Answers

Page 3

1. $4x^2 + 20x$
2. $8x^2 - 12x$
3. $2x^3 + 3x^2$
4. $30a^2 - 18a$
5. $6k^3 + 4k^2 + 2k$
6. $12x^4 + 24x^3 - 28x^2$
7. $x^2 + 8x + 15$
8. $3x^2 + 2x - 1$
9. $6a^2 + 5a + 1$
10. $^-30x^2 + 25x + 30$
11. $15x^2 - 6x - 12$
12. $12x^2 - 9x + 3$
13. $x^2 - 2x - 12$
14. $16x^2 - 4x + 10$
15. $21x^2 - 14x$
16. $x^3 - 21x^2 + 147x - 343$
17. $12x^3 + 24x^2 - 3x - 6$
18. $3x^3 + x^2 - 8x + 4$
19. $a^3 + a^2 - 14a - 24$
20. $6x^3 + 13x^2 - 40x - 75$
21. $x^3 + x^2 - 8x - 12$
22. $6y^3 - 19y^2 + y + 6$
23. $x^3 - 11x^2 + 31x - 29$
24. $64x^3 + 48x^2 + 12x + 1$
25. $3k^3 - 16k^2 + 32k - 17$
26. $^-8y^3 + 12y^2 - 9y - 11$
27. $^-12x^2 - 16$

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28. $8q^3 - 12q^2 + 6q - 1$
29. $^-11m^2 + 5m$
30. $6p^3 - 7p^2 - 6p + 3$
31. $12x^2 - 28x + 8$
32. $m^3 - 7m - 6$
33. $24k^3 - 58k^2 + 23k + 15$
34. a) $2x + 40$

    b) $\pi x^2 + 40\pi x + 400\pi$

    c) $40\pi x + 400\pi$

    d) $x = 5.92$

    Diameter of stage is 11.8 m (1 dp).

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35. $3x(7 - 2x)$
36. $5x(1 - 3x)$
37. $x^3yz\,(z + y + x)$
38. $3b(4a^2 + a - 3b)$
39. $2\pi r(r + h)$
40. $7km(4k - 7m)$
41. $4k^2(1 - 3k)$
42. $12xy(3x - 2y)$
43. $(k - 3)(4 + y)$
44. $(x + 2)(x + 7)$
45. $(x - 7)(x - 1)$
46. $(x - a)(4 - 2y)$

    $= 2(x - a)(2 - y)$
47. $(x + 2)(12x + a)$
48. $(e + 1)(3ac - 6b)$

    $= 3(e + 1)(ac - 2b)$
49. $(4x - 1)(2x + 3z)$

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50. $(x + 6)(x + 1)$
51. $(x + 3)(x + 4)$
52. $x(x + 9)$
53. $(x - 8)(x + 3)$
54. $(a + 6)(a - 4)$
55. $(z + 5)(z - 5)$
56. $(x + 5)(x + 1)$
57. $(x + 8)(x - 3)$
58. $(x - 4)(x - 8)$
59. $(h + 6)(h + 6) = (h + 6)^2$
60. $(p + 1)(p - 1)$
61. $(x + 9)(x - 9)$
62. $(m - 1)(m - 6)$
63. $(x - 9)(x + 5)$
64. $(x + 26)(x - 2)$

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65. $(m + 7)(m - 2)$
66. $(k - 9)(k - 6)$
67. $(r + 7)(r - 4)$
68. $(a - 9)(a + 2)$
69. $(p - 6)^2$
70. $(n + 1)(n + 12)$
71. $(s - 17)(s + 4)$
72. $(x + 14)(x - 8)$
73. $(k + 12)(k - 9)$
74. $(x + 10)(x - 10)$
75. $(4k - 7)(4k + 7)$
76. $(y - 8x)(y + 8x)$

Page 8 cont...

77. $(5 - 6a)(5 + 6a)$
78. $(1 - pq)(1 + pq)$
79. $(b - \dfrac{1}{b})(b + \dfrac{1}{b})$
80. $(x - 10)^2$
81. $(x + 8)^2$
82. $(x + 11)^2$
83. $(x + 13)^2$
84. $(x - 7)^2$
85. $(x - 15)^2$

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86. $2(x - 4)(x + 1)$
87. $3(x - 3)(x - 1)$
88. $3(x + 10)(x - 1)$
89. $(4b + 5)(b + 2)$
90. $(x - 3)(3x + 2)$
91. $(2n - 5)(3n - 4)$
92. $3(x - 8)(x + 5)$
93. $4(x + 7)(x - 2)$
94. $2(n + 1)(n + 12)$

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95. $3(p + 7)(p - 7)$
96. $5(q - 7)(q - 3)$
97. $0.5(n - 8)(n + 6)$
98. $(3x + 1)(x + 1)$
99. $(4x + 1)(x - 1)$
100. $(5x + 1)(x - 2)$
101. $(7x + 3)(x + 2)$
102. $(3x - 1)(3x + 4)$
103. $(2x + 3)(3x - 2)$
104. a) Length $= 60 - 2w$

     Width $= 40 - w$

     b) $2w^2 - 140w + 2400 = 1408$

     $w^2 - 70w + 496 = 0$

     c) $w = 8$ m (reject 62)

     Length 44 m, Width 32 m

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105. $\dfrac{1}{81}$
106. 2
107. $\dfrac{1}{125}$

Page 13 cont...

108. $\dfrac{-1}{4}$

109. 8

110. ⁻8

111. $27x^9$

112. $\dfrac{2}{x^3}$

113. x^2

114. y

115. $\dfrac{1}{m^2}$

116. $\dfrac{k^2}{q^3}$

117. $9a^4b^6$

118. $\dfrac{64}{25k^2}$

119. $\dfrac{q^{3/2}}{p^2}$

120. $\dfrac{9p^4}{4m^2n^2}$

121. $\dfrac{1}{2g^2}$

122. $\dfrac{y^9}{x^6}$

123. $\dfrac{16p^{13}}{q^{11}}$

124. x^8

125. $\dfrac{1}{x^{4b}}$

126. $\dfrac{3b}{a}$

127. $\dfrac{x^{4a}}{y^a}$

128. $\dfrac{1}{q^{2x}}$

129. $\dfrac{2n^{1/4}}{m^{1/2}}$

130. $2x^4$

131. $\dfrac{p^{5/4}}{q^{1/2}}$

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132. $4x^3$

133. x^3

134. $\dfrac{1}{4x}$

135. $\dfrac{x^2}{9}$

136. $\dfrac{1}{p^{10}}$

137. $\dfrac{1}{x^4}$

Page 14 cont...

138. $\dfrac{6n}{m^2}$

139. $4x^{9/2}$

140. $x^{2/3}$

141. a) $A = \dfrac{10p^5}{9}$

   b) $V = \dfrac{80p^4}{27}$

   c) $p = 1.5$ cm

   d) Width = 4.13 cm

   Depth = 2.04 cm

   Height of bottle 5.33 cm

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142. $u = v - at$

143. $I = \dfrac{V}{R}$

144. $c = y - mx$

145. $x = \dfrac{y - c}{m}$

146. $r = \sqrt{\dfrac{A}{2\pi}}$

147. $r = \sqrt[3]{\dfrac{3V}{4\pi}}$

148. $a = \dfrac{2s - 2ut}{t^2}$

149. $R = \sqrt{\dfrac{A + \pi r^2}{\pi}}$

150. $h = \dfrac{2A}{a + b}$

151. $b = \dfrac{2A}{h} - a$

152. $M = \dfrac{4P - 4t}{3}$

153. $R = \dfrac{100I}{PT}$

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154. $b = \pm\sqrt{a^2 - c^2}$

155. $T = \dfrac{100I}{PR}$

156. $h = \dfrac{E}{mg}$

157. $F = \dfrac{9C}{5} + 32$

158. $x = k\left(\dfrac{a}{b} + c\right)$

159. $y = \dfrac{20 - 10x}{3}$

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160. $a = \dfrac{2b + bv}{3v - 1}$

161. $a = \dfrac{5d + 12 - bs}{s}$

162. $x = \dfrac{cy + a}{y - 5}$

163. $b = \dfrac{ar}{a - r}$

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164. $r = \sqrt{\dfrac{A}{\pi}}$ no ± as r is +ve

165. $r = \sqrt[3]{\dfrac{3V}{\pi}}$

166. $x = \left(\dfrac{c - a}{b}\right)^2$

167. $s = \dfrac{v^2 - u^2}{2a}$

168. $r = \sqrt{\dfrac{3S}{4\pi}}$ no ± as r is +ve

169. $x = \dfrac{(y - 4a)^2}{25}$

170. $y = \dfrac{x}{k^2}$

171. $b = \dfrac{12}{15a - 2x^2}$

172. Half sphere $= \dfrac{2}{3}\pi r^3$

   Total Vol. $= \dfrac{2}{3}\pi r^3 + \pi r^2 h$

   Make h the subject

   $\pi r^2 h = V - \dfrac{2}{3}\pi r^3$

   $h = \dfrac{V - \dfrac{2}{3}\pi r^3}{\pi r^2}$

   $h = \dfrac{V}{\pi r^2} - \dfrac{2}{3}r$

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173. $25 = 5^A$

   $A = 2$

174. $512 = 8^B$

   $B = 3$

175. $64 = 2^C$

   $C = 6$

176. $0.5 = 2^D$

   $D = ^-1$

177. $G = 3^3$

   $G = 27$

Page 22 cont...

178. $H = 10^3$

$H = 1000$

179. $M^5 = 32$

$M = 2$

180. $N^1 = 10$

$N = 10$

181. $P^0 = 1$

P = any real number except 0

182. $Q^5 = 243$

$Q = 3$

183. $4y + 4 = 32$

$y = 7$

184. $2x - 1 = 9$

$x = 5$

185. $\dfrac{1}{x} = 2$

$x = \dfrac{1}{2}$

186. $x^2 = 64$

$x = 8, ^-8$

187. $4x - 4 = x^2$

$(x - 2)^2 = 0$

$x = 2$

188. $7z - 10 = z^2$

$z = 2, 5$

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189. $\log 20$

190. $\log 5$

191. $\log 625$

192. $\log 3$

193. $\log 1 \ (= 0)$

194. $\log \dfrac{a^2}{b^2}$

195. $2\log a + \log b + \log c$

196. $\log a - \log b - \log c$

197. $2\log a - \log b$

198. $\dfrac{1}{2}(\log a + \log b + \log c)$

199. $\log(4x - 3) - \log x^2 = \log 1$

$\log\left(\dfrac{4x - 3}{x^2}\right) = 0$

$\left(\dfrac{4x - 3}{x^2}\right) = 1$

$x^2 - 4x + 3 = 0$

$x = 3$ and 1

Page 24 cont...

200. $\log(x^2 + x + 3) = \log(9x - 9)$

$x^2 + x + 3 = 9x - 9$

$x^2 - 8x + 12 = 0$

$(x - 2)(x - 6) = 0$

$x = 2$ and 6

201. $(5x - 6) = x^2$

$x^2 - 5x + 6 = 0$

$(x - 2)(x - 3) = 0$

$x = 2$ and 3

202. $(7z - 12) = z^2$

$z^2 - 7z + 12 = 0$

$(z - 3)(z - 4) = 0$

$z = 3$ and 4

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203. $\log(x + 1)^2 = \log 4$

$\log\left(\dfrac{x^2 + 2x + 1}{4}\right) = 0$

$\left(\dfrac{x^2 + 2x + 1}{4}\right) = 1$

$x^2 + 2x - 3 = 0$

$x = ^-3$ and 1

But $x = ^-3$ leaves $\log(x + 1)$ undefined so

$x = 1$

204. $\log(x^2 + 3x) = \log 10$

$\log\left(\dfrac{x^2 + 3x}{10}\right) = 0$

$\left(\dfrac{x^2 + 3x}{10}\right) = 1$

$x^2 + 3x - 10 = 0$

$x = ^-5$ and 2

But $x = ^-5$ leaves $\log x$ undefined so

$x = 2$

205. $\log_x(2x + 8) = 2$

$2x + 8 = x^2$

$x^2 - 2x - 8 = 0$

$x = ^-2$ and 4

But log base cannot be negative so

$x = 4$

206. $3k + 13 = (k + 1)^2$

$k^2 - k - 12 = 0$

$k = ^-3$ and 4

But log base cannot be negative so

$k = 4$

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207. $x = 7$

208. $x = 3.457$ (3 dp)

Page 26 cont...

209. $x = ^-0.810$ (3 dp)

210. $x = 4.637$ (3 dp)

211. $x = 20.219$ (3 dp)

212. $x = 2.278$ (3 dp)

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213. a) \$13 956 (0 dp)

b) $3000 = 28000(0.87^n)$

$n = \dfrac{\log 0.10714}{\log 0.87}$

$n = 16$ years (0 dp)

214. a) \$3 200 000 (3 sf)

b) $2\,000\,000 = 825\,000\,(1.145^n)$

$n = \dfrac{\log 2.4242}{\log 1.145}$

$n = 6.54$

$n = 7$ years (0 dp)

215. $1\,000\,000 = 125\,000\,(1.045^n)$

$n = \dfrac{\log 8}{\log 1.045}$

$n = 47.24$

Year would be 2048.

216. In thousands

$1200\,(1.084^n) = 825\,(1.145^n)$

$n = \dfrac{\log 1.4545}{\log 1.145 - \log 1.084}$

$n = 6.8435$

Year would be 2023.

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217. $\dfrac{U}{2} = U(1 - \dfrac{r}{100})^T$

$\dfrac{1}{2} = (1 - \dfrac{r}{100})^T$

$\log 0.5 = T \log(1 - \dfrac{r}{100})$

$T = \dfrac{\log 0.5}{\log(1 - \dfrac{r}{100})}$

$T = 17$ years (0 dp), $r = 4\%$

218. $\log U_1(1 + r_1/100)^t =$
$\log U_2(1 + r_2/100)^t$

$\log U_1 + t \log(1 + r_1/100) =$
$\log U_2 + t \log(1 + r_2/100)$

$t = \dfrac{\log U_2 - \log U_1}{\log(1 + \dfrac{r_1}{100}) - \log(1 + \dfrac{r_2}{100})}$

$t = \dfrac{\log\left(\dfrac{U_2}{U_1}\right)}{\log\left(\dfrac{100 + r_1}{100 + r_2}\right)}$

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219. $3b^2$

220. $\dfrac{5}{m^2}$

221. $\dfrac{5x}{4y^3z^3}$

222. $\dfrac{9(x+2)}{4az}$

223. $\dfrac{5n^3p}{9m^5q^3}$

224. $\dfrac{(x+2)}{(x-5)^2}$

225. $\dfrac{x^2+7}{y^2+7}$ no factors

226. $\dfrac{(x-3)}{4}$

227. $\dfrac{m}{5(n+1)}$

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228. $\dfrac{(x+4)}{(x-4)}$

229. $\dfrac{(x-5)}{(x-3)}$

230. $\dfrac{x}{(x+3)}$

231. $\dfrac{(x-3)}{(x-4)}$

232. $\dfrac{2(x+2)}{(x+4)}$

233. $\dfrac{2x+1}{x+2}$

234. x

235. $\dfrac{x-5}{x}$

236. $\dfrac{3}{x+7}$

237. $\dfrac{(6-x)}{(x+2)}$

238. $\dfrac{a(a+5)}{(a+3)}$

239. $\dfrac{5k+3}{k+3}$

240. $\dfrac{x-2}{3x-1}$

241. $\dfrac{z(z+2)}{z-3}$

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242. $\dfrac{ab}{c^2}$

243. $\dfrac{2x}{3}$

Page 32 cont...

244. $\dfrac{1}{c}$

245. $\dfrac{x^3}{y^6}$

246. $\dfrac{d^3}{e^4g^2}$

247. $6x^6y^2$

248. $\dfrac{4x^6y}{9}$

249. 1

250. $\dfrac{x^2y^2z^2}{8}$

251. $\dfrac{c^2}{b^2}$

252. $\dfrac{1}{(x+3)(x+2)}$

253. $\dfrac{y^5}{2x}$

254. $\dfrac{2}{(x+4)}$

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255. $\dfrac{x(a+b)}{ab}$

256. $\dfrac{4b+15a}{10ab}$

257. $\dfrac{6b-5ax}{3x}$

258. $\dfrac{2yz-5xz+6xy}{xyz}$

259. $\dfrac{3xy-4y+5x^2}{x^2y^2}$

260. $\dfrac{6bc+8ac-9ab}{12abc}$

261. $\dfrac{2b+5a}{ab}$

262. $\dfrac{4y-3x}{xy}$

263. $\dfrac{a-5}{(a-2)(a-3)}$

264. $\dfrac{24b^2-10ab-21a}{6a^2b^2}$

265. $\dfrac{5x-7}{(x-3)(x+1)}$

266. $\dfrac{11x-8}{30x}$

267. $\dfrac{3x+7}{(x+3)(x+5)}$

Page 33 cont...

268. $\dfrac{x^2-5x}{4(x-2)(x-3)}$

269. $\dfrac{15x^3+64x^2+4x}{10(x+1)(x+4)}$

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270. $x = {}^-10$

271. $x = 8$

272. $x = {}^-6$

273. $x = 2$

274. $q = {}^-6$

275. $x = {}^-3$

276. $x = 2$

277. $x = 3\dfrac{5}{7}$ (3.714)

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278. $x = 5\dfrac{1}{3}$ (5.333)

279. $x = {}^-17$

280. $q = \dfrac{{}^-1}{10}$ (${}^-0.1$)

281. $k = 4$

282. Any real number

283. No real answer

284. $x = 4$

285. $y = \dfrac{1}{11}$ (0.0909)

286. $x = 0$

287. No real answer

288. $x = 2\dfrac{2}{17}$ (2.118)

289. $z = {}^-2$

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290. $x = 4$

291. $x = {}^-5$

292. x is any real number

293. $x = {}^-7$

294. $x = {}^-38$

295. $x = {}^-3$

296. $x = 1\dfrac{7}{8}$ (1.875)

297. $x = 10$

298. No real solution

299. $x = 2\dfrac{1}{2}$ (2.5)

300. $x = 2\dfrac{7}{9}$ (2.778)

301. $x = \dfrac{1}{2}$ (0.5)

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302. a) Long side x and short side x – 12

b) Long side x + 30 and short side x + 18

c) 2(2x + 2(x – 12)) = 2(x + 18) + 2(x + 30)
or equivalent

d) x = 36
Length 36 m
Width 24 m

303. a) There are 20 who support the idea (19 + 1) and 24 + x who were at the meeting.

b) $\dfrac{1}{x}$

c) $\dfrac{20}{24 + x} - \dfrac{1}{x} = \dfrac{1}{2}$
38x – 48 = x(24 + x)
x = 6, 8

d) Two answers and both possible.
$\dfrac{20}{32} - \dfrac{1}{8} = \dfrac{1}{2}$ and
$\dfrac{20}{30} - \dfrac{1}{6} = \dfrac{1}{2}$

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304. a) 1500x + 800(2x + 3) + 4800x

b) 1.5x + 0.8(2x + 3) + 4.8x = 20
x = 2.228
Order: 2 m³ cement,
(4 + 3) = 7 m³ sand and
6 m³ crushed stone.

305. a) Energy = 11t + 2t x 5

b) Energy = 21t + (60 – 3t) x 8

c) 21t + 8(60 – 3t) = 440
Jogging 13 minutes,
walking 27 minutes and
Cycling 20 minutes.
Answers to 0 dp.

306. Time cafe = t
5(13.5t + 2t x 10 + 20) = 600
t = 2.985
Cafe = 3 hours (0 dp)

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307. x < ⁻9

308. x ≥ ⁻4

309. $r \le 7\dfrac{1}{2}$ (7.5)

310. k < 10

311. x < 7

312. $x < 4\dfrac{1}{4}$ (4.25)

313. k ≥ 5

314. $m \ge {}^-4\dfrac{1}{2}$ (⁻4.5)

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315. y ≤ 2

316. $x < \dfrac{-12}{13}$ (⁻0.9231)

317. x ≥ ⁻11

318. a > ⁻20

319. x > ⁻5

320. x > 4

321. $p \le 2\dfrac{1}{3}$ (2.333)

322. $q \le \dfrac{-1}{13}$ (⁻0.0769)

323. y ≤ ⁻11

324. x > ⁻4

325. y > 4

326. $w \le \dfrac{16}{53}$ (0.3019)

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327. m < ⁻3

328. k < 0

329. a) 0.065(15000 – n) + 0.085n = 0.02n + 975

b) 0.02n + 975 ≥ 1200

c) n ≥ 11 250 ($)

d) Term deposit $3750
Unit trust int. = $956.25
Term deposit int. = $243.75

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330. $x = \dfrac{-1}{2}, 5$

331. x = ⁻5, 5

332. $x = \dfrac{3}{4}, {}^-2\dfrac{1}{2}$

333. x = 0, 2

334. x = 0, ⁻9

335. a = ⁻5

336. q = 11, ⁻12

337. a = ⁻12, 3

338. x = 9, ⁻7

339. x = ⁻3, 11

340. x = 1, 2

341. x = ⁻2, 2

342. x = ⁻3, 8

343. b = ⁻5, 3

344. c = ⁻7, 7

345. x = ⁻1, 10

346. x = ⁻2, 6

347. k = ⁻15, 9

348. p = ⁻11, ⁻4

349. q = ⁻12, 6
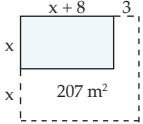
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350. x = ⁻11, 2

351. x = 3

352. $x = \dfrac{-1}{2}, 5$

353. $x = {}^-1, \dfrac{1}{6}$

354. $x = {}^-1\dfrac{1}{2}, \dfrac{1}{4}$

355. $x = 3\dfrac{1}{2}, {}^-1\dfrac{1}{2}$

356. x = ⁻5, 5

357. $k = {}^-1\dfrac{1}{2}, \dfrac{1}{3}$

358. $x = \pm\sqrt{3}$, (±1.732) only

359. $x = \dfrac{1}{25}$ (0.04) only

360. $x = 1\dfrac{1}{2}$ only as x = 1
involves ÷ by 0.

361. $b = \pm 1, \pm\sqrt{\dfrac{5}{4}}$ (±1.118)

362. z = 3 only

363. $p = \sqrt[3]{3}, {}^-\sqrt[3]{6/5}$ or
p = 1.4422, ⁻1.0627 (4 dp)

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364. a) (x + 18)(x + 12) = 432

b) x² + 30x – 216 = 0

c) (x – 6)(x + 36) = 0

d) x = 6 m only
x = ⁻36 is rejected

365. a) (20 – x)(15 – x) = 249.75
x² – 35x + 50.25 = 0
4x² – 140x + 201 = 0

b) (2x – 3)(2x – 67)
x = 1.5, 33.5
Ignore 33.5
Length = 18.5 m
Width = 13.5 m

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366. a)

b) 2x(x + 11) = x(x + 8) + 207
x² + 14x – 207 = 0

c) (x + 23)(x – 9) = 0
x = 9, ⁻23
Ignore ⁻23
Width = 9 m
Length = 17 m

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367. $n(n+2) = 7[n + n + 2] - 1$
$n^2 - 12n - 13 = 0$
$(n - 13)(n + 1) = 0$
$n = 13, n = {}^-1$
Ignore $n = {}^-1$
Numbers 13 and 15.

368. $(x + 3)^2 + x^2 = (2x - 3)^2$
$2x^2 - 18x = 0$
$2x(x - 9) = 0$
$x = 9, 0$
Ignore $x = 0$
Answer:
Lengths 9 cm, 12 cm, 15 cm

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369. $x = 4.562, 0.438$ (3 dp)
370. $x = {}^-0.228, {}^-8.772$ (3 dp)
371. $x = {}^-1, {}^-1.5$
372. $x = 1.212, {}^-3.712$ (3 dp)
373. $x = {}^-0.697, {}^-4.303$ (3 dp)
374. $x = 0.425, {}^-1.175$ (3 dp)

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375. $x = 0.309, {}^-0.809$ (3 dp)
376. $x = {}^-0.775, {}^-3.225$ (3 dp)
377. $x = {}^-1.5$
378. $x = {}^-0.469, {}^-8.531$ (3 dp)
379. $m = 1.644, {}^-0.553$ (3 dp)
380. $y = 2.610, {}^-1.277$ (3 dp)
381. $x = 1.844, {}^-1.844$ (3 dp)
382. $x = 4, {}^-1.5$
383. $p = {}^-0.057, 2.182$ (3 dp)
384. $x = {}^-2.5, 2.333$ (3 dp)
385. $a = 7, {}^-0.5$
386. $q = 2.443, {}^-4.776$ (3 dp)
387. $x = {}^-2, 2$
388. $x = {}^-0.215, 1.549$ (3 dp)

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389. a) $0.70 + 0.10x$
b) $30\,000 - 2\,000x$
c) $y = {}^-200x^2 + 1\,600x + 21\,000$
d) $24\,200 = {}^-200x^2 + 1\,600x + 21\,000$
e) Solve for x to find increase = 40¢ so Price = $1.10

390. a) $x + 12$ and $x + 14$
b) $(x + 14)^2 = (x + 12)^2 + x^2$
c) $x^2 - 4x - 52 = 0$
d) $(x - 2)^2 - 56 = 0$
$x = {}^-5.5, 9.5$ (2 sf)
Length of sides to 2 sf.
are 9.5, 21.5 and 23.5 m.

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391. $\Delta = 0$ one rational root
392. $\Delta = 16$ two rational roots
393. $\Delta = {}^-8$ no real roots
394. $\Delta = 0$ one rational root
395. $\Delta = 49$ two rational roots
396. $\Delta = 37$ two irrational roots
397. $\Delta = 0$ one rational root
398. $\Delta = 0.96$ two irrational roots
399. $\Delta = 169$ two rational roots
400. $\Delta = 8$ two irrational roots
401. $b^2 - 64 = 0$
$b = 8$ or $^-8$

402. $36 - 32a < 0$
$a > 1\frac{1}{8}$ (1.125)

403. $25 - 36p \geq 0$
$36p \geq {}^-25$
$p \leq \frac{25}{36}$ (0.6944)

404. $q^2 - 32 < 0$
$^-\sqrt{32} < q < \sqrt{32}$
$^-5.657 < q < 5.657$

405. $4 - 24k > 0$
$k < \frac{1}{6}$ (0.1667)

406. $36 - 12b < 0$
$b > 3$

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407. $16 - 4p(p + 3) = 0$
$4p^2 + 12p - 16 = 0$
$4(p + 4)(p - 1) = 0$
$p = {}^-4, 1$

408. $\Delta = (2q + 1)^2 - 4q(q + 1)$
$\Delta = 1$ two rational roots

409. $16k^2 - 4(2k + 3)(4) = 0$
$16k^2 - 32k - 48 = 0$
$16(k - 3)(k + 1) = 0$
$k = 3, {}^-1$

410. $(k + 1)^2 - 4(1)({}^-k - 1) = 0$
$k^2 + 6k + 5 = 0$
$(k + 5)(k + 1) = 0$
$k = {}^-5, {}^-1$

411. a, b and c are consecutive arithmetic terms
so $b - a = Z$ and $c - b = Z$
therefore $c - a = 2Z$ and
$a - b$ and $b - c = {}^-Z$.
Discriminant
$(2Z)^2 - 4({}^-Z)({}^-Z) = 0$ so
quadratic has equal roots.

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412. $t^2 - 10t + 11 + k$
Discriminant < 0
$100 - 44 - 4k < 0$
$k > 14$

413. $(p + 3)^2 - 4(p)(p) \geq 0$
$^-3p^2 + 6p + 9 \geq 0$
$^-3(p - 3)(p + 1) \geq 0$
$^-1 \leq p \leq 3$

414. $p({}^-k) = k^2 + 3k + 5$
$p(3k) = 9k^2 - 9k + 5$
Equating
$k^2 + 3k + 5 = 9k^2 - 9k + 5$
$8k^2 - 12k = 0$
$4k(2k - 3) = 0$
Ignoring $k = 0$ so $k = 1.5$

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415. Discriminant must be negative for no roots and below axes so $2q$ must be negative.
$0 > 4(q + 4)^2 - 4.2q.(q + 1)$
$0 > 4q^2 + 32q + 64 - 8q^2 - 8q$
$0 > {}^-4q^2 + 24q + 64$
$0 < 4q^2 - 24q - 64$
$0 < q^2 - 6q - 16$
$0 < (q - 8)(q + 2)$
This is true if $q > 8$ OR $q < {}^-2$.
As $2q$ must be negative then answer $q < {}^-2$.

416. Take logs base 10 of both sides.
$\log(y^{(4x + 1)}) = \log(100y^x)$
$(4x + 1)\log y = \log 100 + \log y^x$
$(4x + 1)\log y = \log 100 + x \log y$
$4x \log y - x \log y = 2 - \log y$
$x(4 \log y - \log y) = 2 - \log y$
$x = \dfrac{2 - \log y}{3 \log y}$
$x = \dfrac{2 - \log y}{\log y^3}$
and $y > 0$, but $y \neq 1$.
When $y > 0$ $\log y$ exists, but if $y = 1$, $\log 1 = 0$ and we cannot divide by zero.

417. Substitute for y in the parabola
$y = 2x^2 - 8x + 4$
$4x + k = 2x^2 - 8x + 4$
$0 = 2x^2 - 12x + 4 - k$
$a = 2, b = {}^-12, c = 4 - k$
$144 - 32 + 8k = 0, k = {}^-14$
$0 = 2x^2 - 12x + 18$
$0 = 2(x - 3)^2$
Intersection is $(3, {}^-2)$.

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418. a) $5000 = 50 (1.095)^T$

$100 = (1.095)^T$

$T = \dfrac{\log 100}{\log 1.095}$

$T = 50.7$ years

$T = 51$ years round up

i.e. in 2031.

b) $500 (1.095)^N = 25 \times 68 (1.030)^N$

$N = 20$ years rounding up

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419. a) $k = 43.527(8)$

$(d + 0.5)^{1.2} = 8.7055$

$d = 5.6$ km

b) $48 = 120 \times 1.0151^{(13 - t)}$

$t = 74.1$ or 74 weeks

c) $t_1 + t_2 = 27.10$

$t_1 = 0.45c$

$t_2 = 5 + 0.015c^2$

$0.45c + 5 + 0.015c^2 = 27.10$

$0.015c^2 + 0.45c - 22.10 = 0$

$c = {}^-56.21, 26.2$

Answer 26 characters

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Practice External Assessment Task

Question One

a) $2x^3 - 5x^2 - 13x + 30$ **A**

b) $\log\left(\dfrac{A^3 B}{\sqrt{C}}\right)$ **A**

c)   i) $x = \dfrac{1}{3}, 1$ **A**

     ii) Discriminant

        $0 = k^2 - 4 \times 3 \times 4$

        $k = \pm 4\sqrt{3}$ (± 6.928)   **E**

d) $x + 3x + (6x - 12) = 128$

     Mary is 72 years   **M**

     A part eqn. or M eqn. & soln.

e)   i) Factorise numerator and
       denominator

        $(x - 4)(x + 1) = 3(x - 4)$

        $x = 4$ **M**

     **A some simplification or
     M complete simplification
     and solution.**

     ii) Both $x = 2$ and $x = {}^-2$
        make the original
        equation undefined.   **E**

Question Two

a) $A = 42.89$ **A**

     $k = 1.118$

     $t = 13.7$ years   **M**

     Sept 2021 or Jan 2022   **E**

     Depends upon whether t
     rounded or not as to the
     month it reaches 200.

b)   i) $x = 1.672$ (4 sf)   **A**

     ii) $x = 5, {}^-2$ **A**

     Excludes $x = {}^-2$ **M**

     as all log expressions

     must be positive.   **E**

c) $2n - 14 > 9n$

       ${}^-7n > 14$

        $n < {}^-2$ **A**

d) $p(x + 2)^2 + q(x + 2) + r$

     $= p(x^2 + 4x + 4) + qx + 2q + r$

     $= px^2 + (4p + q)x + 4p + 2q + r$

     Equating to $2x^2 - 7x - 4$

     $p = 2, q = {}^-15, r = 18$ **M**

Question Three

a)   i) $(18 - 2x)(15 - x) = 162.5$

        $2x^2 - 48x + 107.5 = 0$

        $4x^2 - 96x + 215 = 0$ **A**

     ii) $(2x - 43)(2x - 5)$ **A**

       $x = 2.5, 21.5$

       Ignore 21.5

       Length = 13 m

       Width = 12.5 m   **M**

b) $4x = (x + 1)^2$

     $x^2 - 2x + 1 = 0$

     $x = 1$ **M**

c) $x^2 + ({}^-x - k)^2 = 8$

     $2x^2 + 2kx + k^2 - 8 = 0$

     Discriminant > 0

     $4k^2 - 4 \times 2(k^2 - 8) > 0$

     ${}^-4k^2 + 64 > 0$

     ${}^-4 \le k \le 4$ **E**

     **A correct simplified
     quadratic or M
     discriminant > 0.**

d) $T_1 = a$

     $T_n = a + (n - 1)d$

     $S_n = n \times \left(\dfrac{\text{first} + \text{last}}{2}\right)$

     $S_n = n\left(\dfrac{a + (a + (n - 1)d)}{2}\right)$

     $S_n = \dfrac{n}{2}(a + a + (n - 1)d)$

     $S_n = \dfrac{n}{2}(2a + (n - 1)d)$ **E**

Question Three cont...

e)   25 litres a month at a cost of
     $26   **A**

Sufficiency

**For Question 1 students require
two of A for Achievement or two of
M for Merit or one or two of E for
Excellence.**

**For Question 2 students require
two of A for Achievement or two of
M for Merit or one or two of E for
Excellence.**

**For Question 3 students require
two of A for Achievement or two of
M for Merit or one or two of E for
Excellence.**

Overall students require

**Two or more Achievement
questions or better for overall
Achievement.**

**Two or more Merit questions plus
one Achievement question or
better for overall Merit.**

**Two or more Excellence questions
plus one Achievement question or
better for overall Excellence.**

In the external examinations NZQA
uses a different approach to marking
based on understanding (u),
relational thinking (r) and abstract
thinking (t). They then allocate
marks to these concepts and add
them up to decide upon the overall
grade. This approach is not as easy
for students to self mark as the
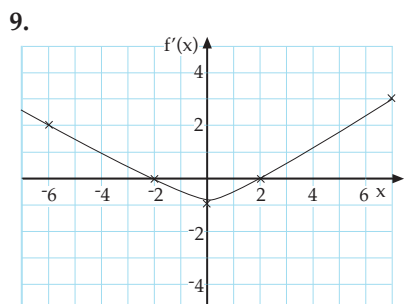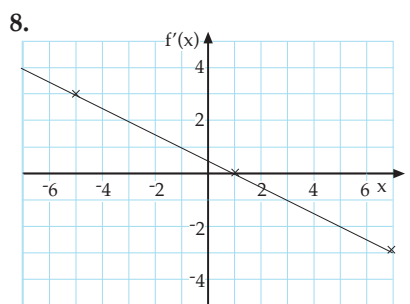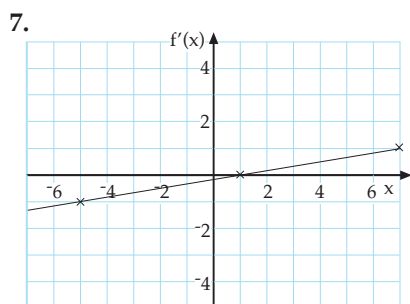NuLake approach, but the results
should be broadly similar.

Answers

Page 68

1. At x = ⁻2, m = ⁻2
   At x = 2, m = 2

2. At x = ⁻4, m = 3
   At x = ⁻1, m = ⁻2
   At x = 1, m = 0
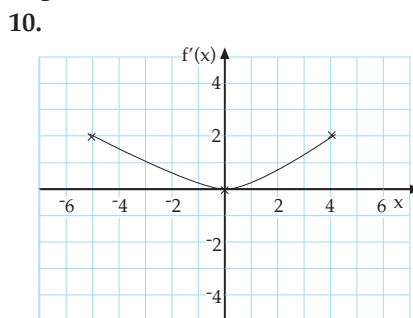
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3. a) m = 1 at (3, 2.5)
   b) m = ⁻2 at (⁻3, 5.5)

4. a) m = 2 at x = ⁻3.5 approx.
      and x = 3.5 approx.
   b) m = 0 at x = ⁻2 and x = 2

5. a) Increasing
      x < ⁻4 or x > ⁻1
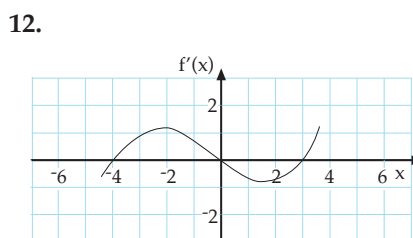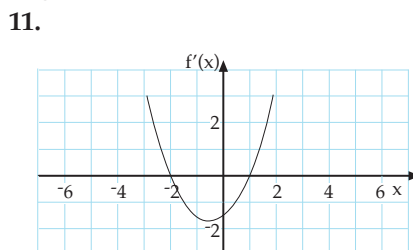   b) Decreasing
      ⁻4 < x < ⁻1

6. At x = ⁻4, ⁻1 and 4

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7.

8.

9.

Page 72 cont...

10.

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11.

12.

13.

Description	Looks like	Gradient function
Increasing	╱	Always +ve
Decreasing	╲	Always –ve
Maximum	⌒	Is zero. Gradient goes from +ve through 0 to –ve
Minimum	⌣	Is zero. Gradient goes from –ve through 0 to +ve

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14. Average rate of change = 4
15. Average rate of change = 3
16. Average rate of change = ⁻2
17. Average rate of change = ⁻4
18. Average rate of change = 4
19. Average rate of change = 3
20. Average rate of change = 2.4
21. Average rate of change = 2.01
22. Average rate of change = 2.001
23. Average rate of change = 2.0001
24. Avg. rate of change = 2.000 01
25. Avg. rate of change = 2.000 001
26. Expect the gradient to be 2

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27. $f'(x) = 2x + 2$
28. $f'(x) = 2x - 6$
29. $f'(x) = 5 - 2x$
30. $f'(x) = 2x + 3$

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31. $f'(x) = 3x^2$
32. $f'(x) = {}^-3$
33. $f'(x) = {}^-4x - 1$
34. $f'(x) = 6x^2 - 1$
35. $f'(x) = 20x$
36. $f'(x) = 8x - 5$
37. $f'(x) = 2x + 1$
38. $f'(x) = 2x - 7$

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39. $f'(x) = 15x^2$
40. $f'(x) = 9$
41. $f'(x) = 0$
42. $f'(x) = 2x + 3$
43. $f'(x) = 4x$
44. $f'(x) = 10x - 10x^4$
45. $f'(x) = 15x^2 + 4x$
46. $f'(x) = 10x + 10$
47. $f'(x) = 55x^{10} - 45x^4$
48. $f'(x) = x - 2$
49. $f'(x) = \dfrac{1}{2}x - \dfrac{1}{5}$
50. $f'(x) = 2x^2 - \dfrac{1}{4}$
51. $f'(x) = 2x^3 - 0.75x^2$
52. $f'(x) = 1.2x^5 + 0.9x^2 - 1.5$
53. $f'(x) = 1.5x^2 + 0.6x - 0.8$
54. $f'(x) = 6x^4 + 7x - 1.4$
55. $f'(x) = \dfrac{3x}{2} - \dfrac{1}{5} - \dfrac{x^2}{2}$
56. $f'(x) = \dfrac{10x^4}{3} - 3x^3 - \dfrac{6x^2}{5} + 8x - 2$
57. $f(x) = x^2 - 2x - 15$
    $f'(x) = 2x - 2$
58. $f(x) = x^3 - 2x^2 + 5x - 10$
    $f'(x) = 3x^2 - 4x + 5$
59. $f(x) = 3x^4 + 5x^3$
    $f'(x) = 12x^3 + 15x^2$
60. $f(x) = x^4 - 2x^2 - 35$
    $f'(x) = 4x^3 - 4x$

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61. $f'(x) = 27x^2 + 2$

62. $f''(x) = 126x^5 - 30x$

63. $\dfrac{dy}{dx} = 5 - 15x^4$

64. $\dfrac{dy}{dx} = 2x$

65. $f''(x) = 60x^3$

66. $\dfrac{d^2y}{dx^2} = 24x - 12$

67. $\dfrac{dy}{dx} = x - \dfrac{9}{4}x^2$

68. $\dfrac{dy}{dx} = {}^-6x^4 + 5x$

69. $f''(x) = \dfrac{9x}{2} - \dfrac{4}{3}$

70. $\dfrac{d^2y}{dx^2} = 192x^2 - 48$

71. $f''(x) = \dfrac{{}^-48x^2}{5} + \dfrac{18x}{5} + \dfrac{8}{3}$

72. $\dfrac{d^2y}{dx^2} = 3 - \dfrac{8x}{3}$

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73. $f'(0) = 2$
74. $f'({}^-2) = {}^-2$
75. $f'({}^-3) = 7$
76. $f'({}^-4) = {}^-4.25$
77. $f'({}^-1) = 2$
78. $f'(4) = 41$
79. $y = x + 1$
80. $y = 2x - 6$
81. $y = {}^-5$
82. $y = {}^-x$
83. $y = {}^-3x - 7$
84. $y = 7x - 4$

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85. Gradient $= 8$
86. Gradient $= {}^-42$
87. $({}^-1, 4)$
88. $(0.5, 1.25)$
89. $(0, {}^-2), (2, 0)$
90. $({}^-3, {}^-1.5), (0, 3)$
91. a) $f'(x) = 0.5x - 1.5$
    b) $f'(1) = {}^-1$ and Grad. $= {}^-1$
         $f'(5) = 1$ and Grad. $= 1$
    c) $y = 6 - x$ for $(1, 5)$
         $y = x$ for $(5, 5)$
    d)   Intersection $(3, 3)$

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92. $f'(x) = 2x + 8$ min. at $({}^-4, {}^-1)$

93. $f'(x) = 2x - 2$ min. at $(1, {}^-4)$

94. $f'(x) = 2x + 1$ min. at $({}^-0.5, 2.75)$

95. $f'(x) = 2x$
    minimum at $(0, {}^-5)$

96. $\dfrac{dy}{dx} = 6 - 2x$
    maximum at $(3, 1)$

97. $f'(x) = 8x - 12$
    minimum at $(1.5, 0)$

98. $\dfrac{dy}{dx} = 6x - 8$
    minimum at $(1.333, 5.667)$ (4 sf)

99. $f'(x) = {}^-4x - 16$
    maximum at $({}^-4, 47)$

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100. $\dfrac{dy}{dx} = x^2 - 4x - 12$
     minimum $(6, {}^-72)$
     maximum $({}^-2, 13.33)$ (4 sf)

101. $f'(x) = 3x^2 - 6x - 9$
     minimum $(3, {}^-17)$
     maximum $({}^-1, 15)$

102. $\dfrac{dy}{dx} = {}^-3x^2 + 6x + 9$
     minimum $({}^-1, {}^-2)$
     maximum $(3, 30)$

103. $f'(x) = 3x^2 - 16x + 5$
     minimum $(5, {}^-52)$
     maximum $(0.333, {}^-1.185)$ (4 sf)

104. $\dfrac{dy}{dx} = 6x^2 + 18x + 12$
     minimum $({}^-1, {}^-10)$
     maximum $({}^-2, {}^-9)$

105. $f'(x) = {}^-3x^2 - 2x + 8$
     minimum $({}^-2, {}^-12)$
     maximum $(1.333, 6.519)$

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106. Turning point $({}^-1, 2)$
     Increasing $x > {}^-1$
     Decreasing $x < {}^-1$

107. Turning point $({}^-3, 13)$
     Increasing $x < {}^-3$
     Decreasing $x > {}^-3$

108. Turning points $({}^-3, {}^-5)$ and
     $({}^-1, {}^-6\frac{1}{3})$.
     Increasing $x < {}^-3$ or $x > {}^-1$
     Decreasing ${}^-3 < x < {}^-1$

109. Turning points $({}^-2, 5\frac{1}{3})$ and
     $(2, {}^-5\frac{1}{3})$.
     Increasing $x < {}^-2$ or $x > 2$
     Decreasing ${}^-2 < x < 2$

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110. Turning points $({}^-4, 51.67)$
     and $(5, {}^-69.83)$
     Increasing $x < {}^-4$ or
     Increasing $x > 5$
     Decreasing ${}^-4 < x < 5$

111. Turning points $({}^-0.5, 2.8)$
     and $(3, {}^-11.5)$
     Increasing $x < {}^-0.5$ or $x > 3$
     Decreasing ${}^-0.5 < x < 3$

112. Turning point $(2, {}^-1)$
     Increasing $x > 2$
     Decreasing $x < 2$

113. Turning point $(1, 2)$
     Increasing $x < 1$
     Decreasing $x > 1$

114. Turning points $(0, 0)$
     and $(\frac{1}{2}, {}^-0.0417)$
     Increasing $x < 0$ or $x > \dfrac{1}{2}$
     Decreasing $0 < x < \dfrac{1}{2}$

115. Turning points $({}^-2, {}^-4)$
     and $(0, 0)$
     Increasing ${}^-2 < x < 0$
     Decreasing $x < {}^-2$ or $x > 0$

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116. Prod. $= x(31 - x)$
     $x = 15.5$
     Max prod. $= 240.25$

117. Area $= w(500 - w)$
     Width $= 250$ m
     Max area $= 62\,500$ metres^2

118. Area $= w(1000 - 2w)$
     Width $= 250$ m
     Height $= 500$ m
     Max area $= 125\,000$ metres^2

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119. Rail: $2x + 2y = 18$
     Area $= {}^-x^2 + 5x + 54$
     $x = 2.5$ m, $y = 6.5$ m
     Area $= 60.25$ m^2

120. Area $= 0.5\,x \times y$
     $x + y = 40$
     Area $= 20x - 0.5x^2$
     Max Area $= 200$ cm^2

121. SA $300 = 4x^2 + 6xh$
        Vol $= 2x^2h$
     width $(x) = 5$ cm
     length $= 10$ cm
     height $= 6\frac{2}{3}$ cm

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122. Link equation

$3y + 2x = 600$

$x = 300 - 1.5y$

Area $= 2xy$

$= 2y(300 - 1.5y)$

$= 600y - 3y^2$

$(Area)' = 600 - 6y$

$y = 100$ m for max

length $= 100$ m

width (x) $= 150$ m

123. $r^2 + h^2 = 22^2$

$V = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi(22^2 - h^2)h$

radius $= 18.0$ cm

height $= 12.7$ cm

124. Link equation

$y = 9 - x^2$

Area $= 2xy$

$= 2x(9 - x^2)$

$= 18x - 2x^3$

$(Area)' = 18 - 6x^2$

$x = \pm\sqrt{3}$

width (2x) $= 3.464$ (4 sf)

height (y) $= 6$

Area $= 20.78$ units^2

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125. Link equation

$\pi x + 4x + 2y = 5$

$y = 0.5(5 - x(\pi + 4))$

Area $= 2xy$

$= x(5 - x(\pi + 4))$

$= 5x - x^2\pi - 4x^2$

$(Area)' = 5 - 2x\pi - 8x$

$x = 0.35$ m for max

height $= 1.25$ m

width (2x) $= 0.70$ m

126. $y_2 - y_1 = D$

$D = (x + 2)(6 - x) - ((x - 1)^2 - 9)$

$D = -x^2 + 4x + 12 - x^2 + 2x + 8$

$D = -2x^2 + 6x + 20$

$D' = -4x + 6$

$D' = 0$ for max / min

$x = 1.5$

$D = 24.5$ units

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127. Corner x units long

$V = x(600 - 2x)(400 - 2x)$

$V = 240\,000x - 2000x^2 + 4x^3$

$V' = 240\,000 - 4000x + 12x^2$

$V' = 0$ for max / min

$x = 78.47, 254.85$

Max $x = 78.5$ mm (3 sf) gives

Vol. $= 8\,450\,000$ mm^3 (3 sf)

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128. a) $v(t) = 60 - 10t$ m/s

b) $a(t) = -10$ m/s^2

c) $a(4) = -10$ m/s^2

129. a) $s(0) = 0$ m

$s(6) = -144$ m

$s(12) = 0$ m

b) $v(0) = -48$ m/s

$v(6) = 0$ m/s

$v(12) = 48$ m/s

130. a) $s(0) = 9$ m

b) $s(2) = 1$ m

$s(3) = 0$ m

$s(6) = 9$ m

c) $v(3) = 0$ m/s

$v(6) = 6$ m/s

d) $a(1) = 2$ m/s^2

131. a) $h(t) = 0$

$t = -10$ and 50 seconds

b) $v(5) = 30$ m/s

c) $v(t) = 0$

$t = 20$ seconds

d) $h(20) = 900$ metres

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132. a) $V'(t) = 12 - 1.6t$

$V'(5) = 4$ m^3/h

b) $12 - 1.6t = 6$

$t = 3.75$ hours

133. a) $v(t) = -360 + 12t$

$v(15) = -180$ m/s

$v(30) = 0$ m/s

$v(40) = 120$ m/s

b) min. when $v(t) = 0$ so

$t = 30$ s

$s(30) = 14\,600$ m

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134. a) $s = 4$ m

b) $t = 1$ second (and $t = 4$).

c) 7 m/s

d) Acceleration $= 2$ m/s^2 for $t = 1, 2$ and 3.

Constant acceleration.

135. a) $s = 30$ m

b) $v = 3$ m/s

c) $a = 0$ m/s^2

d) $t = 1$ second

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136. $V' = 4\pi r^2$ and at $r = 15$ mm

$V' = 2800$ mm^3 per mm increase (2 sf).

137. $H' = 2 - d$ and at $d = 3.5$ m

$H' = -1.5$ m per horizontal m.

138. $V = 6w^3$

$V' = 18w^2$ and at $w = 0.25$ m

$V' = 1.1$ m^3 drop per m decrease (2 sf).

139. $Vol. = \dfrac{\pi d^3}{48\,000}$

$V' = \dfrac{\pi d^2}{16\,000}$ and at $d = 75$ m

$V' = 1.1$ m^3 increase per m travelled (2 sf).

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140. $f(x) = x^3 - 6x^2 + 10x + C$

141. $f(x) = 3x^4 - 2x^3 - 5x + C$

142. $f(x) = 4x^5 - 4x^3 + C$

143. $f(x) = \dfrac{1}{3}x^3 + \dfrac{3}{2}x^2 + 2x + C$

144. $f(x) = 3x^3 + 3x^2 + x + C$

145. $f(x) = \dfrac{1}{6}x^3 - \dfrac{3}{8}x^2 + 5x + C$

146. $f(x) = \dfrac{2x^5}{15} + \dfrac{x^3}{5} - \dfrac{x^2}{8} + x + C$

147. $f(x) = \dfrac{x^4}{4} - \dfrac{5x^3}{3} + \dfrac{7x^2}{2} - 3x + C$

148. $f(x) = \dfrac{x^5}{25} - \dfrac{x^3}{6} - \dfrac{x^2}{6} - 8x + C$

149. $f(x) = \dfrac{3x^5}{20} + \dfrac{5x^3}{9} + \dfrac{7x^2}{4} - 4x + C$

150. $f(x) = 0.5x^5 - 0.8x^4 + 0.5x^3 - 6x + C$

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151. $f(x) = 0.214x^7 - 1.53x^6 + 3x + C$

152. $f(x) = 2x^7 - x^6 + 9x + C$

153. $f(x) = \dfrac{^-3x^4}{10} - \dfrac{4x^3}{9} - \dfrac{2x^2}{5} + C$

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154. $f(x) = 6x^2 - 5x - 3$

155. $f(x) = 4x - 2x^2 + 16$

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156. $y = 3x - \dfrac{x^2}{2} - 4$

157. $f(x) = x - x^3 - 1$

158. $f(x) = x^3 - x^2 + x + 2$

159. $f(x) = x - \dfrac{x^3}{3} + \dfrac{x^2}{2}$

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160. $f(x) = x^3 - \dfrac{9x^2}{2} - 18x + 20$

161. $y = 2x + 3x^2 - \dfrac{x^3}{3} + 2\dfrac{2}{3}$

162. $f(x) = x^4 - 6x^3 + 12x^2 - 10x + 3$

163. $f(x) = x^4 - 5x^3 + 5x^2 + 5x - 6$

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164. $0 = 4 - 2x$ for Max/Min.
Max. at $x = 2$

$f(x) = 4x - x^2 + C$
through (2, 7)

$f(x) = 4x - x^2 + 3$

165. $0 = 4x + 12$ for Max/Min.
Min. at $x = ^-3$.

$f(x) = 2x^2 + 12x + C$
through ($^-3$, 0)

$f(x) = 2x^2 + 12x + 18$

166. $f'(x) = k(x - 1)(x - 3)$

$f'(0) = 3$ gives $k = 1$

$f'(x) = x^2 - 4x + 3$

$f(x) = \dfrac{x^3}{3} - 2x^2 + 3x$

167. $f'(x) = k(x + 2)(x - 3)$

$f'(0) = 12$ gives $k = ^-2$

$f'(x) = ^-2(x^2 - x - 6)$

$f'(x) = 12 + 2x - 2x^2$

$f(x) = 12x + x^2 - \dfrac{2x^3}{3} + C$

through (0, 2)

$f(x) = 12x + x^2 - \dfrac{2x^3}{3} + 2$

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168. a) $3t^2 - 12 = 0$, $t = 2$ seconds ($t = ^-2$)

b) $s(t) = t^3 - 12t + 2$

c) $a(t) = 6t$. After 3 seconds
$a(3) = 18$ m/s^2

169. a) $v(t) = t^2 + t + 6$

b) $s(t) = \dfrac{1}{3}t^3 + \dfrac{1}{2}t^2 + 6t + 5$
$s(2) = 21.67$ m (4 sf)

170. a) $a(t) = 140 - 14t$ When $t = 0$, $a = 140$ m/s^2

b) $s(t) = 70t^2 - \dfrac{7}{3}t^3 + C$
as $s(t) = 0$ when $t = 0$
$s(t) = 70t^2 - \dfrac{7}{3}t^3$
$s(1) = 67.67$ m (4 sf)

c) $v(t) = 0$ when $t = 0$ and 20 seconds.
$s(20) = 9333$ m (4 sf)

171. a) When $v = 0$, $t = 3$

$s(t) = 30t - 5t^2 + 2$

$s(3) = 47$ m

b) $s(0) = 2$, $s(2) = 42$
therefore 40 m in the first 2 seconds.

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172. a) 15 cm/s

b) $a(t) = 6t - 18$
$t = 3$ seconds

c) $s(t) = t^3 - 9t^2 + 15t + 150$

d) $3t^2 - 18t + 15 = 0$
$t = 1$ and $t = 5$ seconds

e) $s(5) = 125$ cm

f) $s(1) = 157$ cm

173. a) 34 m/s

b) $a = ^-6.8$ m/s^2

c) $v(t) = 0$ at $t = 5$ seconds

d) $h(t) = 34t - 3.4t^2$

e) $s(5) = 85$ m

f) $s(4) = 81.6$ m

g) $s(t) = 0$ so
$t = 10$ seconds

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174. a) $v(t) = 2t^2 - t + 3$

b) $v(3) = 18$ m/s

c) $s(t) = \frac{2}{3}t^3 - \frac{1}{2}t^2 + 3t + 4$

d) $s(6) = 148$ m

e) Dist. $= s(3) - s(2)$
$= 13.17$ m (4 sf)

f) $s(0) = 4$, $s(3) = 26.5$ m
velocity $= 7.5$ m/s

175. a) $a(t) = 2t - 14$
$a(3) = {}^-8$ m/s

b) $s(t) = \frac{1}{3}t^3 - 7t^2 + 40t + 20$

c) $v(t) = 0$ at
$t = 4, 10$
$s(10) = 53.3$ m

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176. a) $a(t) = 0$
$t = 40$ s

b) $v(t) = 8t - 0.1t^2 + 0$ (from plane)
$v(40) = 160$ m/s

c) $s(t) = 4t^2 - 0.0333t^3 + 0$ (from plane)
$s(40) = 4270$ m (3 sf)

177. a) $f'(x) = 2ax + b$ at $x = 2$ $f'(2) = 0$
$0 = 4a + b$
$b = {}^-4a$
$f(x) = ax^2 - 4ax - 3$
$3 = 4a - 8a - 3$
$a = {}^-1.5$ and $b = 6$

b) For the parabola to have a maximum 'a' must be negative.
$3 = 4a - 8a + c$ for $a > 0$
$3 - c > 0$
$c < 3$

178. a) $f'(x) = 6(x - 2)(x - 5)$
At turning points $f'(x) = 0$ giving
$x = 2, 5$

b) $f(x) = 2x^3 - 21x^2 + 60x + C$
through (5, 26)
$26 = 2 \times 125 - 21 \times 25 + 60 \times 5 + C$
$C = 1$
$f(x) = 2x^3 - 21x^2 + 60x + 1$
At $x = 2$
Maximum value (2, 53)

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179. a) $a(t) = {}^-12.5$ m/s^2
$v(t) = {}^-12.5t + C$ and (2, 10)
$10 = {}^-25 + C$
$v(t) = {}^-12.5t + 35$

b) $s(t) = {}^-6.25t^2 + 35t + 0$ as distance from $t = 0$.
$v(t) = 0$ at $t = 2.8$ s
Max. distance therefore 49 m.

180. a) $V = x(200 - 2x)(300 - 2x)$
$V = 4x^3 - 1000x^2 + 60\,000x$

b) $V' = 12x^2 - 2000x + 60\,000$
which is equal to 0 when
$x = 39.2$ and 127.4 mm
Discard 127.4 as it is over half the 200 mm of one side.
$V(max) = 1\,060\,000$ mm^3 (3 sf)

181. a) $C'(v) = {}^-0.25 + 0.003\,333v$
Set equal to 0 for max / min
$v = 75$ km/h

b) $C(v) = 2.625$ litres/100 km at minimum
Petrol $= 5.17 \times 2.625$
$= 13.57$ litres
Cost $= \$29.58$

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182. a) Diameter $= 26$, w and h on diagram.

b) $26^2 = w^2 + h^2$
$S = 10wh^2$
$S = 10w(26^2 - w^2)$
$S' = 6760 - 30w^2$
$S' = 0$ for max gives
$w = 15.0$ cm and $h = 21.2$ cm (3 sf)

183. a) Area $= 0.5 \times$ base $\times$ height
Area $= 0.5xy$
Area $= 6x + 2x^2 - 0.5x^3$

b) $A' = 6 + 4x - 1.5x^2$
which is equal to 0 when
$x = 3.737$ (ignore negative answer).
Area $= 24.3$ units^2

184. a) $f'(x) = x^3 - 6x^2 + 8x$
$f'(x) = 0$ when
$x = 0, 2$ or 4

b)

x	-1	0	1	2	3	4	5
f′(x)	–ve	0	+ve	0	–ve	0	+ve
grad.	＼	min	／	max	＼	min	／

Increasing $0 < x < 2$ or $x > 4$.

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Practice External Assessment Task

Question One

a) i) $f'(x) = 3x^2 - 4$

         $f'(2) = 8$ **A**

   ii) $\dfrac{dy}{dx} = 3 - 6x - 3x^2$

         $m = 6$ **(A)**

         $y = 6x + 5$ **M**

b) i) $h'(x) = {}^-x^2 + 3x - 2.$

         Turning points $h'(x) = 0$
         $x = 1, 2$

         Minimum point (1, 1.667)
         Maximum point (2, 1.833)   **(M)**

         Difference = 0.1667 m
                  = 16.7 cm (3 sf)
         which is less than the required 20 cm.   **E**

   ii)   Starts 2.5 m above the ground.
         Hollow is across 1 m and up 1.667 m.   **(A)**
         Lip is across 2 m and up 1.883 m.
         End is across 3 m and 1 m above the ground.
                                         M

c) $f'(x) = 3x^2 - 6x - 6$

         $3 = 3x^2 - 6x - 6$

         $x = {}^-1, 3$

         $({}^-1, 10)$ and $(3, {}^-10)$ **M**

d) i) $C'(s) = \dfrac{2s}{3} - 12$

         $s = 18$ km/h   **A**

   ii) $C_{min} = \$117$ per h

         $t = \dfrac{54}{18}$

           = 3 hours

         Cost = \$351   **E**

Question Two

a) $f'(x) = 6x^3 - 3x^2 + x$

         $f'(2) = 38$ **A**

b) $f'(x) = 6x^4 - 15x^2 - 7$

         $f'({}^-2) = 29$ **A**

c) $f(x) = 1.5x^4 - 2x^3 - 4x^2 + 5x + C$

         $f(x) = 1.5x^4 - 2x^3 - 4x^2 + 5x - 2$ **M**

d) $f'(x) = 4x^3 - 12x^2 - 4x + 12$

         $0 = x^3 - 3x^2 - x + 3$

         $x = {}^-1, 1, 3$

         Min. at $({}^-1, {}^-7)$ and $(3, {}^-7)$ and max. at (1, 9)
         Demonstrate max and min with Calc.   **E**

e) i) $w'(t) = 30t^2 - 570t + 2100$

         $w'(8) = {}^-540$ kg/day   **M**

Question Two cont...

   ii) $w'(t) = 30t^2 - 570t + 2100$

         $0 = t^2 - 19t + 70$

         $t = 5, 14$ days

         Min. (14, 1100)   **(M)**

         Demonstrate this is a minimum.   **E**

Question Three

a) i) $f'(x) = 3x^2 + 3x - 6$

         $f'(1) = 0$ **(A)**

         $x = {}^-2$

         $({}^-2, 12)$ is a max. and $(1, {}^-1.5)$ is a min.   **M**

   ii)   Function is increasing before the min. and
         after the max. points so $x < {}^-2$ or $x > 1$ **M**

b) i) $f(x) = x^3 - 4.5x^2 - 12x + C$

         $f(x) = x^3 - 4.5x^2 - 12x + 2$ **A**

   ii) $f'(x) = 3x^2 - 9x - 12$

         $0 = x^2 - 3x - 4$

         $x = {}^-1, 4$

         Min. $(4, {}^-54)$ **M**

c) i) $a(t) = 6t - 24$

         $a(2) = {}^-12$ m/s²   **A**

   ii)   Flying back towards the radar station when
         velocity is negative, i.e. $2 < t < 6$ seconds. **E**

   iii) $s(t) = t^3 - 12t^2 + 36t + C$

         through (5, 10) gives C = 5 m.   **(M)**

         Furthermost point at $t = 2$ where $s = 37$ m
                                         E

Judgement

The grade in brackets is an alternative if the full grade is not earned. In each question the student needs at least 2 **A** for an **Achievement,** 2 **M** for a **Merit** and 1 **E** plus **1 M** for **Excellence.**

The final grade is found by combining the results of the three questions.

Achievement

Requires two question **Achievements** or better.

Merit

Requires two question **Merits** or better.

Excellence

Requires two question **Excellences.**

NZQA External Examination

In the external examinations NZQA uses a different approach to marking based on understanding (u), relational thinking (r) and abstract thinking (t). They then allocate marks to these concepts (maximum of 8 for a question) and add them up to decide upon the overall grade. This approach is not as easy for students to self mark as the NuLake approach, but the results should be broadly similar.

Answers

Page 133

1. a) P(dissat.) $= \dfrac{60}{290} = 0.207$ (3 dp)

b) P(civil) $= \dfrac{155}{290} = 0.534$ (3 dp)

c) P(civil and dissatisfied)

$= \dfrac{30}{290} = 0.103$ (3 dp)

d) P(satisfied given elect.)

$= \dfrac{57}{75} = 0.76$

e) P(satis. given chem or elect.)

$= \dfrac{105}{135} = 0.778$ (3 dp)

2. a) P(low cal.) $= \dfrac{28}{75} = 0.373$ (3 dp)

b) P(medium fibre)

$= \dfrac{34}{75} = 0.453$ (3 dp)

c) P(low fibre and calories)

$= \dfrac{15}{75} = 0.2$

d) P(medium or high fibre)

$= \dfrac{52}{75} = 0.693$ (3 dp)

e) P(high cal. given low in fibre)

$= \dfrac{8}{23} = 0.348$ (3 dp)

f) P(low cal. given high in fibre)

$= \dfrac{3}{18} = 0.167$ (3 dp)

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3. a) P(under 25 one or more)

$= \dfrac{81}{256} = 0.316$ (3 dp)

b) P(3+ claims and 25 or over)

$= \dfrac{21}{38} = 0.553$ (3 dp)

c) P(1-2 claims and under 25)

$= \dfrac{64}{176} = 0.364$ (3 dp)

Page 134 cont...

4. a) P(migrant went to Aust.)

$= \dfrac{49\ 016}{113\ 362} = 0.432$ (3 dp)

b) P(Pacific | 2015)

$= \dfrac{1\ 756}{57\ 396} = 0.031$ (3 dp)

c) P(2015 | Australia)

$= \dfrac{25\ 246}{49\ 016} = 0.515$ (3 dp)

5. a) P(takes biology)

$= \dfrac{46}{72} = 0.639$ (3 dp)

b) P(physics takes chemsitry)

$= \dfrac{13}{29} = 0.448$ (3 dp)

c) P(biology takes chemistry)

$= \dfrac{16}{46} = 0.348$ (3 dp)

d) P(bio. takes phy. & chem.)

$= \dfrac{5}{46} = 0.109$ (3 dp)

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6. a) P(winning home game)

$= \dfrac{21}{24} = 0.875$

b) P(if won was away game)

$= \dfrac{8}{29} = 0.276$ (3 dp)

c) P(if lost was home game)

$= \dfrac{3}{13} = 0.231$ (3 dp)

7. a) P(under 40 years of age)

$= \dfrac{55}{160} = 0.344$ (3 dp)

b) P(given female ≥ 40)

$= \dfrac{65}{85} = 0.765$ (3 dp)

c) P(given < 40 male)

$= \dfrac{35}{55} = 0.636$ (3 dp)

Page 135 cont...

8. a) P(non-smoker)

$= \dfrac{40}{100} = 0.4$

b) P(cancer or heart disease)

$= \dfrac{52}{100} = 0.52$

c) P(can. or hrt. given smokes)

$= \dfrac{43}{60} = 0.717$ (3 dp)

d) P(can. or hrt. given heavy)

$= \dfrac{28}{35} = 0.80$

e) P(heavy died can. or hrt.)

$= \dfrac{28}{100} = 0.28$

9. a) P(hypertensive)

$= \dfrac{44}{220} = 0.2$

b) P(male given hypertensive)

$= \dfrac{40}{77} = 0.519$ (3 dp)

c) P(normal given female)

$= \dfrac{53}{106} = 0.5$

Page 136

10. a)

	M	F	Total
Coffee	65	140	205
Tea	70	50	120
Other	15	60	75
Total	150	250	400

b) P(male) $= \dfrac{150}{400} = 0.375$

c) P(orders coffee)

$= \dfrac{205}{400} = 0.513$ (3 dp)

d) P(orders tea given male)

$= \dfrac{70}{150} = 0.467$ (3 dp)

e) P($\neq$ tea or coff. given female)

$= \dfrac{60}{250} = 0.24$

Page 136 Q10 cont...

f) P(coffee given a male)

$= \dfrac{65}{150} = 0.433$ (3 dp)

g) P(man given other beverage)

$= \dfrac{15}{75} = 0.2$

11. a) Non-smokers = 210

b) Proportion = 0.475

c) P(smks. and parents non-smk)

$= \dfrac{3}{40} = 0.075$

d) P(fam. smk. given non-smk.)

$= \dfrac{8}{21} = 0.381$ (3 dp)

e) From homes where both parents smoke 60% of individuals ended up being smokers. From homes where one parent smokes only 50% of individuals ended up smoking and from homes where no parent smoked only 25% of individuals ended up smoking. So it appears it is true.

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12. a)

		Red				
		1	2	3	4	5	6
W	1	1,1	1,2	1,3	1,4	1,5	1,6
h	2	2,1	2,2	2,3	2,4	2,5	2,6
i	3	3,1	3,2	3,3	3,4	3,5	3,6
t	4	4,1	4,2	4,3	4,4	4,5	4,6
e	5	5,1	5,2	5,3	5,4	5,5	5,6
	6	6,1	6,2	6,3	6,4	6,5	6,6

b) P(diff.) $= \dfrac{30}{36} = 0.833$ (3 dp)

c) P(R < W) $= \dfrac{15}{36} = 0.417$ (3 dp)

d) P(= 3) $= \dfrac{11}{36} = 0.306$ (3 dp)

e) P(diff = 2) $= \dfrac{8}{36} = 0.222$ (3 dp)

f) P(W fac. R) $= \dfrac{14}{36} = 0.389$ (3 dp)

g) P(tot. ≤ 9) $= \dfrac{30}{36} = 0.833$ (3 dp)

h) P(= 2) $= \dfrac{10}{36} = 0.278$ (3 dp)

Page 139 cont...

13. a)

		Red			
Y		1	2	3	4
e	1	1,1	1,2	1,3	1,4
l	2	2,1	2,2	2,3	2,4
l	3	3,1	3,2	3,3	3,4
o					
w	4	4,1	4,2	4,3	4,4

b) P(= 1) $= \dfrac{6}{16} = 0.375$

c) P(both 1) $= \dfrac{1}{16} = 0.0625$

d) P(both diff.) $= \dfrac{12}{16} = 0.75$

e) P(sum = 5) $= \dfrac{4}{16} = 0.25$

f) P(diff = 2) $= \dfrac{4}{16} = 0.25$

g) P(R > Y) $= \dfrac{6}{16} = 0.375$

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14. a) P(sec. male) $= \dfrac{1}{2} = 0.5$

b) P(both male) $= \dfrac{1}{4} = 0.25$

c) P(both same) $= \dfrac{1}{2} = 0.5$

d) P(third fem.) $= \dfrac{1}{2} = 0.5$

15. a) P(J clubs) $= \dfrac{1}{52} = 0.019$ (3 dp)

b) P(black card) $= \dfrac{26}{52} = 0.5$

c) P(= 5) $= \dfrac{4}{52} = 0.077$ (3 dp)

d) P(J, Q, K) $= \dfrac{12}{52} = 0.231$ (3 dp)

e) P(J giv. bl) $= \dfrac{1}{26} = 0.038$ (3 dp)

f) P(J given J, Q, K)

$= \dfrac{4}{12} = 0.333$ (3 dp)

16. a) P(dark) $= \dfrac{8}{15} = 0.533$ (3 dp)

b) P(first two dark)

$= \dfrac{56}{210} = 0.267$ (3 dp)

Page 140 Q16 cont...

c) P(first dark, sec milk)

$= \dfrac{56}{210} = 0.267$ (3 dp)

d) P(dark, milk) $= \dfrac{7}{14} = 0.5$

e) P(dark, milk any order)

$= \dfrac{8}{15} = 0.533$ (3 dp)

17. a) P(both green)

$= \dfrac{1}{64} = 0.016$ (3 dp)

b) P(first blue, sec red)

$= \dfrac{12}{64} = 0.188$ (3 dp)

c) P(R, G in any order)

$= \dfrac{6}{64} = 0.094$ (3 dp)

d) P(both blue)

$= \dfrac{12}{56} = 0.214$ (3 dp)

e) P(red, green any order)

$= \dfrac{6}{56} = 0.107$ (3 dp)

18. 0.6 x 0.5 = 0.30
60% will not be fine and 50% of these not fine days will have heavy rain.

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19. a) P(blood type O)

$= \dfrac{60}{200} = 0.30$

b) P(blood type A or B)

$= \dfrac{125}{200} = 0.625$

c) P(not AB) $= \dfrac{185}{200} = 0.925$

d) P(both O)

$= \dfrac{177}{1990} = 0.089$ (3 dp)

20. a) P(both miss) = 0.08

b) P(both hit) = 0.48
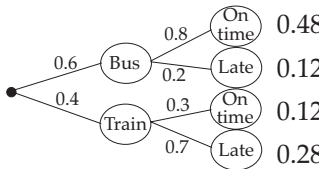
c) P(only one hits)
= (0.8 x 0.4) + (0.2 x 0.6)
= 0.44
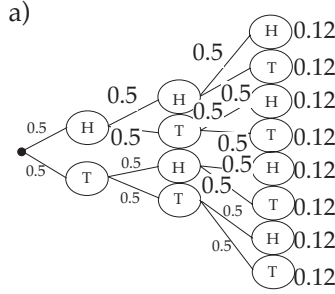
d) P(hits only once)
= (0.8 x 0.2) + (0.2 x 0.8)
= 0.32

Page 141 cont...

21. a) P(both defective)

$$= \frac{1}{300} = 0.003 \ (3 \text{ dp})$$

b) P(both okay)

$$= \frac{253}{300} = 0.843 \ (3 \text{ dp})$$

c) P(one defective)

$$= \frac{23}{150} = 0.153 \ (3 \text{ dp})$$
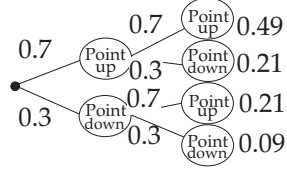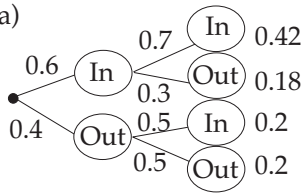
22. a) P(all green)

$$= \frac{1}{56} = 0.018 \ (3 \text{ dp})$$

b) P(two red, third green)

$$= \frac{1}{56} = 0.018 \ (3 \text{ dp})$$

c) P(none are white)

$$= \frac{5}{28} = 0.179 \ (3 \text{ dp})$$

d) P(only red is the last one)

$$= \frac{5}{28} = 0.179 \ (3 \text{ dp})$$

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23. a)

Bus 0.6 → On time 0.8 → 0.48
Bus → Late 0.2 → 0.12
Train 0.4 → On time 0.3 → 0.12
Train → Late 0.7 → 0.28
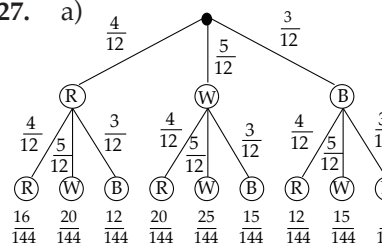
b) P(bus and late) = 0.12

c) P(train on time) = 0.12

d) P(on time bus or train)

= 0.6

24. a)

Tree with all branches 0.5, giving 8 outcomes each 0.125.
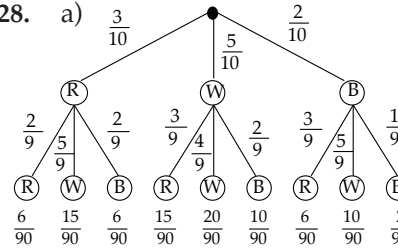
b) P(3H) = 0.125

c) P(2H1T) = 0.375

d) P(1H+) = 0.875

Page 143 cont...

25. a)

Point up 0.7 → Point up 0.7 → 0.49
Point up → Point down 0.3 → 0.21
Point down 0.3 → Point up 0.7 → 0.21
Point down → Point down 0.3 → 0.09
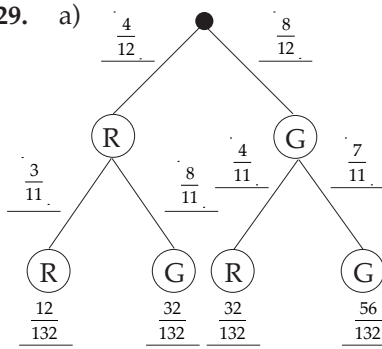
b) P(one pt. up) = 0.3

c) P(two pt. up) = 0.49

d) P(two pt. down) = 0.09

e) P(1 pt. up 1 pt dn.) = 0.42

f) 5 drawing pins

26. a)

In 0.6 → In 0.7 → 0.42
In → Out 0.3 → 0.18
Out 0.4 → In 0.5 → 0.2
Out → Out 0.5 → 0.2

b) P(Both in) = 0.42

c) P(Both miss) = 0.2

d) P(1+ in) = 0.8

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27. a)

Tree: R ($\frac{4}{12}$), W ($\frac{5}{12}$), B ($\frac{3}{12}$)

From R: R $\frac{16}{144}$, W $\frac{20}{144}$, B $\frac{12}{144}$
From W: R $\frac{20}{144}$, W $\frac{25}{144}$, B $\frac{15}{144}$
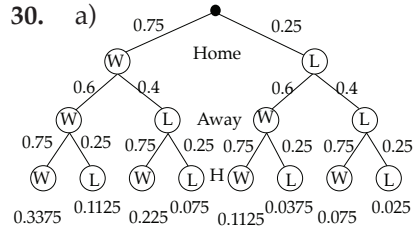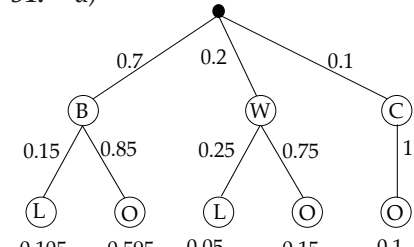From B: R $\frac{12}{144}$, W $\frac{15}{144}$, B $\frac{9}{144}$

b) $P(R \text{ and } B) = \frac{24}{144}\left(\frac{1}{6}\right)$

$= 0.167$

c) P(2 same)

$$= \frac{16}{144} + \frac{25}{144} + \frac{9}{144}$$
$$= \frac{25}{72}$$
$$= 0.347 \ (3 \text{ dp})$$

28. a)

Tree: R ($\frac{3}{10}$), W ($\frac{5}{10}$), B ($\frac{2}{10}$)

From R: R $\frac{6}{90}$, W $\frac{15}{90}$, B $\frac{6}{90}$
From W: R $\frac{15}{90}$, W $\frac{20}{90}$, B $\frac{10}{90}$
From B: R $\frac{6}{90}$, W $\frac{10}{90}$, B $\frac{2}{90}$

b) $P(R \text{ and } B) = \frac{12}{90}\left(\frac{2}{15}\right)$

$= 0.133 \ (3 \text{ dp})$

Page 144 Q28 cont...

c) P(2 same)

$$= \frac{28}{90}\left(\frac{14}{45}\right) = 0.311 \ (3 \text{ dp})$$

29. a)

Tree: R ($\frac{4}{12}$), G ($\frac{8}{12}$)

From R: R $\frac{3}{11}$, G $\frac{8}{11}$ → R $\frac{12}{132}$, G $\frac{32}{132}$
From G: R $\frac{4}{11}$, G $\frac{7}{11}$ → R $\frac{32}{132}$, G $\frac{56}{132}$

b) P(both green)

$$= \frac{56}{132} = 0.424 \ (3 \text{ dp})$$

c) P(RG or GR)

$$= \frac{64}{132} = 0.485 \ (3 \text{ dp})$$

d) P(sec. red given first red)

$$= \frac{3}{11} = 0.273 \ (3 \text{ dp})$$

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30. a)

Home → W 0.75, L 0.25
W 0.6 → W 0.75 (0.3375), L 0.25 (0.1125)
W 0.4 → W 0.75 (0.225), L 0.25 (0.075)
(Away) L 0.6 → W 0.75 (0.1125), L 0.25 (0.0375)
L 0.4 → W 0.75 (0.075), L 0.25 (0.025)

b) P(win 3) = 0.3375

c) P(win 2) = 0.1125 + 0.225 + 0.1125

= 0.45

d) P(Home given 2 games)

$$= \frac{0.225}{0.45} = 0.5$$

31. a)

Root → B 0.7, W 0.2, C 0.1
B → L 0.15 (0.105), O 0.85 (0.595)
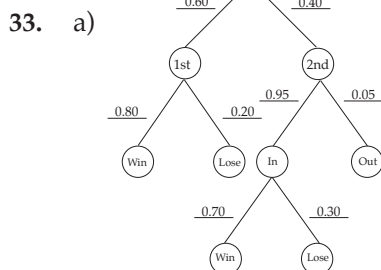W → L 0.25 (0.05), O 0.75 (0.15)
C → O 1 (0.1)

Page 145 Q31 cont...

b) P(late) $= 0.105 + 0.05$

$= 0.155$

c) P(bike given late)

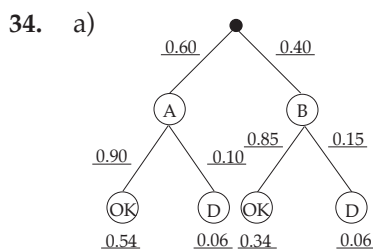$= \dfrac{0.105}{0.155} = 0.677$

d) Days $= 27.9$

$= 28$ days

32. a)
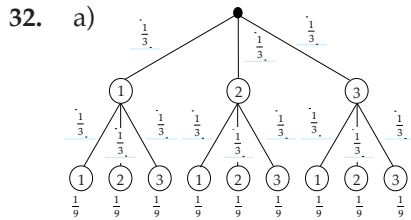
b) P(sum even)

$= \dfrac{5}{9} = 0.556$ (3 dp)

c) P(sum ≥ 4)

$= \dfrac{6}{9} = 0.667$ (3 dp)

d) P(sum even given first 1)

$= \dfrac{2}{3} = 0.667$ (3 dp)

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33. a)

b) P(double faults) $= 0.02$

c) P(loses on first serve) $= 0.12$

d) P(wins point) $= 0.746$
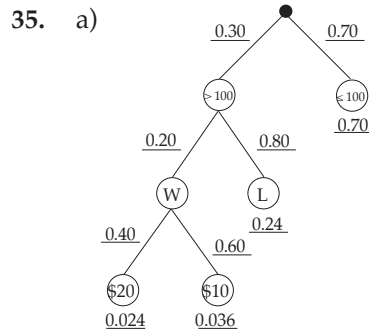
e) P(given wins on 2nd serve)
$= 0.357$ (3 dp).

34. a)

b) P(B and defective) $= 0.06$

Page 146 Q34 cont...
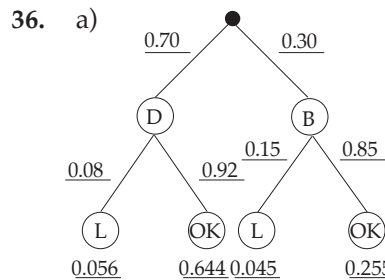
c) P(defective) $= 0.12$

d) P(given defective prod. A)

$= 0.5$

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35. a)

b) P(not win) $= 0.94$

c) P($20 voucher) $= 0.024$

d) P($10 vouch. given wins)
$= 0.036 \div 0.06 = 0.6$

36. a)

b) P(bike and gets flat) $= 0.045$

c) P(late) $= 0.101$

d) P(car breaks down given
late) $= 0.056 \div 0.101 = 0.554$

e) P(late two successive days)
$= 0.101^2 = 0.010$ (3 dp)

Page 150

37. a) P(dying)

$= \dfrac{215}{605} = 0.355$ (3 dp)

b) P(< 50 dies)

$= \dfrac{180}{320} = 0.563$ (3 dp)

c) P(≥ 50 dies)

$= \dfrac{35}{285}$ 0.123 (3 dp)

d) RR $= \dfrac{\frac{180}{320}}{\frac{35}{285}} = 4.6$

e) A person under 50 who
contracts the disease is 4.6
times more likely to die
within one year of
diagnosis.

Page 150 cont...

38. a) P(sex content)

$= \dfrac{150}{350} = 0.429$ (3 dp)

b) P(comedy contain sex)

$= \dfrac{70}{180} = 0.389$ (3 dp)

c) P(drama contain sex)

$= \dfrac{80}{170} = 0.471$ (3 dp)

d) RR $= \dfrac{\frac{70}{180}}{\frac{80}{170}} = 0.83$

e) A comedy show is 0.83
times less likely than
a drama show to contain
sexual content.

Page 151

39. a) Absolute risk $= 0.0003$

b) RR $= 4$

c) RR $= 0.25$

d) That the relative risk in
Russia of birth deformities
is four times greater if you
are situated in a town near
a nuclear facility.

40. a) Absolute risk $= 0.125$

b) RR $= 0.9$

c) RR $= 1.1$

d) That you are 1.1 times
more likely to pass the
course by tutoring
than CAI. Assuming the
figures calculated
are unbiased,
i.e. more brighter
students may decide to
study the course by tutoring.

41. a) Absolute risk $= 0.2$

b) RR $= 1$. No difference
between the two factories.

c) RR $= 1.5$,
i.e. 1.5 times more likely to
have an accident in factory
two than factory one.

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42. a) Absolute risk $= 0.36$

b) RR $= 6.6$

c) RR $= 0.15$

Page 152 Q42 cont...

    d) RR = 4.1

    e) RR = 0.24

    f) An obese person is 6.6 times more likely to have diabetes than a person who is not obese. There appears to be causal link between obesity and diabetes.

43. RR = 9

44. a) Cycling

    b) They have a fatality rate less than cycling.

    c) 3.3 times

Page 153

45. a) i) $\dfrac{306}{435}$ (0.70)

    ii) 7034 per 10 000

    b) $\dfrac{180}{225}$ (0.8)

    c) $\dfrac{126}{210}$ (0.6)

    d) $\dfrac{0.6}{0.8}$ (0.75)

    e) The risk of having an asthma attack for those taking the trialled drug is 0.75 times the risk of those taking the placebo.

    f) $\dfrac{0.8}{0.6}$ (1.33 (3 sf))

    g) The risk of having an asthma attack for those on the placebo is 1.3 times the risk of those taking the trialled drug.

    h) Placebo. It is better to compare the risk of the drug group (treatment) with the placebo group (non-treatment).

    i) $\dfrac{0.6 - 0.8}{0.8}$ x 100% = ⁻25%

    j) There is a 25% decrease in the chance of a patient having an asthma attack if they are taking the trialled drug compared to the placebo.

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Answer from tables (calculator Ans. in brackets).

46. a) 0.4554

    b) 0.4222 + 0.4032 = 0.8254

Page 158 Q46 cont...

    c) 0.5 – 0.4538 = 0.0462

    d) 0.5 – 0.4268 = 0.0732 = (0.0733)

    e) 0.5 + 0.2197 = 0.7197

    f) 0.4505 – 0.4066 = 0.0439

47. a) 0.5 – 0.2190 = 0.2810

    b) 0.4896 – 0.4525 = 0.0371 = (0.0370)

Page 159 Q47 cont...

    c) 0.5 + 0.4694 = 0.9694 = (0.9693)

    d) 0.4332 + 0.4332 = 0.8664

    e) 0.5 + 0.2245 = 0.7245 = (0.7244)

    f) 0.4756 + 0.1646 = 0.6402

48. a) 0.4500

    b) 0.0500

    c) 0.4276

    d) 0.3530

    e) 0.3149

    f) 0.9483

Page 160 Q48 cont...

    g) 0.2180

    h) 0.4712

    i) 0.8901 (0.8900)

    j) 0.1806

Page 163

49. a) 0.4522

    b) 0.0478

    c) 0.3944

    d) 0.2906 (0.2907)

    e) 0.9522

50. a) 0.2412

    b) 0.2247

    c) 0.2016 (0.2017)

    d) 0.5932 (0.5934)

    e) 0.9559 (0.9560)

51. a) Z = 1.000 P(85 < X < 115) = 0.6826 (0.6827)

    b) Z = 1.333 to Z = 2 P(120 < X < 130) = 0.0685

    c) P = 0.0478. Expect 16 or 17.

    d) P = 0.0310. Expect 10 or 11.

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52. a) p = 0.3545 (0.3546)

    b) p = 0.0391 (0.0392)

    c) p = 0.0391 (0.0392)

    d) p = 0.8545 (0.8546)

    e) p = 0.2752

53. a) p = 0.4136 (0.4137)

    b) p = 0.0864 (0.0863)

    c) p = 0.7932 (0.7934)

54. a) p = 0.0863

    b) p = 0.3247

    c) p = 0.3132

    d) p^2 = 0.0075

    e) p = 0.0363

    f) p^3 = 0.0342

    g) Expect 38 or 39.

Page 165 Tables ans. (Calc. ans.)

55. a) P(X > 42 000) = 0.0766
        = (0.0766)

  b) P(X < 27 000) = 0.2377
        = (0.2375)

  c) P(25 000 < X < 39 000)
        = 0.6826
        = (0.6827)

  d) P(X > 52 000) = 0.0022
        = (0.0021)

    Managers= 0.0021 x 800
    = 1.71
    = 2 (rounding up)

Page 166 Tables ans. (Calc. ans.)

56. a) P(X > 4.2) = 0.0228

  b) P(X < 3.4) = 0.1151

  c) P(3.8 < X < 4.5) = 0.3439

  d) P(X < 3.0) = 0.0026

    Trees = 0.0026 x 2000
        = 5.2
        = 5 (round down)

Page 167 Tables ans. (Calc. ans.)

57. a) P(X < 30 000) = 0.0433
        = (0.0432)

  b) P(X > 45 000) = 0.334
        = (0.3341)

  c) P(X > 43 800) = 0.4052

    Drives OK = 0.3985 x 150
        = 59.78
        = 60 HDs
      (rounding 0 dp)

  d) P(X < 30 000) = 0.0433

    Two fail = 0.0432 x 0.0432
        = 0.0019 (4 dp)

Page 168 Tables ans. (Calc. ans.)

58. a) P(X > 180) = 0.0303
        = (0.0304)

    = 3.0%

  b) P(X < 143) = 0.0030

    = 0.3 %

  c) P(150 < X < 160)
        = 0.2357
        = (0.2356)

    = 23.6 %

  d) P(X < 190) = 0.9991

    Two days = 0.9991 x 0.9991
        = 0.9982 (4 dp)

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59. a) $z = 0.579$

  b) $z = 1.501$

  c) $z = 1.137$

  d) $z = 0.393$

  e) $z = ^-0.985$

60. a) $z = 1.01$ $x = 79.2$

  b) $z = 1.375$ $x = 84.9$

  c) $z = 0.78$ $x = 75.6$

  d) $z = 0.407$ $x = 69.7$

Page 172 Q60 cont...

  e) $z = ^-0.444$ $x = 56.5$

  f) $z = ^-0.892$ $x = 49.5$

61. a) $z = ^-1.716$ $p = 0.0431$ (0.0430)

  b) $z = ^-2.054$ $x = 747.7$

  c) Mean = 763.8

62. $\dfrac{x - 21.5}{2.4} = 0.385$

      $x = 22.424$

    mass = 22.4 kg

63. $\dfrac{x - 3}{0.75} = 1.645$

      $x = 4.234$

    time = 4.23 min.

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64. $\dfrac{x - 55}{12.5} = ^-0.842$

      $x = ^-10.52 + 55$

      $x = 44.48$

      $x = 44.5\%$

65. a) $\dfrac{x - 55}{16} = 1.751$

        $x = 83.016$

        $x = 83\%$

  b) $\dfrac{x - 55}{16} = 1.175$

        $x = 73.8$

        $x = 74\%$

    Interval 74% to 83%

66. $\dfrac{x - 43.33}{13.25} = 1.645$

      $x = 65.13$

      $x = 65$ minutes 8 sec.

Page 173 cont...

67. a) $z = ^-1.786$

        P(x < 200) = 0.037
            = (0.037)
      Two less than 200 g
        P(x < 200)² = 0.037²
            = 0.0014

  b) $\dfrac{x - 207.5}{4.2} = ^-2.327$

        $x = 197.7$ g

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68. $\dfrac{5 - M}{0.150} = ^-1.555$

      M = 5.23 kg

69. a) $\dfrac{38 - 32}{\sigma} = 0.613$

      σ = 9.8 wpm (1 dp)

  b) $\dfrac{x - 32}{9.79} = ^-0.674$

      x = 25.4 wpm

  c) $\dfrac{x_1 - 32}{9.79} = ^-0.755$

      x_1 = 24.6 wpm

    $\dfrac{x_2 - 32}{9.79} = 0.755$

      x_2 = 39.4 wpm

    Interval 24.6 to 39.4

Page 179

70. a) First minute 22.5%

  b) No. 85% of customers
      were served within
      2 minutes.

  c) A Normal distribution is
      a symmetrical bell shaped
      distribution about the mean.
      In the experiment there is a
      longer tail on the left of the
      mean and an unexpected
      peak in the 120+ class.
      Expt. P(X < 40) = 0.1375
      N.D. P(X < 40) = 0.094

71. a) 50 students

  b) 78%

  c) Mean = 90 seconds
      P(x < 90) = 0.68

  d) The distribution is not
      symmetrical. It is right
      skewed with a large group
      of students (9) taking over
      150 seconds to complete the
      level.

Page 180

72. a) $P(x > 3) = 0.42$

b) No. It could be that there are balls with the number 5, but by chance none were selected.

c) The teacher's graph has the probability that a ball with 4 on it is 0.5 but in our experiment we only got 18 out of 50 which seems very low. The opposite occurs for the 2 ball. The teacher's graph says the probability is 0.075 but we selected 10 out of 50 which is a probability of 0.20. The teacher's graph is symmetrical while the experimental distribution is skewed to the left. It seems unlikely that the teacher's graph is correct.

73. a) 77 boys

b) 33.8%

c) The distribution of distances is almost symmetrical with a slight peak at over 70 m. The normal distribution would predict that for a throw over 50 m the probability would be 0.3938 which is close to the figure from the experiment.
In the experiment a random thrower had a 0.364 chance of throwing between 40 and 50 metres. The normal distribution predicts 0.298 for the same region. This normal distribution is similar to our experimental distribution.

Pages 181 – 187

Practice External Assessment Task

Question One

a) i) $P(30 < X < 35) = P(^-1.25 < Z < 0.3125)$

$= 0.5172 \ (0.5170)$ **A**

ii) $P(X < 40) = P(Z < 1.875)$
$= 0.9697 \ (0.9696)$ **(A)**

Need the probability for two successive days of only being late once. Let L = Late
$P(L \text{ and } \neq L \text{ OR } \neq L \text{ and } L)$
$= 0.0304 \times 0.9696 + 0.9696 \times 0.0304$
$= 0.0590$ **M**

iii) $\dfrac{x - 34}{3.2} = 1.645$ **(A)**
$x = 39.264$ **(M)**

Can't really leave any later as she has effectively only 44 seconds to spare otherwise she will be late 5% of the time.   **E**

b) i) $\dfrac{30 - 31.1}{\sigma} = {}^-0.385$ **(A)**
$\sigma = 2.857$
$\sigma = 2.86 \ (3 \text{ sf})$ **M**

Question One b) cont...

b) ii) The normal curve is symmetrical about 31 minutes while the histogram also has a mean of about 31 minutes but is skewed to the left (tail on left) with no results over 36 minutes while the normal curve exists beyond 36 minutes. The probability for both the histogram and the normal distribution having a result under 30 minutes is both 35%. The probability of a time over 34 minutes on the histogram is 5% but with the normal curve it is 15.5%.   **E**

iii) $P(X < 28) = 0.125$
Over 240 days expect 30 days under 28 minutes.   **A**

For Question One students require three of A for Achievement or two of M for Merit or one of E and one M for Excellence.

Question Two

a) $P(R \text{ or } Y \text{ and } 6) = (\dfrac{3}{6} \times \dfrac{1}{6}) + (\dfrac{1}{6} \times \dfrac{1}{6})$

$= \dfrac{1}{9} = 0.111 \ (3 \text{ dp})$ **A**

b) $P(\text{no } 6) = \dfrac{85}{108}$

$\text{Cost (no 6)} = \dfrac{85}{108} \times 540 \times \$1$
$= \$425$

$P(\text{one } 6) = \dfrac{22}{108}$

$\text{Return (one 6)} = \dfrac{22}{108} \times 540 \times \$5$
$= \$550$

$P(\text{two } 6) = \dfrac{1}{108}$

$\text{Return (one 6)} = \dfrac{1}{108} \times 540 \times \$10$
$= \$50$

$\text{Nett} = \$175 \ (550 + 50 - 425)$ **E**

Or **A for 1 probability, M for one return.**

c) $P(G \text{ and } 6) = (\dfrac{2}{6} \times \dfrac{1}{6}) + (\dfrac{2}{6} \times \dfrac{5}{6} \times \dfrac{1}{6})$

$= \dfrac{11}{108} \ (0.1019)$ **A**

d) $P(G \mid 6) = \dfrac{P(G \text{ and } 6)}{P(6)}$

$P(\text{six}) = \dfrac{23}{108}$

$P(G \mid 6) = \dfrac{11}{23}$ **M**

Question Two cont...

e) $P(R, G, Y) = (\frac{3}{6} \times \frac{2}{6} \times \frac{1}{6}) \times 6$ **M**

$= \frac{1}{6}$

A for $(\frac{3}{6} \times \frac{2}{6} \times \frac{1}{6})$

f) P(green and 1st > 2nd)

$= (\frac{2}{6} \times \frac{15}{36})$

$= \frac{5}{36}$

P(other colour > 4)

$= (\frac{3}{6} \times \frac{1}{3}) + (\frac{1}{6} \times \frac{1}{3})$

$= \frac{2}{9}$

Don't agree as $\frac{2}{9} > \frac{5}{36}$.   **E**

A for one probability M for both.

For Question Two students require three of A for Achievement or two of M for Merit or one of E and one M for Excellence.

Question Three

a) (i) $P(\text{risk F}) = \frac{200}{550}$

$= 0.364 \text{ (3 dp)}$ **A**

(ii) $P(F, Tut, P) = \frac{95}{160}$

$= 0.594 \text{ (3 dp)}$ **A**

b) $P(M, CAI, P) = \frac{84}{149}$

$= 0.564 \text{ (3 dp)}$ **M**

c) (i) $P(T, P) = \frac{160}{288}$

$= 0.556 \text{ (3 dp)}$ **M**

(ii)   Risk / 100 $= 44.4 \text{ (44)}$ **A**

(iii) Pass tutoring $= \frac{160}{288}$

Pass CAI $= \frac{190}{262}$

Relative risk $= \frac{190}{262} \div \frac{160}{288}$

$= 1.3$

So 1.3 times more likely to pass course by CAI than tutoring.   **E**

(A) risk CAI or (M) relative risk.

Question Three c) cont...

(iv) Male pass with tutoring or CAI.

$P(M, P, Tut.) = \frac{65}{128}$

$P(M, P, CAI) = \frac{84}{110}$

Female pass with tutoring or CAI.

$P(F, P, Tut.) = \frac{95}{160}$

$P(F, \text{pass CAI}) = \frac{106}{152}$

Relative risk of females passing by tutoring compared to males.

Relative risk $= \frac{95}{160} \div \frac{65}{128}$

$= 1.2$

Females are 1.2 times more likely to pass by tutoring than males.

Relative risk of males passing by CAI compared to females.

Relative risk $= \frac{84}{110} \div \frac{106}{152}$

$= 1.1$

Males are 1.1 times more likely to pass by CAI than females.   **E**

(A) probability calculation or (M) relative risk or E answer plus explanation.

For Question 3 students require three of A for Achievement or two of M for Merit or one of E and one M for Excellence.

Overall students require:

Two or more Achievement questions or better for overall Achievement.

Two or more Merit questions or better for overall Merit.

Two or more Excellent questions or better for overall Excellent.

NZQA External Examination

In the external examinations NZQA uses a different approach to marking based on understanding (u), relational thinking (r) and abstract thinking (t). They then allocate marks to these concepts (maximum of 8 for a question) and add them up to decide upon the overall grade. This approach is not as easy for students to self mark as the NuLake approach but the results should be broadly similar.