + α correct.
THREE	No integ's. correct.	1 Integ. with error.	1 A or 1 integ. correct.	2A or 2 integ's. correct.	3A or 3 integ's. correct.	1M + 1M minor error.	2M all correct.	Integ. $v^2(t)$ correct.	1E all correct.

Cut Scores

Not Achieved	Achievement	Achievement with Merit	Achievement with Excellence	
0 – 6	7 – 13	14 – 20	21 – 24	