Answers

Page 3

1. 14

2. $2\sqrt{2}$

3. $22\sqrt{2}$

4. 1

5. $3\sqrt{2}-3\sqrt{5}$

6. $^{-}17$

7. $52+6\sqrt{35}$

8. $\frac{2}{3}$

Page 4

9. $\frac{2\sqrt{15}}{5}$

10. $\frac{^{-}\sqrt{15}-5\sqrt{3}+3\sqrt{5}+15}{20}$

11. $\frac{2\sqrt{6}+3\sqrt{2}}{6}$

12. $\frac{4\sqrt{14}-3\sqrt{10}}{10}$

13. $\frac{11+6\sqrt{2}}{7}$

14. $^{-}8-10\sqrt{2}$

15. $\frac{^{-}3\sqrt{14}+9\sqrt{2}+5\sqrt{7}-15}{4}$

16. $\frac{12\sqrt{10}+4\sqrt{15}-6\sqrt{2}-2\sqrt{3}}{15}$

17. $\frac{a\sqrt{6}+3\sqrt{2}}{6}$

18. $\frac{2\sqrt{5}a+\sqrt{5b}}{5}$

19. $\frac{6+3\sqrt{a}}{4-a}$

20. $\frac{4\sqrt{b}+2b}{4b-b^2}$

Page 5

21. $\frac{a\sqrt{b}-b\sqrt{a}}{a-b}$

22. $\frac{^{-}2\sqrt{a}}{4-a}=\frac{2\sqrt{a}}{a-4}$

23. $\frac{1+6\sqrt{a}+9a}{1-9a}$

24. $\frac{2}{3x-1}$

25. $34+9\sqrt{2}$

26. $\frac{4}{1-a}$

Page 5 cont...

27. a) $\frac{a\sqrt{a}+a-2}{a-1}$

b) $\frac{a\sqrt{a}-2\sqrt{a}+a}{a-1}$

Page 8

28. $(x+10(x-2)=0$
$x=^{-}10, 2$

29. $(2x+1)(3x-4)=0$
$x=\ ^{-}\frac{1}{2},\ \frac{4}{3}\left(1\frac{1}{3}\right)$

30. $(4k-7)(4k+7)=0$
$k=\frac{7}{4}\left(1\frac{3}{4}\right),\ \frac{^{-}7}{4}\left(^{-}1\frac{3}{4}\right)$

31. $(x-4)(2x+3)=0$
$x=4,\ \frac{^{-}3}{2}\left(^{-}1\frac{1}{2}\right)$

32. $6x(3-4x)=0$
$x=0,\ \frac{3}{4}$

33. $(5x-4)(2x-3)=0$
$x=\frac{4}{5},\frac{3}{2}\left(1\frac{1}{2}\right)$

34. $(4x-1)(3x+5)=0$
$x=\frac{1}{4},\frac{^{-}5}{3}\left(^{-}1\frac{2}{3}\right)$

35. $(k-6)(9k+2)=0$
$k=\ 6,\frac{^{-}2}{9}$

36. $(x-7)(x+1)=0$
$x=7,\ ^{-}1$

37. $2(1-2k)(1+2k)=0$
$k=\frac{1}{2},\frac{^{-}1}{2}$

38. $(3x-1)(x-5)=0$
$x=\ 5,\frac{1}{3}$

39. $3(a-3)(a+3)=0$
$a=3,\ ^{-}3$

40. $(h-3)(3h+5)=0$
$h=\ 3,\frac{^{-}5}{3}\left(^{-}1\frac{2}{3}\right)$

41. $(x-\frac{1}{2})(x+\frac{1}{2})=0$
$x=\frac{1}{2},\frac{^{-}1}{2}$

42. $(x-a)(x-a)=0$
$x=a$

43. $(x-a)(x+2a)=0$
$x=a,\ ^{-}2a$

44. $(x-a)(x-3a)=0$
$x=a,\ 3a$

45. $(x-2a)(x+2a)=0$
$x=2a,\ ^{-}2a$

Page 10

46. $(x+2)^2-5=0$
$x=^{-}2\pm\sqrt{5}$

47. $(x-3)^2-18=0$
$x=3\pm3\sqrt{2}$

48. $(x+4)^2-22=0$
$x=^{-}4\pm\sqrt{22}$

49. $(x+2.5)^2-5.25=0$
$x=\frac{^{-}5}{2}\pm\sqrt{\frac{21}{4}}$

50. $(x+2)^2-7=0$
$x=^{-}2\pm\sqrt{7}$

51. $(x-4)^2-19=0$
$x=4\pm\sqrt{19}$

52. $(x-1)^2-11=0$
$x=1\pm\sqrt{11}$

53. $(x-5)^2-12=0$
$x=5\pm\ 2\sqrt{3}$

54. $(x+3)^2-9+k=0$
$x=\ ^{-}3\pm\sqrt{9-k}$

55. $(x-5)^2-25+k=0$
$x=\ 5\pm\sqrt{25-k}$

56. $(x-k)^2-k^2+5=0$
$x=\ k\pm\sqrt{k^2-5}$

57. $(x+2k)^2-4k^2+1=0$
$x=\ ^{-}2k\pm\sqrt{4k^2-1}$

Page 11

58. $6(x-1)^2-24=0$
$x=^{-}1,\ 3$

59. $5(x-3)^2-35=0$
$x=3\pm\sqrt{7}$

60. $4(x-2)^2-24=0$
$x=2\pm\sqrt{6}$

61. $3(x+2)^2-6=0$
$x=^{-}2\pm\sqrt{2}$

62. $3(m+4)^2-3=0$
$m=^{-}3,\ ^{-}5$

63. $3(x+2)^2-14=0$
$x=\ ^{-}2\pm\sqrt{\frac{14}{3}}$

64. $3(x-1)^2-4=0$
$x=1\pm\sqrt{\frac{4}{3}}\ \text{ or }\ 1\pm\frac{2}{\sqrt{3}}$

Page 11 cont...

65. $2(k+2)^2 - 1 = 0$

$k = -2 \pm \sqrt{\frac{1}{2}}$ or $-2 \pm \frac{1}{\sqrt{2}}$

66. $x = -1 \pm \sqrt{1 + \frac{k}{2}}$

67. $x = 2 \pm \sqrt{4 - \frac{k}{2}}$

68. $x = -1 \pm \sqrt{1 + \frac{6}{k}}$

69. $x = \frac{-1 \pm \sqrt{7}}{k}$

Page 13

70. $x = -0.146, -6.854$

71. $x = 1.854, -4.854$

72. $x = 5, -6$

73. $x = -1, 2.5$

Page 14

74. $x = 1.143, 0.180$

75. $x = 3.886, -0.886$

76. $x = -3 \pm \sqrt{10}$

77. $x = 3 \pm \sqrt{6}$

78. $x = \frac{-5 \pm \sqrt{29}}{2}$

79. $x = 2 \pm \sqrt{14}$

80. $x = -4 \pm \sqrt{16 + k}$

81. $x = \frac{-2 \pm \sqrt{13}}{k}$

82. $x = \frac{(k+2) \pm k}{2}$ or $k + 1$, 1

83. $x = 5k \pm \sqrt{26k}$

Page 16

84. $\Delta = 89$. Roots are **unequal**, **real** and **irrational**.

85. $\Delta = 25$. Roots are **unequal**, **real** and **rational**.

86. $\Delta = 0$. Roots are **equal** and **real**.

87. $\Delta = -23$. Roots are **unequal** and **complex**.

88. $4 - 12c \geq 0$ so $c \leq \frac{1}{3}$. Includes equal as equal roots are real.

89. $4 + 16d < 0$ so $d < -\frac{1}{4}$.

Page 16 cont...

90. $e^2 - 144 = 0$ so $e = \pm 12$.

91. $f^2 - 8 < 0$ so $-\sqrt{8} < f < \sqrt{8}$.

92. $9k^2 - 32k < 0$ so
$k(9k - 32) < 0$

$0 < k < \frac{32}{9}$

93. $9k^2 - 60k + 96 < 0$ so
$(3k - 8)(k - 4) < 0$

$\frac{8}{3} < k < 4$

Page 18

94. $p(-1) = -3$

95. $p(2) = 15$

96. $p(-1) = 3$

97. $p(-0.5) = -3.4375$

98. $p(-2) = -35$

99. $p(\frac{1}{3}) = 3.691$

100. $p(3) = 27 + 63 - 18 - 72 = 0$

101. $k = 22$

Page 19

102. $p(-1.5) = 2(-1.5)^3 + 9(-1.5)^2 - 1.5 - 12 = 0$
hence $(2x + 3)$ is a factor.

103. $k = 7$

104. $k = 12.5$

105. $k = 3, -6$

106. $k = 19$

107. $q = 2$

108. $p(2a) = 16a^3 - 4a^3 - 6a^3 - 6a^3$
$p(2a) = 0$ hence a factor

109. $m = 2, n = 5$

110. $a = 3, b = -7$

111. $a = 1, b = -8$

Page 22

112. $(x + 1)(2x - 1)(2x + 1)$
$x = -1, 0.5, -0.5$

113. $(x - 4)(x - 2)(x + 1)$
$x = 4, 2, -1$

114. $(x + 1)(2x + 1)(3x - 2)$
$x = 0.667, -1, -0.5$

115. $(x - 1)(2x - 5)(2x + 3)$
$x = -1.5, 2.5, 1$

Page 23

116. $(x + 3)(x - 3)(4x - 1)$
$x = -3, 3, 0.25$

117. $(x - 2)(x + 4)(2x - 1)$
$x = 2, -4, 0.5$

118. $(x - 4)(3x - 1)(2x + 3)$
$x = 4, 0.333, -1.5$

119. $(2x + 1)(2x - 1)(x - 1)$
$x = -0.5, 0.5, 1$

120. $(3x - 1)(x - 2)(5x - 1)$
$x = 0.333, 2, 0.2$

121. $(x + 4)(x + 2)(x - 6)$
$x = -4, -2, 6$

122. $(3x - 1)(4x - 3)(2x - 3)$
$x = 0.333, 0.75, 1.5$

123. $(x - 3)(2x - 5)(3x - 2)$
$x = 3, 2.5, 0.667$

124. $(2x - 1)^2(2 - x)$
$x = 0.5, 2$

125. $(-3x - 2)^3$
$x = -0.667$

Page 24

126. a) $p(-3) = -275$

b) $p(2) = 48 - 52 + 4 = 0$

c) $x = \frac{2}{3}, \frac{-1}{2}, 2$

127. a) $p(-1) = 3$

b) $(x + 2)$

128. a) $p(-1) = -1 + 4 - 8 + 5 = 0$

b) $p(x) = (x + 1)(x^2 + 3x + 5)$

129. a) $(6 - k)$

b) $k = 6$

c) $(x + 2)(2x + 1)(x - 3)$

130. $p(x) = (x + 3)(x - 4)(x + 1)$

131. $a = 13, b = 8$

Page 27

132. $x = 1$

133. $x = 3$

134. $x = 2$

135. $x = 8$

136. $x = 6, 5$

137. $x = 16$

138. $x = 9$

139. $x = 0.333$

Page 28

140. $x = 0.589$

141. $x = 12.685$

142. $x = 1.804$

143. $x = 2.303$

144. $x = \dfrac{(4-t)^2}{16}$

145. $x = \dfrac{(t+16)^2}{64}$

146. $x = \dfrac{^-q^2}{q^2-9}, q \neq \pm 3$

147. $x = \dfrac{k^2}{k^2-16}, k \neq \pm 4$

Page 33

148. $6 + 3i$

149. $5 - 3i$

150. $5 + 3i$

151. $22 + 31i$

152. $1 + 3i$

153. $10 - 10i$

154. $23 + 2i$

155. $^-5 + 14i$

156. $11 + 13i$

157. $^-1 - 9i$

158. $21 - i$

159. $^-14 + 44i$

160. ^-i

161. 2

162. $^-5 + 12i$

163. $^-2 - 2i$

164. $10 + 6i$

165. $^-46 - 9i$

Page 34

166. 17

167. $2 + 6i$

168. $\dfrac{3+2i}{13}$

169. $\dfrac{^-36-52i}{25}$

170. $\dfrac{4+3i}{5}$

171. $\dfrac{4-\sqrt{5}i}{21}$

Page 34 cont...

172. $\dfrac{^-4+i}{17}$

173. $\dfrac{7-3i}{3}$

174. $\dfrac{^-1+2\sqrt{2}i}{3}$

175. $\dfrac{12+5i}{2}$

176. $\dfrac{9+i}{5}$

177. $\dfrac{3-i}{2}$

178. $\dfrac{8-14i}{5}$

179. $\dfrac{58+4i}{169}$

Page 36

180. $z = 3 \pm \sqrt{2}i$

181. $z = \dfrac{3 \pm i}{2}$

182. $z = \dfrac{5 \pm \sqrt{83}i}{6}$

183. $z = \dfrac{^-2 \pm \sqrt{11}i}{3}$

184. $z = \dfrac{1 \pm \sqrt{55}i}{14}$

185. $z = \dfrac{5 \pm \sqrt{7}i}{4}$

Page 37

186. $z = 2 \pm \sqrt{2}i$

187. $z = 5 \pm 2i,$

188. $z = \dfrac{4 \pm \sqrt{92}i}{6}$

189. $z = \dfrac{^-3 \pm \sqrt{31}i}{4}$

190. $z = \dfrac{4 \pm \sqrt{104}i}{10}$

191. $z = \dfrac{1 \pm \sqrt{3}i}{2}$

192. $z = \dfrac{6 \pm \sqrt{60}i}{6}$

193. $z = \dfrac{1 \pm \sqrt{23}i}{4}$

Page 37 cont...

194. $z = a \pm 2ai$

195. $z = ^-a \pm 3ai$

196. $z^2 - 8z + 17 = 0$

197. $z^2 - 2z + 5 = 0$

198. $1 - \sqrt{3}i$, $k = 4$

199. $^-2 + \sqrt{2}i$, $k = 6$

Page 39

200. $x = ^-1,$
$x = 0.75 + 1.561i,$
$x = 0.75 - 1.561i$

201. $x = 2,$
$x = 1 + 1.732i,$
$x = 1 - 1.732i$

Page 40

202. $x = ^-1$, $x = ^-2 + 1.414i,$
$x = ^-2 - 1.414i$

203. $x = 2$, $x = ^-0.5 + 0.866i,$
$x = ^-0.5 - 0.866i$

204. a) $p(2) = 8 + 20 - 2 - 26 = 0$

b) $x = 2,$
$x = ^-3.5 + 0.866i,$
$x = ^-3.5 - 0.866i$

205. $x = 2$, $x = ^-3 + 1.732i,$
$x = ^-3 - 1.732i$

206. $A = 8$ and $x = 2$, $x = 3 + 2i,$
$x = 3 - 2i$

207. $A = 2$, $x = ^-4$, $x = 1 + 3i,$
$x = 1 - 3i$

208. $x = 1$, $x = ^-1 + 2i,$
$x = ^-1 - 2i$

209. $x = 2,$
$x = ^-2 + i,$
$x = ^-2 - i$

210. $z = ^-2,$
$z = ^-3 + i,$
$z = ^-3 - i$

211. $x = 1,$
$x = 2.5 + 1.658i,$
$x = 2.5 - 1.658i$

Page 41

212. a) $2 + 3i$
b) $z^2 - 4z + 13$
c) $A = 7, z = 3$

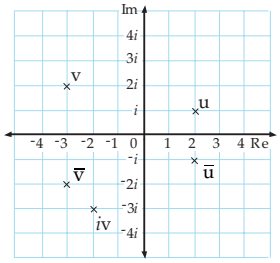
213. a) $p(^-1) = ^-1 + 4 - 8 + 5 = 0$
b) $(x + 1)(x^2 + 3x + 5)$
c) $x = ^-1$,
$x = ^-1.5 + 1.658i$,
$x = ^-1.5 - 1.658i$

214. a) $^-k^2 + 5k + 6$
b) $k = 6, ^-1$
c) $k = 6$
d) $x = ^-0.375 + 2.09i$
$x = ^-0.375 - 2.09i$
$x = 1$

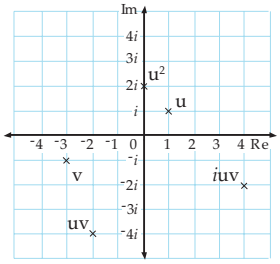
215. a) $1 - i$
b) $(z - (1 - i))(z - (1 + i))$
$= z^2 - 2z + 2$
$= (z + 2)(z^2 - 2z + 2)$
$= z^3 - 2z + 4$
so $a = ^-2, b = 4$

Page 43

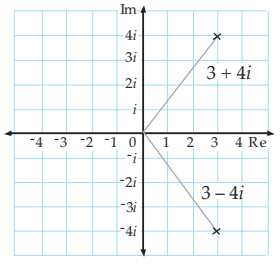
216. a) $u = 2 + i, v = ^-3 + 2i$
b) $\overline{u} = 2 - i, \overline{v} = ^-3 - 2i$
c) $iv = i(^-3 + 2i) = ^-2 - 3i$



217. a) $uv = ^-2 - 4i$
b) $iuv = 4 - 2i$
c) $u^2 = 2i$

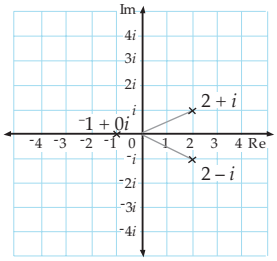


218.



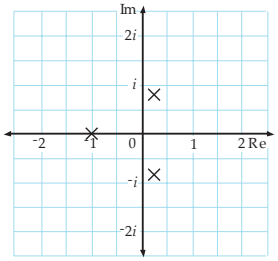
Page 43 cont...

219.



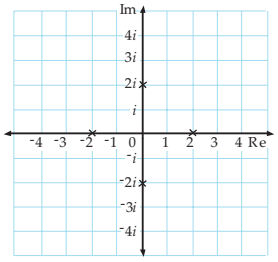
Page 44

220. $z^3 + 1 = 0$
$(z + 1)(z^2 - z + 1) = 0$
$z = ^-1, \frac{1}{2} \pm \frac{\sqrt{3}}{2}i$



All are 1 unit from the centre and are symmetrically spaced every $\frac{2\pi}{3}$ radians (120°).

221. $z^4 - 16 = 0$
$(z^2 + 4)(z^2 - 4) = 0$
$(z + 2i)(z - 2i)(z - 1)(z + 2) = 0$
$z = 2, ^-2, 2i, ^-2i$



All are 2 units from the centre and are symmetrically spaced every quarter turn. The roots sum to 0.

222. $z = 3, ^-1 \pm i$

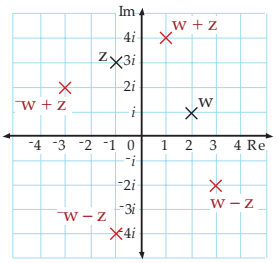
i) $(z - 3)(z + 1 - i)(z + 1 + i) = 0$
$z^3 - z^2 - 4z - 6 = 0$

ii) Sum = 1

iii) Product = 6

223. $w = 2 + i, z = ^-1 + 3i$
$w + z = 1 + 4i$
$w - z = 3 - 2i$
$^-w + z = ^-3 + 2i$
$^-w - z = ^-1 - 4i$

Page 44 Q223 cont...



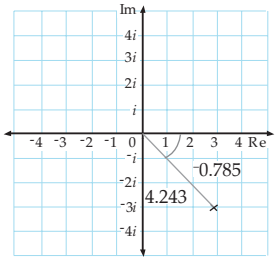
The resulting shape is a parallelogram.

Page 47

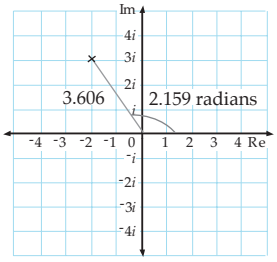
224. a) $u = 2.236 \text{ cis } 0.464$
$v = 3.606 \text{ cis } 2.554$
b) $\overline{u} = 2.236 \text{ cis } ^-0.464$
$\overline{v} = 3.606 \text{ cis } ^-2.554$
c) $uv = 8.062 \text{ cis } 3.017$

225. a) $2 \text{ cis } 0.5236$
b) $4 \text{ cis } ^-1.047$
c) $4 \text{ cis } ^-1.571$
d) $3 \text{ cis } 3.142$
e) $\sqrt{2}k \text{ cis } 0.7854$
f) $\sqrt{5}k \text{ cis } ^-0.4636$
g) $k \text{ cis } 1.571$

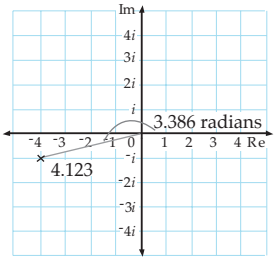
226. $z = 4.243 \text{ cis } ^-0.785$



227. $z = 3.606 \text{ cis } 2.159$

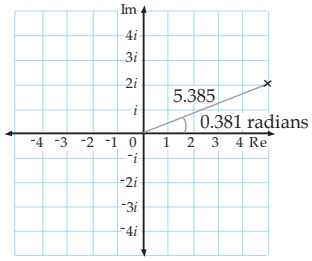


228. $z = 4.123 \text{ cis } ^-2.897$ or
$z = 4.123 \text{ cis } 3.386$



Page 47 cont...

229. z = 5.385 cis 0.381



Page 48

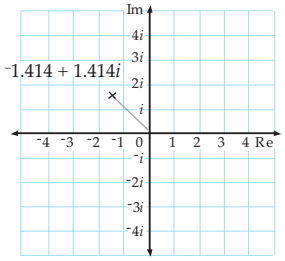
230 a) $1 + i$ b) $^{-}3$

c) $^{-}1.732 + i$ d) $3.464 - 2i$

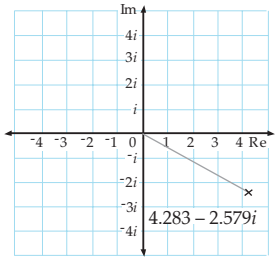
e) $0.707 - 1.225i$

f) $^{-}4.807 - 3.591i$

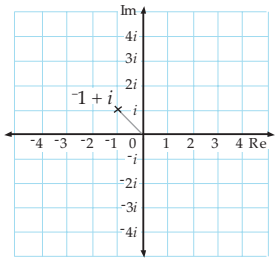
231. $z = ^{-}1.414 + 1.414i$



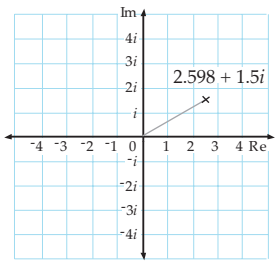
232. $z = 4.283 - 2.579i$



233. $z = ^{-}1 + i$

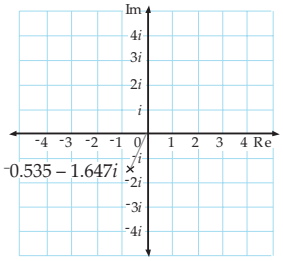


234. $z = 2.598 + 1.500i$

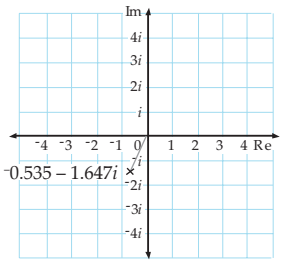


Page 48 cont...

235. $z = ^{-}1.848 - 0.765i$



236. $z = ^{-}0.535 - 1.647i$



Page 51

237. a) 108 cis 1.12

b) 324 cis $^{-}1.16$

c) 3 cis $^{-}2.283$

d) 2 cis 0.84

238. a) 7.616 cis 1.976

b) 9.849 cis $^{-}1.153$

c) 75 cis 0.823

d) 0.7733 cis 3.128

e) 441.7 cis $^{-}0.356$

239. a) 16 cis 1.15

b) 0.25 cis $^{-}0.575$

c) 0.0625 cis $^{-}1.15$

d) 64 cis 1.725

240. a) 0.2 cis 0.6435

b) 0.04 cis 1.287

c) 25 cis $^{-}1.287$

d) 125 cis $^{-}1.9305$

241. a) 28.285 cis $^{-}0.142$

b) 800 cis $^{-}0.284$

c) 0.884 cis $^{-}1.712$

d) 4.419 cis $^{-}2.639$

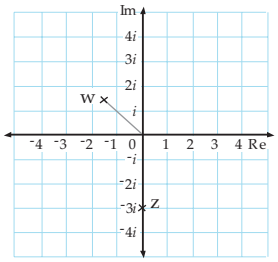
242. a) $6 + 10i$

b) 11.66 cis 1.030

c) 20.58 cis $^{-}1.030$

Page 52

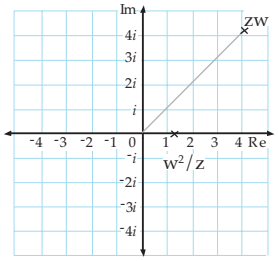
243. a)



b) 6 cis 0.785

c) 1.333 cis 0 (2π)

d)



244. a) $\dfrac{1}{z_{comb.}} = 0.28 + 0.04i$

$z_{comb.} = 3.5 - 0.5i$

b) $z_{comb.} = 3.54$ cis $^{-}0.142$

c) I = 36.77 cis 0.2869 (amps)

245. a) V = 161.8 cis 1.700

b) Z = 17.68 cis $^{-}1.429$ or
Z = 17.68 cis 4.854

246. a) z = 1 cis 2π

b) $z^3 = 1$ cis 2π
$z^3 = 1 + 0i$ in rect. form,
hence $z^3 - 1 = 1 - 1 = 0$

247. I = 15.18 cis $^{-}0.9653$ amps

Page 54

248. a) 8 cis 180° = $^{-}8$

b) 625 cis 120° = $^{-}312.5 + 541i$

c) 5.196 cis π = $^{-}5.196$

d) 64 cis $^{-}4\pi = 64$

249. a) $^{-}597 - 122i$

b) $64 - 110.85i$

c) $^{-}240.1 - 218.1i$

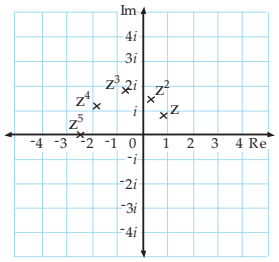
d) $5.657 + 5.657i$

Page 54 cont...

250. $z^2 = 4 \text{ cis } 2.094$
$= -2 + 3.464i$
$z^3 = 8 \text{ cis } 3.142$
$= -8 + 0i$

251. $z^2 = 20 \text{ cis } -2.214$
$= -12 - 16i$
$z^3 = 89.44 \text{ cis } 2.96$
$= -88 + 16i$

252. $z^2 = 1.44 \text{ cis } 1.256$
$z^3 = 1.73 \text{ cis } 1.885$
$z^4 = 2.07 \text{ cis } 2.513$
$z^5 = 2.49 \text{ cis } \pi$



Values appear to spiral out.

253. $$\frac{(\sqrt{3}-i)^9}{(1+i)^7} = \frac{\left(2\text{cis}\frac{-\pi}{6}\right)^9}{\left(\sqrt{2}\text{cis}\frac{\pi}{4}\right)^7}$$

$$= \frac{2^9\text{cis}\frac{-3\pi}{2}}{2^{7/2}\text{cis}\frac{7\pi}{4}}$$

$$= 2^{11/2}\text{cis}\frac{-13\pi}{4}$$

$= -32(1 - i)$

Page 56

254. $3 \text{ cis } 0.7854 = 2.12 + 2.12i$
$3 \text{ cis } 2.880 = -2.90 + 0.776i$
$3 \text{ cis } 4.974 = 0.776 - 2.90i$

255. $1.732 \text{ cis } 0.262 = 1.673 + 0.448i$
$1.732 \text{ cis } 1.833 = -0.448 + 1.673i$
$1.732 \text{ cis } 3.403 = -1.673 - 0.448i$
$1.732 \text{ cis } 4.974 = 0.448 - 1.673i$

256. $1.523 \text{ cis } -0.2975$
$1.523 \text{ cis } 1.273$
$1.523 \text{ cis } 2.844$
$1.523 \text{ cis } 4.415 (-1.868)$

Page 56 cont...

257. $1.378 \text{ cis } 0.1470$
$1.378 \text{ cis } 1.718$
$1.378 \text{ cis } 3.289 (-2.994)$
$1.378 \text{ cis } 4.859 (-1.424)$

258. $1.800 \text{ cis } 0.3435$
$1.800 \text{ cis } 2.438$
$1.800 \text{ cis } 4.532 (-1.751)$

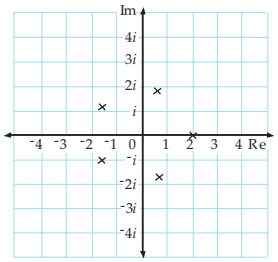
259. $1 \text{ cis } 0 = 1$
$1 \text{ cis } 2.094 = -0.5 + 0.886i$
$1 \text{ cis } 4.189 = -0.5 - 0.886i$

260. $1.414 \text{ cis } 0.349$
$1.414 \text{ cis } 2.443$
$1.414 \text{ cis } 4.538 (-1.745)$

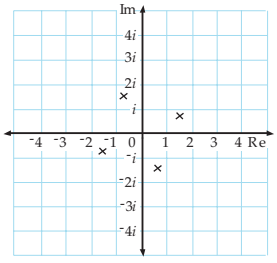
261. $1.378 \text{ cis } -0.2457$
$1.378 \text{ cis } 1.325$
$1.378 \text{ cis } 2.896$
$1.378 \text{ cis } 4.467 (-1.816)$

Page 57

262. $2 + 0i$
$0.618 + 1.902i$
$-1.618 + 1.176i$
$-1.618 - 1.176i$
$0.618 - 1.902i$



263. $1.643 \text{ cis } 0.4623$
$1.47 + 0.733i$
$1.643 \text{ cis } 2.033$
$-0.733 + 1.47i$
$1.643 \text{ cis } 3.604$
$-1.47 - 0.733i$
$1.643 \text{ cis } 5.175$
$0.733 - 1.47i$

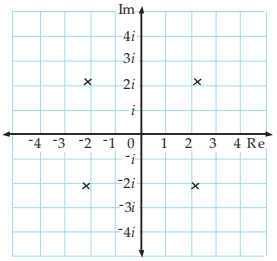


Page 57 cont...

264. $1.378 \text{ cis } -0.5398$
$1.378 \text{ cis } 1.031$
$1.378 \text{ cis } 2.602$
$1.378 \text{ cis } 4.173 (-2.110)$

265. $(2 - i)^3 =$
$2^3 + 3(2)^2(-i) + (3)(2)(-i)^2 + (-i)^3$
$= 2 - 11i$
$z_1 = 2.24 \text{ cis } -0.464$
$= 2 - i$
$z_2 = 2.24 \text{ cis } 1.63$
$= -0.134 + 2.23i$
$z_3 = 2.24 \text{ cis } -2.56$
$= -1.87 - 1.23i$

266. $2.121 + 2.121i$
$-2.121 + 2.121i$
$-2.121 - 2.121i$
$2.121 - 2.121i$



267. $1.495 \text{ cis } 1.339$
$(0.343 + 1.455i)$
$1.495 \text{ cis } 2.910$
$(-1.455 + 0.343i)$
$1.495 \text{ cis } 4.481$
$(-0.343 - 1.455i)$
$1.495 \text{ cis } 6.051 (-0.2318)$
$(1.455 - 0.343i)$

Page 58

268. $1.395 + 1.061i$
$-1.617 + 0.6774i$
$0.2217 - 1.739i$

269. $-1.414 - 1.414i$
$-1.414 + 1.414i$
$1.414 - 1.414i$
$1.414 + 1.414i$

270. $1 \text{ cis } 0.2618$
$1 \text{ cis } 2.3562$
$1 \text{ cis } 4.4506 (-1.8326)$

271. $1.414 \text{ cis } 0.7854$
$1.414 \text{ cis } 2.3562$
$1.414 \text{ cis } 3.9270 (-2.3562)$
$1.414 \text{ cis } 5.4978 (-0.7854)$

Page 58 cont...

272. 2 cis 0.1309, 2 cis 1.7017
2 cis 3.2725 ($^-$3.010)
2 cis 4.8433 ($^-$1.440)

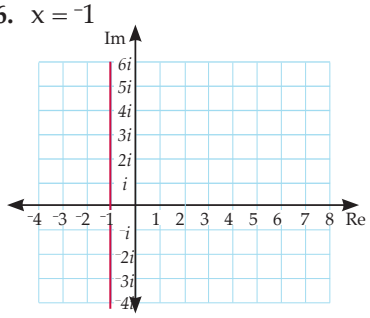
273. $z_1 = \sqrt{k}\text{cis}\frac{\pi}{8}$
$z_2 = \sqrt{k}\text{cis}\frac{5\pi}{8}$
$z_3 = \sqrt{k}\text{cis}\frac{9\pi}{8}$
$z_4 = \sqrt{k}\text{cis}\frac{13\pi}{8}$

Page 61

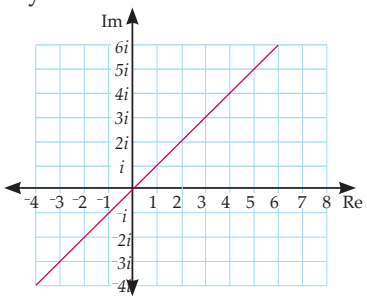
274. $(x-1)^2 + (y-2)^2 = 9$
Circle centre $1 + 2i$, radius 3

275. $x^2 + (y+3)^2 = 16$
Circle centre $0 - 3i$, radius 4

276. $x = ^-1$



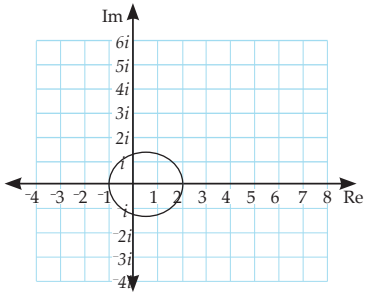
277. $y = x$



Page 62

278. $\frac{4(x-\frac{1}{2})^2}{9} + \frac{y^2}{2} = 1$

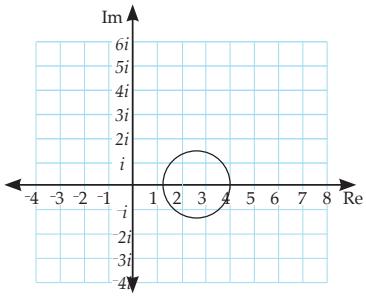
Ellipse centre $\frac{1}{2} + 0i$.
Major axis length = 3
Minor axis length = $2\sqrt{2}$



Page 62 cont...

279. $(x-\frac{8}{3})^2 + y^2 = \frac{16}{9}$

Circle centre $\frac{8}{3} + 0i$, radius $\frac{4}{3}$



280. $y^2 - \frac{x^2}{8} = 1$

Hyperbola centre (0, 0).
Vertices (0, 1) and (0, $^-$1)

Page 63 Excellence Questions

281. a) $(a + bi)^3 = a^3 - 3ab^2 + i(3a^2b - b^3)$
so $3a^2b - b^3 = 0$
so $3a^2 - b^2 = 0$ or $b = 0$

b) If $b \neq 0$ then $3a^2 = b^2$
$(a + bi)^3 = a^3 - 3ab^2$
$= a^3 - 9a^3$
$= ^-8a^3$
so $k = ^-8a^3$

282. $(z - (1 - i))(z - (1 + i))$
$= z^2 - 2z + 2$
$(z - k)(z^2 - 2z + 2)$
$= z^3 - 2z^2 - kz^2 + 2z + 2zk - 2k$
Equating
$2 + 2k = 6$ so $k = 2$
$b = ^-2k$ so $b = ^-4$
$a = 2 + k$ so $a = 4$.

283. $a^2 + 2abi - b^2 = 48 + 14i$
equating real parts
$a^2 - b^2 = 48$
and imaginary parts
$2ab = 14$ gives
$a = \pm 7$, $b = \pm 1$
or $a = \pm i$, $b = \mp 7i$
That is
(a, b) = (7, 1), ($^-$7, $^-$1), (i, ^-7i)
or (^-i, $7i$)

Page 64

284. $z_1.z_2 = (a_1 + b_1i)(a_2 + b_2i)$
$= a_1a_2 - b_1b_2 + i(a_1b_2 + a_2b_1)$
$\overline{z_1.z_2} = a_1a_2 - b_1b_2 - i(a_1b_2 + a_2b_1)$
$\overline{z_1}.\overline{z_2} = (a_1 - b_1i)(a_2 - b_2i)$
$= a_1a_2 - b_1b_2 - ia_1b_2 - ia_2b_1$
$= a_1a_2 - b_1b_2 - i(a_1b_2 + a_2b_1)$
$= z_1.z_2$

285. $k = \frac{z^2 - 4z + 5}{z - 2}$
$0 = z^2 - (4 + k)z + 5 + 2k$
Complex solutions when
$b^2 - 4ac < 0$
$k^2 + 8k + 16 - 20 - 8k < 0$
$k^2 - 4 < 0$
$(k + 2)(k - 2) < 0$
$^-2 < k < 2$

286. $\frac{x - yi + 2x + 2yi}{x^2 + y^2} = 1 + i$
$\frac{3x + yi}{x^2 + y^2} = 1 + i$
Equating
$\frac{3x}{x^2 + y^2} = 1$ and $\frac{y}{x^2 + y^2} = 1$
gives $y = 3x$ and substitution back gives $x = 0.3$ and $y = 0.9$
Note: $x = 0$ and $y = 0$ is NOT a solution.

Page 65

287. Finding the differences between vertices so we can work out the length of each side.

$\text{Line}_1 = v - u$
$= 6 - 3i$
$|L_1| = \sqrt{45}$
$L_2 = v - w$
$= 2 + 4i$
$|L_2| = \sqrt{20}$
$L_3 = w - u$
$= 4 - 7i$
$|L_3| = \sqrt{65}$

As $|L_3|^2 = |L_1|^2 + |L_2|^2$ triangle u, v and w must be right angled. Area 15 units^2.

Page 65 cont...

288. $z = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

If $\Delta \geq 0$ then real roots
If $\Delta < 0$ then complex roots as $b^2 - 4ac$ is negative (and $4ac - b^2$ is positive). Therefore:

$$z = \frac{-b \pm \sqrt{(^-1)}\sqrt{4ac - b^2}}{2a}$$

$$z = \frac{-b \pm i\sqrt{4ac - b^2}}{2a}$$

As the imaginary component is ± the two roots are conjugates of each other.

289. If $z = u + iv$ is a root then the conjugate $\overline{z} = u - iv$ is a root.
$(z - (u + iv))(z - (u - iv))$
$= z^2 - 2uz + (u + iv)(u - iv)$
$= z^2 - 2uz + u^2 + v^2$
Therefore $a = 1$, $b = {}^-2u$ and $c = u^2 + v^2$.

Page 66

290. $v = a + ib$, $w = c + id$
$z = ac - bd + i(ad + bc)$
$|z|^2 = (ac - bd)^2 + (ad + bc)^2$
$= (ac)^2 - 2abcd + (bd)^2 + (ad)^2 + 2abcd + (bc)^2$
$= (ac)^2 + (bd)^2 + (ad)^2 + (bc)^2$
$= (ac)^2 + (ad)^2 + (bc)^2 + (bd)^2$
$= a^2(c^2 + d^2) + b^2(c^2 + d^2)$
$= (a^2 + b^2)(c^2 + d^2)$
As a, b, c and d are non zero integers $(a^2 + b^2)$ and $(c^2 + d^2)$ are whole numbers so $|z|^2$ is a product of whole numbers and therefore NOT prime.

291. $z^{-1} = \dfrac{1}{a + ib}$

$$= \frac{a - ib}{a^2 + b^2}$$

$$|z^{-1}| = \sqrt{\frac{a^2}{\left(a^2 + b^2\right)^2} + \frac{b^2}{\left(a^2 + b^2\right)^2}}$$

$$|z^{-1}| = \frac{1}{\sqrt{\left(a^2 + b^2\right)}}$$

$$|z| = \sqrt{\left(a^2 + b^2\right)}$$

$$|z|^{-1} = \frac{1}{\sqrt{\left(a^2 + b^2\right)}} = |z^{-1}|$$

Page 66 cont...

292. $|x - 1 + i(y + 1)| = 2$

$(x - 1)^2 + (y + 1)^2 = 2^2$

Circle centre $(1, {}^-i)$ radius 2.
point on circle (many different answers but 2 from centre).
$(3, {}^-i)$.

Pages 67 – 72

Practice Assessment – Complex Numbers

In the external examinations NZQA uses a different approach to marking based on understanding (u), relational thinking (r) and abstract thinking (t). They then allocate marks to these concepts and add them up to decide upon the overall grade. This approach is not as easy for students to self mark as the NuLake approach but the results should be broadly similar.

Question One

(a) $(p + 2i)^3$
$= p^3 + 3p^2.2i + 3p(2i)^2 + (2i)^3$
$= p^3 + 6p^2i - 12p - 8i$
$= p^3 - 12p + i(6p^2 - 8)$ **A**
Working must be shown.

(b) $54 + 9A + 3B - 105 = 0$
or $3A + B - 17 = 0$
and
$^-250 + 25A - 5B - 105 = 0$
or $5A - B - 71 = 0$
$A = 11$, $B = {}^-16$ **A**
Working must be shown.

(c) $z^2 = 1 \text{ cis } \pi/2$
$z = 1 \text{ cis } \pi/4,\ 1 \text{ cis } 5\pi/4$ **A**
$\mathbf{z = \dfrac{1}{\sqrt{2}} + \dfrac{i}{\sqrt{2}},\ \dfrac{^-1}{\sqrt{2}} - \dfrac{i}{\sqrt{2}}}$ **M**
Or decimal equivalent.

(d) $\left|\dfrac{(2+i)^3}{2-i+1}\right| = \left|\dfrac{2+11i}{3-i}\right|$ **A**

$$\left|\frac{(2+11i)(3+i)}{(3-i)(3+i)}\right| = \left|\frac{^-5+35i}{10}\right|$$

$$= \frac{5\sqrt{2}}{2}$$ **M**

Working must be shown.

Question One cont...

(e) $z^n + \dfrac{1}{z^n} = (\text{cis } \theta)^n + (\text{cis } \theta)^{-n}$
$= \cos n\theta + i\sin n\theta + \cos({}^-n\theta) + i\sin({}^-n\theta)$ **M**
$= \cos n\theta + i\sin n\theta + \cos n\theta - i\sin n\theta$
$= 2 \cos n\theta$
$= \text{RHS}$ **E**
Working must be shown.

Question Two

(a) $64 \text{ cis } {}^-\pi = 64 \text{ cis } \pi$ **A**

(b) $2z + 1 = (z + 3)^2$
$z^2 + 6z + 9 = 2z + 1$
$z^2 + 4z + 8 = 0$

$$z = \frac{^-4 \pm \sqrt{4^2 - 32}}{2}$$

$z = {}^-2 \pm 2i$ **A**
Working must be shown.

(c) Conjugate $2 + i$ is also a root.
So polynomial is:
$(mz + n)(z - (2 - i))(z - (2 + i))$
$= (mz + n)(z^2 - 4z + 5)$ **A**
$m = 2$ and $n = {}^-1$ by inspection.
third factor is $(2z + 1)$ so
$z = (2 - i)$, $(2 + i)$ and 0.5
and $a = {}^-9$ and $b = 14$ **M**
Working must be shown.

(d) $z = (4.5 + 2.5981i)^{1/3}$

$$= \sqrt{3}\text{cis}\left(\frac{\pi + 12k\pi}{18}\right)$$ **A**

$$= \sqrt{3}\text{cis}\ \frac{\pi}{18},\ \sqrt{3}\text{cis}\ \frac{13\pi}{18},\ \sqrt{3}\text{cis}\ \frac{25\pi}{18}$$

M
Working must be shown.

(e) $|x + iy + 1| = |x + iy - 3i|$
$|x + 1 + iy| = |x + i(y - 3)|$
$(x + 1)^2 + y^2 = x^2 + (y - 3)^2$
$x^2 + 2x + 1 + y^2 = x^2 - 6y + 9 + y^2$
$3y + x - 4 = 0$ **M**
The equation represents the line $y = \dfrac{^-x + 4}{3}$ (the perpendicular bisector of points $({}^-1, 0i)$ and $(0, 3i)$). **E**
Working must be shown.
Geometrical interpretation required as well as solution.

Question Three

(a) $z^2 = 3 - 4i$

$$\frac{5}{3-4i}\frac{3+4i}{3+4i} - \frac{6}{2-i}\frac{2+i}{2+i} + 2$$

$0.2 - 0.4i$ **A**

Working must be shown

(b) $z_1 = 2\text{cis}(\pi/6)$, $z_2 = 2\text{cis}(7\pi/6)$

$z_1 = \sqrt{3} + i$, $z_2 = {}^{-}\sqrt{3} - i$ **A**

(c) $z^4 = n \text{ cis } 0$

$$z = n^{1/4}\text{cis}\frac{(0+2k\pi)}{4}$$

$z_1 = \sqrt[4]{n} \text{ cis } 0 = \sqrt[4]{n}$ **A**

$z_2 = \sqrt[4]{n} \text{ cis } \frac{\pi}{2} = \sqrt[4]{n}i$

$z_3 = \sqrt[4]{n} \text{ cis } \pi = {}^{-}\sqrt[4]{n}$

$z_4 = \sqrt[4]{n} \text{ cis } \frac{3\pi}{2} = {}^{-}\sqrt[4]{n}\, i$ **M**

Working must be shown.

(d) $z^3 - 4z^2 + 14z - 20 = 0$
Use Factor Theorem to find $(z - 2)$

$(z - 2)(z^2 - 2z + 10) = 0$ **A**

$z = 2, 1 + 3i, 1 - 3i$ **M**

(e) $$w = \frac{(x-1)+yi}{(x+1)+yi} \times \frac{(x+1)-yi}{(x+1)-yi}$$

$$w = \frac{x^2+y^2-1+2yi}{x^2+2x+1+y^2}$$ **M**

since $x^2 + y^2 = 1$

$$w = \frac{yi}{x+1}$$

hence w is purely imaginary.
E

Sufficiency. For each question award yourself a score out of 8 using this table. Add the three scores for a score out of 24 and compare to the cut scores. All answers must show working where appropriate.

Quest.	N0	N1	N2	A3	A4	M5	M6	E7	E8
ONE	None correct	1A with error	1A correct	2A correct.	3A correct.	1M + 3A	2M correct.	1E almost	1E all correct
TWO	None correct	1A with error	1A correct	2A correct.	3A correct.	1M + 3A	2M correct.	1E almost	1E all correct
THREE	None correct	1A with error	1A correct	2A correct.	3A correct.	1M + 3A	2M correct.	1E almost	1E all correct
Cut Scores									

Not Achieved	Achievement	Achievement with Merit	Achievement with Excellence	
0 – 8	9 – 14	15 – 20	21 – 24	

Answers

Note: Undef. means Undefined.

Page 78

1. 9

2. 1

3. No limit as approaching 5 from above and below gives different results.

4. 0

5. $^{-}2$

6. 7

7. 8

8. 5

9. 2x

10. 4x

11. $3a^2$

12. 2x + 5

Page 79

13. a) 1.5
b) Undef.
c) 2
d) 5
e) 4

14. a) 0
b) 2
c) Undef.
d) $^{-}1$
e) $^{-}2$
f) 2
g) 1

15. a) 0 b) 1
c) $^{-}2$ d) 0
e) x = 0.5, 1, 2

16. a) 1 b) ∞
c) Undef. d) 1

Page 80

17. a) 2 b) 0
c) $^{-}3$ d) Undef.

18. a) 3 b) $^{-}2$
c) Undef. d) 3
e) 1 f) 2

19. a) Undef. b) 1
c) Undef. d) 1
e) 3 f) Undef.

Page 82

20. a) 0
b) $^{-}1$, 1 and 2
c) $x < ^{-}1$ and $x \geq 3$
d) $\{x: x \neq 1, 2, x \in R\}$
e) Undef.

Page 82 cont...

21. a) $\{x: x \neq ^{-}1, 0, x \in R\}$
b) $^{-}1$, 0, 1 and 2
c) x = 1
d) $\{x: x \geq 2, x \in R\}$
e) Approximately 2.8
f) 0, 1, 2

22. a) f(2.5) = 1.75, f(4.5) = 4
b) $\{x: x \neq 2, 3, x \in R\}$
c) x = 4
d) 2, 3, 4.5
e) 5.5

23. a) $\{x: x \neq 2, x \in R\}$
b) 0.5
c) $^{-}1$, 2, 4
d) $^{-}1$, 2, 4, 6
e) 6

Page 83

24. a) 3
b) f(3) = 4
c) $^{-}1 < x < 1$
d) $x = ^{-}1, 3$
e) $x = ^{-}1, 1, 3$
f) f′(2) = 1

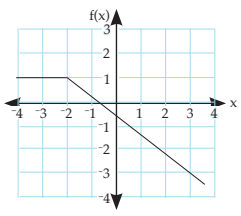
25. a) x > 5 (possibly $5 < x < 6$)
b) $f(^{-}2) = 2$
c) $f'(^{-}2) = 1$
d) Maximum at (1, 4)
e) Inflection (5, 4)
f) x = 3

26. a) Maximum at ($^{-}1$, 3) and (4, 2.5)
b) f′(2) = 4 approx.
c) $f'(^{-}1) = 0$
d) x = 6
e) x = 1.5, 6
f) $^{-}3 \leq x \leq 6, x \neq 1.5$

27. a) Minimum at (1, $^{-}4$)
b) $x = ^{-}0.5$ approx.
c) $x = ^{-}2.25$, 1.5, 2.4, 5.3 all approximate.
d) $^{-}0.5 < x < 2$ or $4 < x < 6$
e) $x = ^{-}0.5$, 2 and 4
f) $x = ^{-}2$, 1 and 4

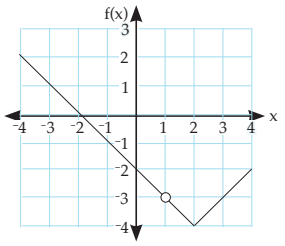
Page 84 (Other answers possible)

28.

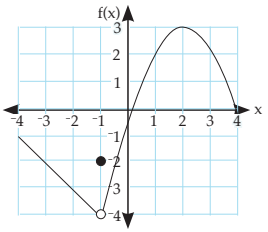


Page 84 cont...

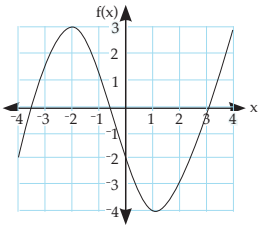
29.



30.



31.



Page 86 (Needs multiple steps)

32. $f'(x) = \lim_{h \to 0} \frac{4x + 4h - 4x}{h}$
f′(x) = 4

33. $f'(x) = \lim_{h \to 0} \frac{4x + 4h + 5 - 4x - 5}{h}$
f′(x) = 4

34. $f'(x) = \lim_{h \to 0} \frac{^{-}8x - 8h + 8x}{h}$
$f'(x) = ^{-}8$

35. $f'(x) = \lim_{h \to 0} \frac{7x + 7h - 7x}{h}$
f′(x) = 7

36. $f'(x) = \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - x^2}{h}$
f′(x) = 2x

37. $f'(x) = \lim_{h \to 0} \frac{x^2 + 2xh + h^2 + 5 - x^2 - 5}{h}$
f′(x) = 2x

38. $f'(x) = \lim_{h \to 0} \frac{2x^2 + 4xh + 2h^2 - 2x^2}{h}$
f′(x) = 4x

39. $f'(x) = \lim_{h \to 0} \frac{2xh + h^2}{h}$
f′(x) = 2x

Page 87 (Needs multiple steps)

40. $f'(x) = \lim_{h \to 0} \frac{2xh + h^2 + h}{h}$
f′(x) = 2x + 1

41. $f'(x) = \lim_{h \to 0} \frac{^{-}2xh - h^2}{h}$
$f'(x) = ^{-}2x$

Page 87 cont...

42. $f'(x)=\lim_{h\to0}\frac{2xh+h^2+5h}{h}$
$f'(x) = 2x + 5$

43. $f'(x)=\lim_{h\to0}\frac{3x^2+6xh+3h^2-3x^2}{h}$
$f'(x) = 6x$

44. $f'(x)=\lim_{h\to0}\frac{ax^2+2axh+ah^2-ax^2}{h}$
$f'(x) = 2ax$

45. $f'(x)=\lim_{h\to0}\frac{x^3+3x^2h+3xh^2+h^3-x^3}{h}$
$f'(x) = 3x^2$

46. $f'(x)=\lim_{h\to0}\frac{3x^2h+3xh^2+h^3+h}{h}$
$f'(x) = 3x^2 + 1$

47. $f'(x)=\lim_{h\to0}\frac{2axh+ah^2+bh}{h}$
$f'(x) = 2ax + b$

Page 89

48. $f'(x) = 12x^3 + 18x^2 - 14x - 2$

49. $\frac{dy}{dx} = 20x^3 - 6x + 6$

50. $f'(x) = \frac{8}{3}x^3 + \frac{3}{2}x - 1$
$f'(3) = 75.5$

51. $f'(x) = \frac{16}{5}x^3 - \frac{12}{5}x^2 + 6x$
$f'(^-1) = ^-11.6$

52. $f'(x) = \frac{7}{2}x^6 + 10x^5 - 2x$
$f'(2) = 540$

53. $\frac{dy}{dx} = 2.5x^4 - 9.6x^3 - 10.5x^2 + 3.2x$

54. $f'(x) = 4x + 5$

55. $\frac{dy}{dx} = 3x^2 - 2x - 6$

56. $f'(x) = 18x^2 + 20x$

57. $f'(x) = 60x^4 - 32x^3 + 12x^2$

58. $f'(x) = 2ax + b$

59. $\frac{dy}{dx} = 3a^2x^2 - 2abx$

60. $f'(x) = 28x^6 + 2 - \frac{1}{x^2}$

61. $f'(x) = \frac{4}{3}x^2 - \frac{2}{x^2} + 2x$

62. $f'(x) = 6x^3 - \frac{5}{x^2} - \frac{4}{x^3} + 1$
$f'(^-1) = ^-6$

63. $\frac{dy}{dx} = \frac{^-3}{x^2} - \frac{2}{x^3} + \frac{27}{x^4}$

Page 90

64. $f'(x) = 4 + \frac{6}{x^2} + \frac{10}{x^3}$

65. $\frac{dy}{dx} = 5 - \frac{2}{x^2} - \frac{6}{x^3}$

66. $f'(x) = \frac{^-2a}{x^3} + \frac{b}{x^2}$

67. $\frac{dy}{dx} = 6x + 2 + \frac{1}{2\sqrt{x}}$

68. $f'(x) = 4 + \frac{5}{x^2} + \frac{1}{2\sqrt{x}}$

69. $f'(x) = \frac{^-5}{3\sqrt[3]{x^4}}$

70. $\frac{dy}{dx} = \frac{^-1}{x^2} + \frac{1}{\sqrt{x^3}} - \frac{1}{\sqrt[3]{x^4}}$

71. $f'(x) = \frac{^-3}{2\sqrt{x^3}} + \frac{2}{5\sqrt[5]{x^6}} - \frac{5}{4\sqrt[4]{x^5}}$

72. $f'(x) = \frac{^-a}{2\sqrt{x^3}} + \frac{b}{3\sqrt[3]{x^4}}$

Page 93

73. $\frac{dy}{dx} = 15(5x - 1)^2$

74. $\frac{dy}{dx} = 4(4x + 3)(2x^2 + 3x - 5)^3$

75. $\frac{dy}{dx} = ^-10x(4 - x^2)^4$

76. $\frac{dy}{dx} = 6ax(ax^2 + b)^2$

77. $g'(x) = 12(2x - 1)^5$

78. $f'(x) = x(x^2 - 12)^{-1/2}$

79. $\frac{dy}{dx} = ^-0.5(24x - 5)(12x^2 - 5x + 8)^{-3/2}$

80. $\frac{dy}{dx} = ^-x(9 - x^2)^{-1/2}$

81. $\frac{dy}{dx} = ax(ax^2 - b)^{-1/2}$

82. $k'(x) = (2x + 8)^{-1/2}$

83. $k'(x) = ^-2x(x^2 - 2)^{-2}$

84. $h'(x) = ^-24(2x + 1)^{-5}$

85. $\frac{dy}{dx} = 6x(6x^2 - 5)^{-1/2}$

86. $k'(x) = (3x - 4)(3x^2 - 8x + 2)^{-1/2}$

87. $f'(x) = ^-4ax(ax^2 - b)^{-2}$

88. $f'(x) = 3(x + \frac{2}{x})^2(1 - \frac{2}{x^2})$

89. $\frac{dy}{dx} = \frac{1}{5}(10x - 2)(5x^2 - 2x)^{^-4/5}$

90. $\frac{dy}{dx} = ^-3(6x^2 - x)(4x^3 - x^2)^{^-3}$

Page 94

91. $\frac{dy}{dx} = \frac{a}{b^2}$

92. $\frac{dP}{dr} = \frac{3}{2}(r - 2)^{1/2}$

93. $\frac{dz}{dt} = ^-3(t - 1)^{-4}$

94. $\frac{dy}{dx} = \frac{3b}{x^2}(a + \frac{b}{x})^{-4}$

95. $\frac{dA}{db} = 2b(1 - b^2)^{-2}$

96. $\frac{dk}{dx} = (3 - x)^{-5/4}$

97. $\frac{dy}{dx} = \frac{3}{4}(x - 1)^5$

98. $\frac{dy}{dx} = \frac{2a}{b^2}(x + 1)$

Page 96

99. $f'(x) = 4e^{4x}$

100. $y' = ^-2e^{-2x}$

101. $g'(x) = 6e^{6x - 1}$

102. $y' = ^-3e^{8 - 3x}$

103. $k'(x) = 6e^{3x}$

104. $y' = 12e^{3x - 1}$

105. $f'(x) = 20e^{-4x + 3}$

106. $m'(x) = ^-6e^{-8x}$

Page 97

107. $f'(x) = \frac{^-3e^{2x+1}}{4}$

108. $h'(x) = \frac{^-8e^{5-4x}}{7}$

109. $y' = 12e^{4x} + 10e^{-5x + 3}$

110. $k'(x) = 10e^{-2x + 1} - 7e^{4 - x}$

111. $y' = 42xe^{3x^2-4} - 16xe^{1-4x^2}$

112. $f'(x) = ^-168x^2e^{7x^3-1} - 6e^{1+2x}$

113. $g'(x) = \frac{e^{\sqrt{x}}}{2\sqrt{x}}$

114. $m'(x) = \frac{2e^{\sqrt{x}}}{\sqrt{x}}$

115. $f'(x) = \frac{^-e^{1/x}}{x^2}$

116. $p'(x) = \frac{3e^{1/x}}{x^2}$

Page 97 cont...

117. $\frac{dy}{dx} = \frac{-5e^{1-x}}{4}$

118. $f'(x) = ae^{ax+b}$

119. $\frac{dy}{dx} = 3(2x+e^{3x})^2(2+3e^{3x})$

120. $\frac{dy}{dx} = \frac{4e^{-2x}}{\sqrt{1-4e^{-2x}}}$

121. $v'(x) = \frac{3e^{\sqrt{2x-3}}}{\sqrt{2x-3}}$

122. $\frac{dy}{dx} = \frac{-6a}{e^{2x}}$

123. $f'(x) = 2(a + be^{bx})(ax + e^{bx})$

124. $\frac{dy}{dx} = \frac{ae^{ax}}{\sqrt{1+2e^{ax}}}$

Page 99

125. $f'(x) = \frac{7}{7x+2}$

126. $f'(x) = \frac{1}{x}$

127. $f'(x) = \frac{-5}{9-5x}$

128. $m'(x) = \frac{6x}{3x^2-1}$

129. $\frac{dy}{dx} = \frac{-8x}{1-4x^2}$

130. $f'(x) = \frac{5}{x}$

131. $g'(x) = \frac{12x^2}{4x^3-3}$

132. $g'(x) = \frac{a}{ax+b}$

133. $\frac{dy}{dx} = \frac{2x+5}{x^2+5x+5}$

Page 100

134. $\frac{dy}{dx} = 3e^{3x} + \frac{1}{x}$

135. $f'(x) = \frac{2ax-b}{ax^2-bx-c}$

136. $p'(x) = \frac{6x-9}{x^2-3x}$

137. $k'(x) = \frac{16x+4}{2x^2+x}$

138. $q'(x) = \frac{-6x^2}{x^3-1}$

Page 100 cont...

139. $k'(x) = \frac{5}{1-x}$

140. $q'(x) = \frac{5-20x}{2x^2-x+1}$

141. $k'(x) = \frac{2x-3}{1-x^2+3x}$

142. $f'(x) = \frac{4x}{x^2-1}$

143. $f'(x) = \frac{6}{1-x}$

144. $f'(x) = \frac{48x}{2x^2+1}$

145. $\frac{dy}{dx} = \frac{108x}{3x^2-4}$

146. $f'(x) = \frac{1}{2x}$

147. $f'(x) = \frac{1}{x+1}$

148. $f'(x) = \frac{-3}{2x-5}$

149. $\frac{dy}{dx} = \frac{-1}{x}$

150. $\frac{dy}{dx} = \frac{1}{2(x-3)}$

151. $g'(x) = \frac{-1}{x(3x+1)}$

152. $\frac{dy}{dx} = \frac{9(\ln(3x+2))^2}{3x+2}$

153. $\frac{dy}{dx} = \frac{1}{4x\sqrt[4]{(\ln(x))^3}}$

Page 105

154. $\frac{dy}{dx} = 3\sec^2 3x$

155. $f'(x) = 5\cos 5x$

156. $\frac{dy}{dx} = -12\sin 4x$

157. $f'(x) = -6\cos 3x$

158. $\frac{dy}{dx} = 10\sec 2x\tan 2x$

159. $f'(x) = -6\cos(2x+1)$

160. $\frac{dy}{dx} = -6\sec^2(1-3x)$

161. $f'(x) = 4\sin(4x-1)$

162. $f'(x) = \frac{-3}{4}\sin\frac{1}{4}x$

Page 105 cont...

163. $f'(x) = -5\operatorname{cosec}^2(5x-2)$

164. $f'(x) = -4\operatorname{cosec}^2 x\cot x$

165. $\frac{dy}{dx} = \frac{-2\cos\left(\frac{2}{x}\right)}{x^2}$

166. $\frac{dy}{dx} = -\tan x$

167. $\frac{dy}{dx} = \frac{3\cos\sqrt{x}}{\sqrt{x}}$

168. $\frac{dy}{dx} = 2\sec 2x\tan 2x - 12\operatorname{cosec}(3x+1)\cot(3x+1)$

169. $\frac{dy}{dx} = 4\sec^2 4x - 3\cos 3x$

170. $\frac{dy}{dx} = -a\sin(ax+b)$

171. $\frac{dy}{dx} = 6ax\cos(ax^2-b)$

Page 108

172. $f'(x) = (6x+5)e^{2x}$

173. $q'(x) = (-9x^2-9x+5)e^{-3x}$

174. $n'(x) = (12x^3+6x)e^{2x^2}$

175. $q'(x) = 2x(15x^2-8)e^{5-3x^2}$

176. $f'(x) = (12x^2+8x-27)e^{3x}$

177. $\frac{dy}{dx} = 3e^{2x}(4x^2-14x-9)$

178. $u'(x) = (12x^2+8x)e^{3x-1}$

179. $p'(x) = (6x^2+6x-2)e^{2x+1}$

180. $p'(x) = 2\ln\sqrt{x} + \frac{2x+1}{2x}$

$p'(x) = \ln(x) + \frac{2x+1}{2x}$

181. $n'(x) = 3\ln 5x^2 + \frac{6x-2}{x}$

182. $v'(x) = 2x\ln\sqrt{x-2} + \frac{x^2-3}{2x-4}$

183. $n'(x) = 4\ln\left(\frac{1}{x^2}\right) - 8 - \frac{2}{x}$

184. $f'(x) = e^x\ln(x^2+1) + \frac{2xe^x}{x^2+1}$

185. $f'(x) = 2e^{2x-1}\ln(\sqrt{x}) + \frac{e^{2x-1}}{2x}$

$= e^{2x-1}\ln(x) + \frac{e^{2x-1}}{2x}$

Page 109

186. $\frac{dy}{dx} = 2x\cos x - x^2\sin x$

187. $\frac{dy}{dx} = 3\tan 3x + (9x-6)\sec^2 3x$

188. $\frac{dy}{dx} = 2x\cot(5x-1) - 5(x^2-1)\operatorname{cosec}^2(5x-1)$

189. $\frac{dy}{dx} = 3x^2\operatorname{cosec} 2x - 2x^3\operatorname{cosec} 2x\cot 2x$

190. $h'(x) = 4\cos^2 x - 4\sin^2 x$

191. $f'(x) = 24\sin x\cos^2 x - 12\sin^3 x$

192. $h'(x) = \sec x\tan^2 x + \sec^3 x$

193. $k'(x) = \sin x(\sec^2 x + 1)$

194. $f'(x) = -24\sin 4x\cos 4x$

195. $q'(x) = 3x^4\cos x + 12x^3\sin x$

196. $q'(x) = (12x^3 + 6x^2 + 4x + 1)e^{3x^2+2}$

197. $q'(x) = 6x\ln(3x-1) + \frac{9x^2}{3x-1}$

198. $\frac{dy}{dx} = (a + 2a^2x^2 + 2abx)e^{ax^2+b}$

199. $f'(x) = ae^{ax+b}\ln(ax+b) + \frac{ae^{ax+b}}{ax+b}$

Page 112

200. $g'(x) = \frac{6x^2-6x-1}{(2x-1)^2}$

201. $g'(x) = \frac{30x^2-16x-17}{(2x^2+8x-1)^2}$

202. $\frac{dy}{dx} = \frac{2x-3x^2}{2\sqrt{x}(x^2-2x)^2}$

203. $\frac{dy}{dx} = \frac{-(4x-5)}{2\sqrt{x}(4x+5)^2}$

204. $g'(x) = \frac{12x+1-15x^2}{3x^{2/3}(3x^2-6x+1)^2}$

205. $h'(x) = \frac{x^{1/3}+3}{6\sqrt{x}(x^{1/3}+1)^2}$

Page 113

206. $\frac{dy}{dx} = \frac{60x^2+64x+8}{e^{3x}(5x^2+2x)^2}$

207. $q'(x) = \frac{8x^2+8x-2}{e^x(4x^2-1)^{3/2}}$

208. $\frac{dy}{dx} = \frac{(24x^3-18x)e^{x^2+1}}{(4x^2-1)^{3/2}}$

209. $f'(x) = \frac{(2x^3-8x)e^{x^2}}{(x^2-3)^2}$

Page 113 cont...

210. $\frac{dy}{dx} = \frac{(48x^2-12x-3)e^{4x}}{x^2(4x^2+1)^2}$

211. $\frac{dy}{dx} = \frac{1-2\ln 2x}{x^3}$

212. $\frac{dy}{dx} = \frac{2ae^{ax}(ax^2+b-2x)}{(ax^2+b)^2}$

213. $\frac{dy}{dx} = \frac{(x-b)-2x\ln(x+b)}{(x^2-b^2)^2}$

214. $\frac{dy}{dx} = \frac{-1}{1+\sin x}$

215. $f'(x) = 8x\cos 4x - 4(4x^2-1)\sin 4x$

Page 114

216. $m'(t) = \frac{2\cos 2t(1-t^2)+2t\sin 2t}{(1-t^2)^2}$

217. $f'(x) = \frac{a(1+\sin x-x\cos x)}{(1+\sin x)^2}$

Page 115

218. $f'(x) = \frac{-8}{x^3} - 10x$

219. $g'(x) = 15x^2 + 7 - \frac{3}{x^2}$

220. $\frac{dy}{dx} = 4x^3$

221. $h'(r) = 4\pi r + \frac{3}{\pi}$

222. $\frac{dy}{dx} = -10e^{2x}(1-e^{2x})^4$

223. $\frac{dy}{dx} = \frac{1}{x} + 2e^{2x}$

224. $m'(x) = \frac{\sec^2 x}{2\sqrt{\tan x}}$

225. $p'(x) = \frac{-11}{(2x-3)^2}$

226. $k'(x) = \frac{2x}{x^2+2}$

227. $\frac{dy}{dx} = \frac{\sec^2 x}{\tan x} = \operatorname{cosec} x\sec x$

228. $f'(x) = \frac{6}{5}(3x+5)^{-3/5}$

229. $\frac{dy}{dx} = \frac{-4}{(3x-1)^2}$

230. $k'(x) = \frac{-8}{x^3} + \frac{2}{x^2} - \frac{9}{x^4}$

231. $\frac{dy}{dx} = 4(6x-2)(3x^2-2x+1)^3$

Page 116

232. $f'(x) = (10x^2+5)e^{x^2}$

233. $g'(x) = \frac{e^{2x}}{2\sqrt{x}} + 2\sqrt{x}e^{2x}$

$g'(x) = \frac{e^{2x}(1+4x)}{2\sqrt{x}}$

234. $\frac{dy}{dx} = 10(2x^2+x-3)(4x^2+2x-1)$

235. $g'(x) = 3e^{3x}\sin x + e^{3x}\cos x$

$g'(x) = e^{3x}(3\sin x + \cos x)$

236. $h'(x) = 3e^{3x}\ln(2x) + \frac{e^{3x}}{x}$

237. $j'(x) = \frac{(x^2+6)\sec x\tan x - 2x\sec x}{(x^2+6)^2}$

238. $k'(x) = \frac{12x^2(1+2\ln x)-8x^2}{(1+2\ln x)^2}$

$k'(x) = \frac{4x^2(1+6\ln x)}{(1+2\ln x)^2}$

239. $\frac{dy}{dx} = \frac{2(1-x^2)\cos x + 4x\sin x}{(1-x^2)^2}$

240. $\frac{dy}{dx} = \frac{2\cos^2 x + 2\sin x + 2\sin^2 x}{\cos^2 x}$

$\frac{dy}{dx} = \frac{2}{1-\sin x}$

or $\frac{dy}{dx} = 2 + 2\sec x\tan x + 2\tan^2 x$ if using the product rule.

241. $h(x)' = \frac{-(\cos x+1)}{\sin^2 x}$

242. $g'(x) = \frac{2(2x-1)^{-1/2}e^{3x} - 6e^{3x}(2x-1)^{1/2}}{(2e^{3x})^2}$

$g'(x) = \frac{(2-3x)e^{-3x}}{\sqrt{2x-1}}$

243. $k'(x) = 12(3x+1)^3(x-2)^{1/2} + \frac{1}{2}(3x+1)^4(x-2)^{-1/2}$

$k'(x) = \frac{(3x+1)^3(27x-47)}{2(x-2)^{0.5}}$

Page 118

244. $\frac{dy}{dx} = 2x - \frac{1}{x}$

Setting $2x - \frac{1}{x} = 1$

$x = 1, x = {}^{-}0.5$

Coordinates (1,1)

245. $\frac{dy}{dx} = \frac{{}^{-}3}{x^2} + \frac{1}{3}$

Setting $\frac{{}^{-}3}{x^2} + \frac{1}{3} = {}^{-}1$

$x = {}^{-}1.5, x = 1.5$

246. $\frac{dy}{dx} = \frac{{}^{-}1}{(x+1)^2}$

When $x = {}^{-}3, \frac{dy}{dx} = \frac{{}^{-}1}{4}$

Gradient of normal $= 4$

247. $6y - x - 31 = 0$

248. $y = 5x$

249. $y = {}^{-}0.3296x + 6.397$

Page 119

250. $y = 2x + 8$

251. $\frac{dy}{dx} = {}^{-}2x + 2$. At $x = 2$

$\frac{dy}{dx} = {}^{-}2$ Grad. of the line is ${}^{-}2$.

252. $4y - 65x = {}^{-}68$

253. $y = {}^{-}3$

254. $y = 3x + 12$

255. $y = {}^{-}3x + 15$

256. $\frac{dy}{dx} = \frac{{}^{-}k}{(x-1)^2} - 2$

$1 = \frac{{}^{-}k}{4} - 2$

$k = {}^{-}12$

257. $\frac{dy}{dx} = \frac{k}{kx-1} + 3$

$5 = \frac{k}{2k-1} + 3$

$k = \frac{2}{3}$

258. $y = \frac{{}^{-}1}{4}x + \frac{15}{4}$

259. $y = 4x + 1.369$

or $y - 2\sqrt{3} = 4(x - \frac{\pi}{6})$

Page 123

260. Minimum at (3, ${}^{-}1$)

261. Minimum at (1.5, ${}^{-}20.25$)

262. Maximum at (1.5, 12.25)

263. a) Minimum at (${}^{-}3$, ${}^{-}32$)
Maximum (1, 0)

b) Decreasing:
$x < {}^{-}3$ or $x > 1$

264. Maximum at (${}^{-}2$, 16.33)
Minimum at (4, ${}^{-}19.67$)
Increasing: $x < {}^{-}2$ and $x > 4$

265. Maximum at (5, 98)
Minimum at (${}^{-}1$, ${}^{-}10$)
Increasing: ${}^{-}1 < x < 5$

Page 124

266. Maximum at (0, 4)
Minimum at (2, 0)

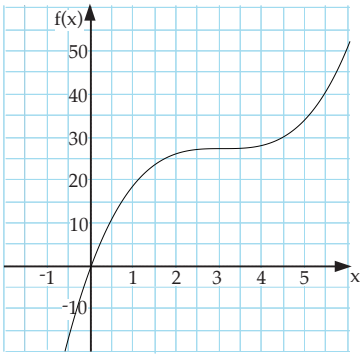
267. Maximum at (${}^{-}2$, ${}^{-}4$)
Minimum at (2, 4)

268. $\frac{dy}{dx} = 3x^2 - 18x + 27$
$= 3(x-3)^2$

Stationary point (3, 27)

$\frac{d^2y}{dx^2} = 6x - 18$

$\frac{d^2y}{dx^2} = 0$ when $x = 3$,
so (3, 27) is a point of inflection.



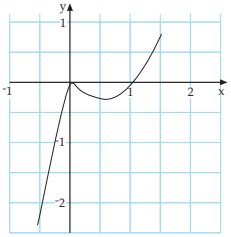
269. a) $x = 0, 1$

b) $f'(x) = 5x^{2/3} - 4x^{1/3}$

c) Maximum at (0, 0)
Minimum at (0.512, ${}^{-}0.25$)

d) Increasing $x < 0$ or
$x > 0.512$

e)

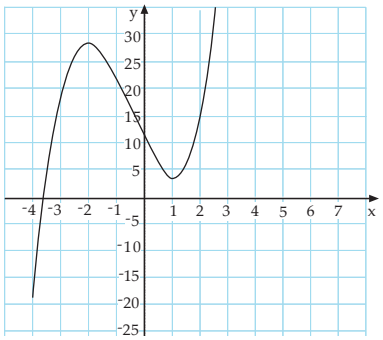


Page 128

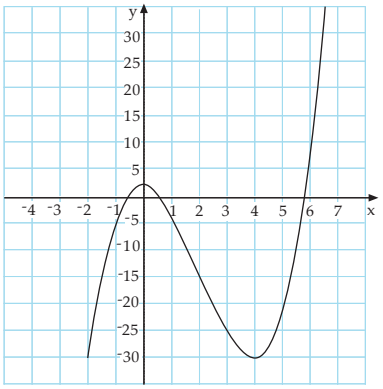
270. Maximum (${}^{-}2$, 41)
Minimum at (0.5, 9.75)
Inflection (${}^{-}0.75$, 25.375)

271. Maximum (${}^{-}2$, 53)
Minimum at (3, ${}^{-}72$)
Inflection (0.5, ${}^{-}9.5$)

272. Maximum (${}^{-}2$, 27)
Minimum at (1, 0)
Inflection (${}^{-}0.5$, 13.5)
Concave down $x < {}^{-}0.5$



273. Maximum (0, 2)
Minimum at (4, ${}^{-}30$)
Inflection (2, ${}^{-}14$)
Concave up $x > 2$



Page 129

274. $f'(x) = \frac{1}{x} - 0.25x$

Stationary points $x = 2$ and ${}^{-}2$, but ignore ${}^{-}2$ as x must be positive for ln to exist.
Maximum at (2, 3.193)
No point of inflection as $f''(x) = 0$ has no real solutions.

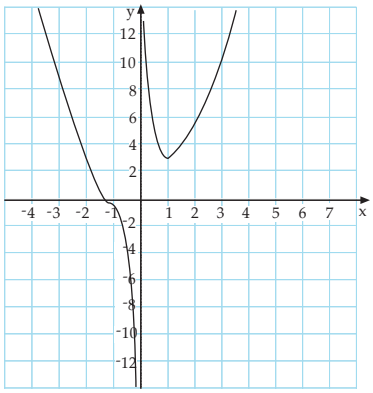
275. $f'(x) = e^x - 4$

$f'(x) = 0$ at $x = 1.386$.
Minimum (1.386, 0.455).
Concave up for all x (no point of inflection).

Page 129 cont...

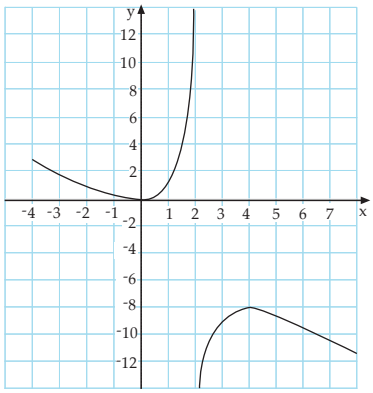
276. $f'(x) = 2x - \frac{2}{x^2}$

Minimum (1, 3)
Inflection (⁻1.26, 0)
Concave up when $x < ⁻1.26$ and $x > 0$.



277. $f'(x) = \frac{4x - x^2}{(2-x)^2}$

Minimum (0, 0)
Maximum at (4, ⁻8)
Decreasing $x < 0$ or $x > 4$



Page 130

278. A = 4, B = 3

279. A = 2, B = 3, C = ⁻1

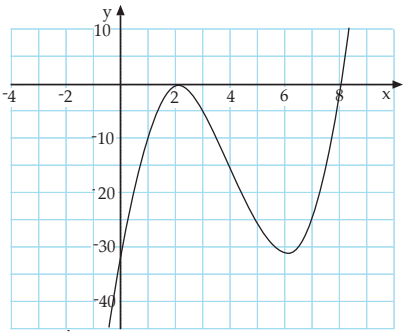
280. k = 3 and $k = 1\frac{1}{3}$

281. $A = \frac{2}{3}$, $B = ⁻1\frac{1}{2}$, C = ⁻2

Page 134

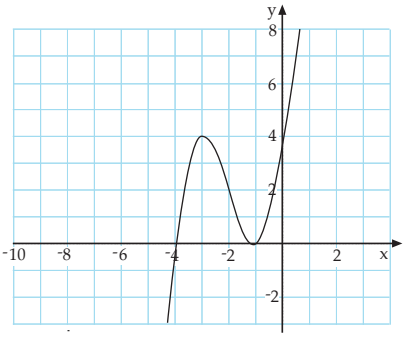
282. $\frac{dy}{dx} = 3x^2 - 24x + 36$

Intercepts (0, ⁻32), (2, 0), (8, 0)
Maximum (2, 0)
Minimum (6, ⁻32)
Inflection (4, ⁻16)



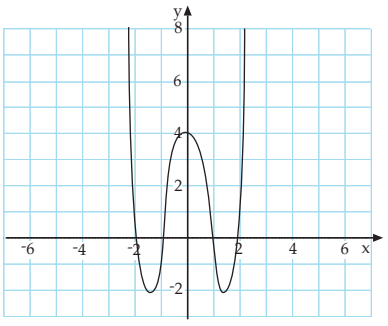
283. $\frac{dy}{dx} = 3x^2 + 12x + 9$

Intercepts (0, 4), (⁻4, 0), (⁻1, 0)
Maximum (⁻3, 4)
Minimum (⁻1, 0)
Inflection (⁻2, 2)



284. $\frac{dy}{dx} = 4x^3 - 10x$

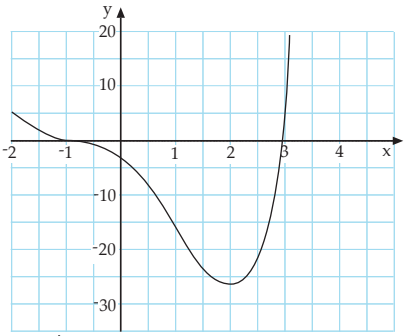
Intercepts (0, 4), (⁻2, 0), (⁻1, 0), (1, 0) and (2, 0)
Maximum (0, 4)
Minimum (⁻1.58, ⁻2.25) and (1.58, ⁻2.25)
Inflection (⁻0.913, 0.527), (0.913, 0.527)



Page 135

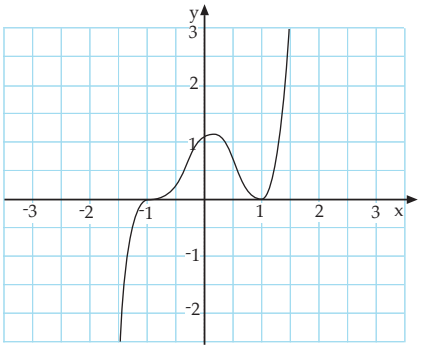
285. $\frac{dy}{dx} = 4x^3 - 12x - 8$

Intercepts (0, ⁻3), (⁻1, 0), (3, 0)
No maximum
Minimum (2, ⁻27)
Points of inflection (⁻1, 0) and (1, ⁻16)



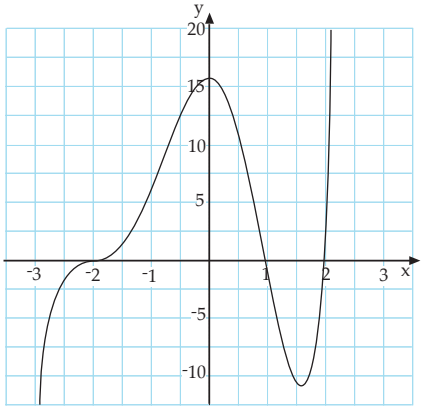
286. $\frac{dy}{dx} = 5x^4 + 4x^3 - 6x^2 - 4x + 1$

Intercepts (0, 1), (⁻1, 0), (1, 0)
Maximum (0.2, 1.106)
Minimum (1, 0)
Points of inflection (⁻1, 0), (⁻0.29, 0.60) and (0.69, 0.46)



287. $\frac{dy}{dx} = 5x^4 + 12x^3 - 12x^2 - 32x$

Intercepts (0, 16), (⁻2, 0), (1, 0) and (2, 0)
Maximum (0, 16)
Minimum (1.6, ⁻11.197)
Points of inflection (⁻2, 0), (⁻0.8, 8.71) and (1, 0)

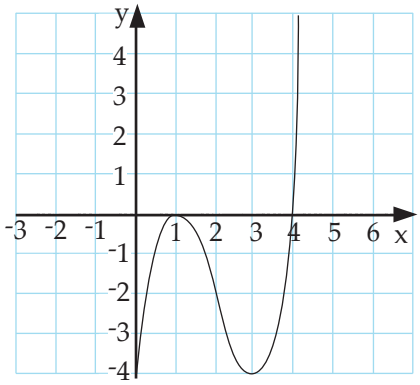


Page 136

288. a) Maximum (1, 0)
Minimum (3, ⁻4)

b) Inflection (2, ⁻2)

c)



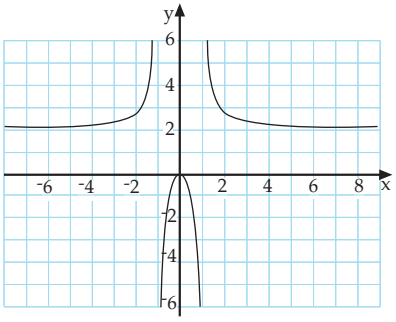
d) $x > 2$

289. a) A = (⁻0.645, 0)
B = (⁻0.5, 0.25)
C = (0, 0)
D = (0.5, ⁻0.25)
E = (0.645, 0)

b) At the points of inflection
$x = 0, \pm 0.3536$
Gradient when x = 0.3536 is ⁻0.938 (3 sf)

290. a) Maximum point (0, 0)

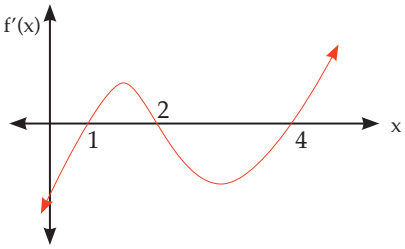
b)



c) limit = 2

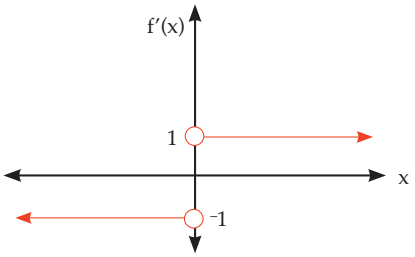
Page 139

291.

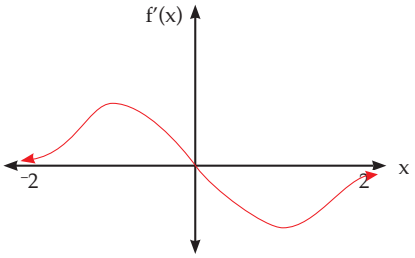


Page 139 cont...

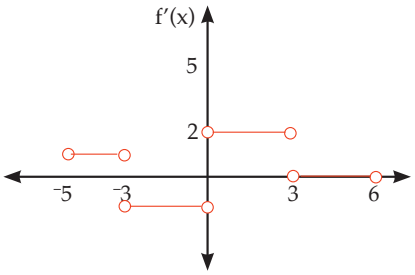
292.



293.

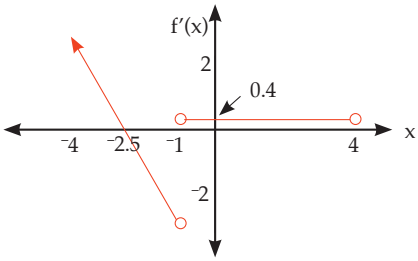


294.

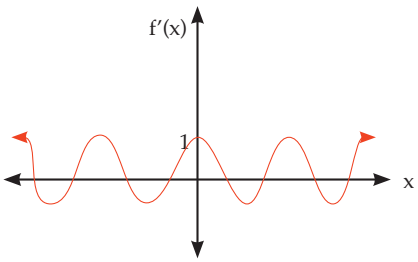


Page 140

295.



296.



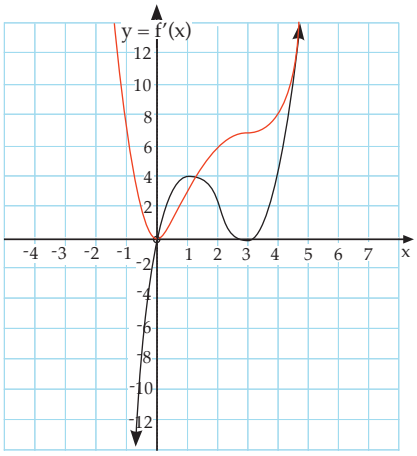
297. a) x = 0

b) x = 1 and 3

c) Stationary point of inflection because at x = 3, y = f'(x) is both an intercept and a stationary point.

Page 140 cont...

297. d)



Page 142

298. $\frac{dy}{dx} = \frac{3}{10t}$

299. $\frac{dy}{dx} = \frac{t}{2}$

300. $\frac{dy}{dx} = \frac{-2}{5}\tan t$

301. $\frac{dy}{dx} = \frac{\cos t}{e^t}$

302. $\frac{dy}{dx} = 4t\sqrt{t}$

303. $\frac{dy}{dx} = 2t^2(1-t)$

304. $\frac{dy}{dx} = \frac{3t^2-1}{2t-3}$

305. $\frac{dy}{dx} = \frac{2t^2+1}{t^2-1}$

306. $\frac{d^2y}{dx^2} = \frac{3}{4t}$

307. $\frac{d^2y}{dx^2} = \frac{-5}{16\cos^3 t}$

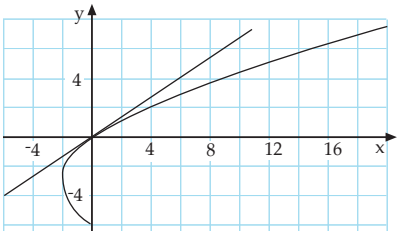
Page 143

308. $\frac{d^2y}{dx^2} = -3\sec^3 t$

309. $\frac{d^2y}{dx^2} = \frac{-(t+1)}{t^2e^{2t}}$

310. $\frac{dy}{dx} = \frac{2}{2t+3}$

At t = 0 the tangent is $3y - 2x = 0$



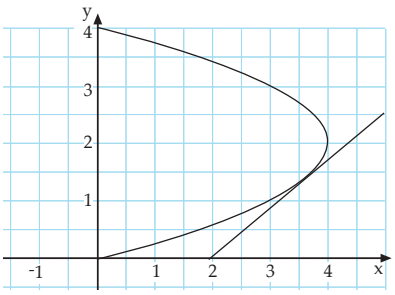
Page 143 cont...

311. $\frac{dy}{dx} = \frac{-1}{2t}$

At t = −0.6 the tangent is

$y = \frac{1}{30}(25x - 49)$

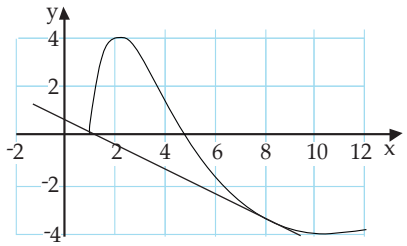
$= 0.833x - 1.6333$



Page 144

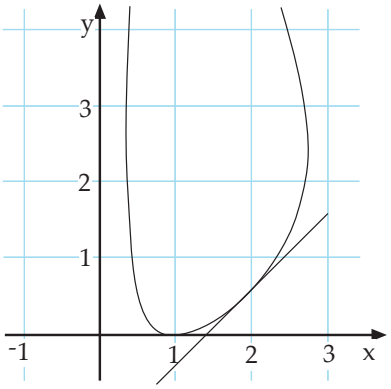
312. $\frac{dy}{dx} = \frac{4\cos t}{0.5e^{t/2}}$

$y = -0.524x + 0.790$



313. $\frac{dy}{dx} = \frac{2t}{\cos t \times e^{\sin t}}$

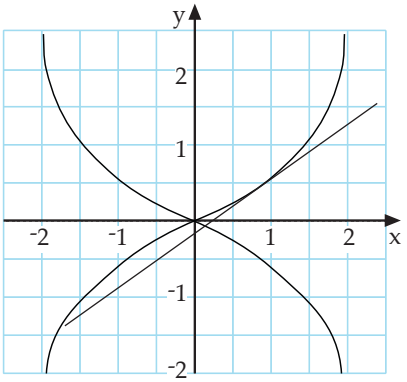
$y = 1.061x - 1.537$



Page 144 cont...

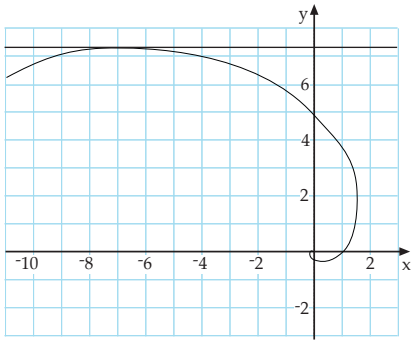
314. $\frac{dy}{dx} = \frac{1}{2\cos^3 t}$

$y = 0.740x - 0.163$



315. $\frac{dy}{dx} = \frac{\cos t + \sin t}{\cos t - \sin t}$

$y = 7.461$



Page 147

316. $\frac{dy}{dx} = \frac{1}{y}$

$= \frac{1}{3}$

317. $\frac{dy}{dx} = \frac{-y}{x + 2y}$

$= \frac{-1}{3}$

318. $\frac{dy}{dx} = \frac{4x}{3y}$

319. $\frac{dy}{dx} = \frac{-x - 2y}{2x + 3y}$

320. $\frac{dy}{dx} = \frac{3x^2 - y^2}{2xy - 2y}$

321. $\frac{dy}{dx} = \frac{-3x^2 - 2y}{3y^2 + 2x}$

322. $\frac{dy}{dx} = \frac{-2x}{e^y}$

323. $\frac{dy}{dx} = 3x^2y + 4y$

Page 147 cont...

324. $\frac{dy}{dx} = \frac{-2xy^2}{1 + x^2y}$

325. $\frac{dy}{dx} = \frac{3y^2 - 8xy}{4x^2 - 6xy}$

326. $\frac{dy}{dx} = \frac{-\sqrt{y}}{\sqrt{x}}$

327. $\frac{dy}{dx} = \frac{-3x^2 + 6xy^3 - 3y^2}{-9x^2y^2 + 6xy - 2y}$

Page 150

328. $\frac{dh}{dt} = \frac{1}{\pi(0.45)^2} \times 0.123$

$= 0.193 \text{ m/min}$

329. $\frac{dh}{dt} = \frac{1}{4\pi(3.25)^2} \times 1.45$

$= 0.0109 \text{ m/s}$

330. a) 0.94 cm/s (2 sf)

b) 3.3 cm^2/s (2 sf)

Page 151

331. $\frac{dh}{dt} = \frac{9}{4\pi h^2} \times \frac{dV}{dt}$

$= 0.895 \text{ cm/m}$

332. a) 9.425 mm^2/s

b) 1.696 mm^2/s

333. $\frac{5.26}{x + y} = \frac{1.71}{y}$

$y = \frac{1.71x}{3.55}$

$\frac{dy}{dt} = 2.2 \text{ m/s}$

Page 152

334. $x = 1000 \cot \theta$

$\frac{dx}{dt} = \frac{-1000}{(\sin x)^2} \times \frac{d\theta}{dt}$

$= -43.91 \text{ m/s}$

335. $\frac{dx}{dt} = \frac{s}{\sqrt{s^2 - 4.2^2}} \times \frac{ds}{dt}$

$= 1050 \text{ km/h}$

Page 152 cont...

336. $\frac{dA}{dt} = 0.00263\ m^2/s$

337. r = 9.35 cm, $\frac{dV}{dt} = 125\ cm^2/s$

$$\frac{dSA}{dt} = \frac{dSA}{dr} \times \frac{dr}{dV} \times \frac{dV}{dt}$$

$$= 8\pi r \times \frac{1}{4\pi r^2} \times \frac{dV}{dt}$$

$= 26.7\ cm^2/s$

Page 156

338. $Cost = 150w^2 + \frac{2400}{w}$

$Cost' = 300w - \frac{2400}{w^2}$

Minimum cost when w = 2, h = 2.5 m giving Cost = $1800

339. $SA = \frac{500000}{r} + 2\pi r^2$

$SA' = \frac{-500000}{r^2} + 4\pi r$

Minimum cost when r = 34.1 cm, h = 68.3 cm

340. a) Maximum height v = 0.
t = 2 s, h(2) = 27 m

b) h(t) = 0 when t = 5 s
v(t) = 12 – 6t
v(5) = −18 m/s

Page 157

341. a) $S = 150h^2 - h^3$
$S' = 300h - 3h^2$
h = 100 mm, w = 50 mm

b) $S = 112^2w - w^3$
$S' = 112^2 - 3w^2$
h = 91.4 mm,
w = 64.7 mm

342. $Area = -y^2 + 3y + 25$
$Area' = -2y + 3$
y = 1.5 m, x = 3.5 m
Area of deck = 27.25 m^2

343. a) $v = -3t^2 + 30t - 60$
$0 = -3t^2 + 30t - 60$
t = 2.764, 7.236 seconds
Minimum h = 27.6 m
Maximum h = 72.4 m

b) h = 0, t = 10 seconds
v = −60 m/s
a = −30 m/s^2

Page 158

344. $P = x(450 - 9.5x) - (2500 + 15x + 0.25x^2)$
$P' = 435 - 19.5x$
x = 22.3
= 22 puppies per year

345. a) $C = 9.125x + \frac{5000}{x} + 250$

b) $C' = \frac{-5000}{x^2} + 9.125$
x = 23 rats
C″(23) > 0 to show min.

Page 159

346. $V = \frac{1}{3}\pi r^2 h$ and $R^2 = r^2 + h^2$

$V = \frac{1}{3}\pi R^2 h - \frac{1}{3}\pi h^3$

$V' = \frac{1}{3}\pi R^2 - \pi h^2$

h = 0.5774R, r = 0.8165R
$2\pi r = R(2\pi - A)$
A = 1.153 radians
$V_{max} = 0.404R^3$

347. $Save = \frac{x}{6} + \frac{32}{1.8} - \frac{\sqrt{x^2 + 32^2}}{1.8}$

$Save' = \frac{1}{6} - \frac{x}{1.8(x^2 + 32^2)^{0.5}}$

x = 10.06 m

Page 160

348. Since $\frac{3}{x-7} = \frac{y-3}{7}$

then $y = \frac{21}{(x-7)} + 3$

$A = 0.5xy - 21$

so $A = \frac{3x^2}{2(x-7)} - 21$

and $A' = \frac{12x(x-7) - 6x^2}{(2(x-7))^2}$

so x = 14, y = 6

and $m = \frac{-3}{7}$

3x + 7y = 42

Page 160 cont...

349. h = u + v

$h = \frac{v^2}{v - f}$

$h' = \frac{2v(v-f) - v^2}{(v-f)^2}$

$v^2 - 2vf = 0$

v = 2f

Substituting in h

h = 4f

Pages 161 – 166

Practice External Assessment – Apply differentiation methods in solving problems.

In the external examinations NZQA uses a different approach to marking based on understanding (u), relational thinking (r) and abstract thinking (t). They then allocate marks to these concepts and add them up to decide upon the overall grade. This approach is not as easy for students to self mark as the NuLake approach but the results should be broadly similar.

Question One

a) $\frac{dy}{dx} = 6x^2 \tan 3x + 3(2x^3 + 3)\sec^2 3x$ **A**

b) $f'(x) = 4\sin(2x + \pi)\cos(2x + \pi)$

$f'(\frac{\pi}{8}) = 2$

$y - \frac{1}{2} = 2\left(x - \frac{\pi}{8}\right)$ **A**

c) (i) 1. $x = 0, 4$

2. $x = -4, 0, 4$

3. $x = -6, 2$

4. $-8 < x < -4$ **At least 2 answers correct A**

(ii) Does not exist. **At least 4 answers correct M**

d) $\frac{dy}{dx} = 2e^x \cos 2x + e^x \sin 2x$ **A**

$\tan 2x = -2$

$x = 1.017$ (4 sf) **M**

e) $\frac{dy}{dx} = 8e^{-x} - 8xe^{-x}$ **A**

Turning point $x = 1$ **M**

$\frac{d^2y}{dx^2} = -8e^{-x} - 8e^{-x} + 8xe^{-x}$

At $x = 1$, this is negative so maximum point.

Required proof E

Question Two

a) $f'(x) = \frac{1}{3}(4x + x^2)^{-2/3}(4 + 2x)$ **A**

b) $f'(x) = 3x^2 - 8x + 3$

$f'(3) = 6$

$m_N = \frac{-1}{6}$

Equation of normal $6y + x - 3 = 0$ **A**

c) Area = xy

$= x.8x\, e^{-x}$ **A**

$A' = 16x\, e^{-x} - 8x^2 e^{-x}$

Max at $x = 2$

Area = 4.331 **M**

d) $\text{Cost} = (32 - x)k + 3k\sqrt{16^2 + x^2}$ **A**

$\text{Cost}' = -k + \frac{3xk}{\sqrt{16^2 + x^2}}$

$x = 5.66$ km **M**

e) $\frac{dH}{dt} = \frac{dH}{dr} \cdot \frac{dr}{dV} \cdot \frac{dV}{dt}$

$V = \frac{4\pi R^3}{3}$. After 40 seconds, $V = 3.68\ m^3$

$R = 0.9577$ m

$\frac{dH}{dt} = 3 \times \frac{1}{4\pi R^2} \times 0.0920$ **M**

$= 0.0239$ m/s **E**

Question Three

a) $f'(x) = \frac{2e^{2x}(1 + \tan 2x) - 2e^{2x}\sec^2 2x}{(1 + \tan 2x)^2}$ **A**

b) $f(x) = e^{-(x+k)^2}$

$f'(x) = -2(x + k)e^{-(x+k)^2}$ **A**

$f''(x) = e^{-(x+k)^2}(4(x + k)^2 - 2)$ **M**

Setting $f''(x) = 0$

$4(x + k)^2 - 2$

$x = -k \pm \frac{1}{\sqrt{2}}$

Correct solution with f'(x) and f''(x) E

c) $f'(x) = 0.5\,(3 + x^2)^{-0.5} \times 2x$ **A**

$f'(x) = 0.5$ at $x = 1$, $f(1) = 2$, so coordinates (1, 2)

Derivative set equal to 0.5 and answer of (1, 2) found. **M**

d) $\frac{dy}{dt} = 2t$

$\frac{dx}{dt} = \frac{0.25}{(t + 2)^{0.75}}$

$\frac{dy}{dx} = 8t(t + 2)^{0.75}$ **A**

Turning points $t = 0, -2$.

Minimum $t = 0$.

Coordinates (1.189, 0)

Turning points found (t = 0 and t = −2).
Minimum identified (t = 0) and justified by use of the second derivative $\frac{d^2y}{dx^2} = 8(t + 2)^{0.5}(7t + 8)$ **M**

e) $\tan\theta = \frac{h}{x}$, $L = \frac{x + d}{\cos\theta}$

$L = h\,\text{cosec}\,\theta + d\sec\theta$

$L' = -h\,\text{cosec}\,\theta\cot\theta + d\sec\theta\tan\theta$ **M**

$L' = 0$

$\frac{h}{\sin\theta\tan\theta} = \frac{d\tan\theta}{\cos\theta}$

$\tan\theta = \sqrt[3]{\frac{h}{d}}$ **E**

Sufficiency. For each question award yourself a score out of 8 using this table. Add the three scores for a score out of 24 and compare to the cut scores. All answers must include derivatives where appropriate.

Quest.	N0	N1	N2	A3	A4	M5	M6	E7	E8
ONE	No diff's. correct.	1 diff. with error.	1 A or 1 diff. correct.	2A or 2 diff's. correct.	3A or 3 diff's. correct.	1M + 1M minor error.	2M all correct.	1E minor error.	1E all correct.
TWO	No diff's. correct.	1 diff. with error.	1 A or 1 diff. correct.	2A or 2 diff's. correct.	3A or 3 diff's. correct.	1M + 1M minor error.	2M all correct.	1E minor error.	1E all correct.
THREE	No diff's. correct.	1 diff. with error.	1 A or 1 diff. correct.	2A or 2 diff's. correct.	3A or 3 diff's. correct.	1M + 1M minor error.	2M all correct.	1E minor error.	1E all correct.
Cut Scores									
Not Achieved		**Achievement**		**Achievement with Merit**		**Achievement with Excellence**			
0 – 6		7 – 13		14 – 20		21 – 24			

Answers

Page 169

1. $\frac{1}{2}x^4 + \frac{4}{3}x^3 - \frac{5}{2}x^2 + 7x + C$

2. $5x^3 - \frac{35}{2}x^2 + C$

3. $\frac{Ax^3}{3} - 3Bx + C$

4. $3x^4 + 2x^3 + 5x + 3x^{-2} + C$

$= 3x^4 + 2x^3 + 5x + \frac{3}{x^2} + C$

5. $2x - x^{-1} + x^{-3} + C$

$= 2x - \frac{1}{x} + \frac{1}{x^3} + C$

6. $6x^2 + 4x^{3/2} + 2x^{5/2} + C$

7. $\frac{2}{3}x^{3/2} - 2x^{5/2} + C$

$= \frac{2}{3}\sqrt{x^3} - 2\sqrt{x^5} + C$

8. $\frac{3}{16}x^4 - \frac{2}{15}x^3 + \frac{1}{10}x^2 - 7x + C$

9. $\frac{2}{3}x^{3/2} - x^{-1} + C$

$= \frac{2}{3}\sqrt{x^3} - \frac{1}{x} + C$

10. $9x^{4/3} + 16x^{1/2} + C$

$= 9\sqrt[3]{x^4} + 16\sqrt{x} + C$

11. $2x^{0.5} + 8x^{1.25} + C$

$= 2\sqrt{x} + 8\sqrt[4]{x^5} + C$

12. $12x^{1/2} + C = 12\sqrt{x} + C$

Page 170

13. $y = 18x^{4/3} + C = 18\sqrt[3]{x^4} + C$

14. $21x^{5/3} + C = 21\sqrt[3]{x^5} + C$

15. $\frac{4}{5}\sqrt[4]{x^5} + 3x + C$

$= 0.8x^{1.25} + 3x + C$

16. $\frac{1}{5}x^5 + \frac{1}{4}x^4 + \frac{1}{3}x^3 + C$

17. $\int x^2 - 10x + 25 \ dx$

$= \frac{1}{3}x^3 - 5x^2 + 25x + C$

18. $\int x^2 - 4 \ dx = \frac{1}{3}x^3 - 4x + C$

19. $\int 4x^2 + 4x + 1 \ dx$

$= \frac{4}{3}x^3 + 2x^2 + x + C$

20. $= B\int x^2 + 8x + 12 \ dx$

$= B(\frac{1}{3}x^3 + 4x^2 + 12x) + C$

Page 170 cont...

21. $\int 2x^2 - 6x^{-2} \ dx$

$= \frac{2}{3}x^3 + 6x^{-1} + C$

$= \frac{2x^3}{3} + \frac{6}{x} + C$

22. $\int x^{1.5} - 4x^{-0.5} \ dx$

$= \frac{1}{2.5}x^{2.5} - \frac{4}{0.5}x^{0.5} + C$

$= \frac{2}{5}\sqrt{x^5} - 8\sqrt{x} + C$

Page 172

23. $\frac{1}{7}e^{7x} + C$

24. $\frac{5}{4}e^{4x} + C$

25. $^-2e^{-3x} + C$

26. $\frac{A^2}{4}e^{4x} + C$

27. $16e^{x/2} + C$

28. $\frac{1}{3}e^{3x+4} + C$

29. $\int e^{-2x} \ dx = \frac{^-1}{2}e^{-2x} + C$

30. $\int e^{0.5x} \ dx = 2e^{0.5x} + C$

$= 2\sqrt{e^x} + C$

31. $\frac{e^{4x}}{4} + e^{2x} + x + C$

32. $\frac{^-1}{9}e^{-3x} - \frac{2}{5}e^{-5x} + C$

33. $\frac{x^2}{2A} + \frac{e^{-3x}}{3} + C$

34. $\frac{^-e^{-2x}}{2} + e^x + C$

Page 175

35. $\frac{^-1}{3}\cos 3x + C$

36. $2 \sin 2x + C$

37. $4 \tan 3x + C$

38. $\frac{3}{5}\sec 5x + C$

39. $\frac{^-4}{3}\operatorname{cosec} 3x + C$

40. $^-3\sin(4x + 3) + \frac{e^{2x+1}}{2} + C$

41. $\frac{1}{AB}\tan(Bx) + C$

42. $\frac{^-2}{3}\operatorname{cosec}(3x + 1) + C$

43. $\frac{^-3}{2}\cot(2x - 1) + C$

Page 175 cont...

44. $\frac{2}{3}\sin 3x - \frac{5}{2}\cos 2x + C$

45. $\frac{^-e^{-2x}}{2} + \frac{3}{2}\sin 2x + C$

46. $\frac{^-3}{2}\cos 4x - \frac{3}{2}\sin 2x + C$

Page 176

47. $3 \ln|x| + C$

48. $\frac{1}{4}\ln|x| + C$

Page 177

49. $\frac{2}{3}\ln|x| + C$

50. $^-B \ln|x| + C$

51. $\frac{A}{B}\ln|x| + C$

52. $A\ln|x| + \frac{e^{Bx}}{B} + C$

53. $A \ln|x + 1| + C$

54. $\frac{x^2}{2A} + A\ln|x| + C$

55. $\frac{1}{2}\ln|2x + 1| + C$

56. $\ln|6x - 5| + C$

57. $\frac{A}{4}\ln|4x - 1| + C$

58. $\frac{3}{5}\ln|5x + 2| + C$

59. $3x - 2\ln|x| + C$

60. $\frac{^-1}{4}\ln|3 - 4x| + C$

Page 178

61. $\int \frac{3}{x} + 1 \ dx = 3 \ln|x| + x + C$

62. $\int \frac{4}{x} - 1 \ dx = 4 \ln|x| - x + C$

Page 179

63. $\int 3 + \frac{5}{x} \ dx = 3x + 5 \ln|x| + C$

64. $\int 2 - \frac{7}{x} \ dx = 2x - 7 \ln|x| + C$

65. $\int 3x + 7 - \frac{4}{x} \ dx$

$= \frac{3}{2}x^2 + 7x - 4 \ln|x| + C$

66. $\frac{Ax^2}{2} + Bx - D \ln|x| + C$

67. $\frac{5}{3}x^3 - 3x^2 + 7x + 3 \ln|x| + C$

68. $\frac{2}{3}x^3 + \frac{3}{2}x^2 + \frac{1}{2}x + 3 \ln|x| + C$

69. $2x^2 + x - A \ln|x| + C$

Page 179 cont...

70. $\frac{Ax^2}{2} + B\ln|x| + \frac{D}{x} + C$

71. $\frac{1}{3}x^3 - \frac{3}{2}x^2 + 5\ln|x| + C$

72. $x^5 + 2\ln|x| - \frac{3}{x} + C$

73. $\frac{1}{5}x^5 + \frac{1}{6}x^2 - \frac{5}{3}\ln|x| + C$

74. $\ln|x| + \frac{B}{Ax} + C$

Page 181

75. $3x - 11\ln|x+1| + C$

76. $3x + 9\ln|x-2| + C$

77. $2x - \frac{7}{4}\ln|4x+5| + C$

78. $3x + 18\ln|x-6| + C$

79. $3x + \frac{1}{2}\ln|2x-1| + C$

80. $^-6x - 6\ln|1-x| + C$

81. $2x + \frac{1}{4}\ln|4x+3| + C$

82. $\frac{x}{2} + \frac{1}{2}\ln|2x-2| + C$

Page 182

83. $2x - \ln|x-1| + C$

84. $^-3x + 13\ln|x+4| + C$

85. $x - 4\ln|2x+3| + C$

86. $3x + 4\ln|3-2x| + C$

87. $4x + 8\ln|x-2| + C$

88. $\frac{x}{3} - \frac{2}{9}\ln|3x+2| + C$

89. $^-7x - 32\ln|x-5| + C$

90. $2x - \ln|2-x| + C$

Page 184

91. $\ln|6x+3| + C$

92. $3\ln|x-2| + C$

93. $2\ln|x^2+1| + C$

94. $\frac{A}{3}\ln|3x-1| + C$

95. $\ln|x^2-3x+1| + C$

96. $2\ln|x^2+x-2| + C$

97. $2\ln|x^2-1| + C$

98. $\ln|e^{2x}-5| + C$

Page 185

99. $\ln|\tan 2x| + C$

100. $\ln|\cos 3x| + C$

101. $2\ln|\operatorname{cosec} 5x| + C$

102. $2\ln|7-\sin 4x| + C$

103. $\ln|\ln|x|| + C$

104. $\ln|e^{3x}+7| + C$

105. $2\ln|e^{x^2}-3| + C$

106. $^-\ln\left|\cos\left(x-\frac{\pi}{4}\right)\right| + C$

107. $\ln|\cos x + \sin x| + C$

108. $\frac{1}{2}\ln|e^{2x}+2x| + C$

Page 187

109. $\int 4(\sin 6x + \sin 4x)\,dx$
$= \frac{^-2}{3}\cos 6x - \cos 4x + C$

110. $\int 4(\sin 4x - \sin 2x)\,dx$
$= ^-\cos 4x + 2\cos 2x + C$

111. $6\sin 2x - 2\sin 6x + C$

112. $^-0.4\cos 10x - 2\cos 2x + C$

113. $\frac{5}{8}\sin 8x + \frac{5}{2}\sin 2x + C$

114. $\frac{^-3}{22}\cos 11x + \frac{3}{2}\cos x + C$

Page 188

115. $4\tan x - 4x + C$

116. $\frac{1}{4}\sin 2x + \frac{1}{2}x + C$

117. $\sin 2x + 2x + C$

118. $^-20\cot x - 20x + C$

119. $\frac{1}{2}\sin 2x + C$ OR
$\sin x \cos x + C$

120. $2x - \sin 2x + C$ OR
$^-2\sin x\cos x + 2x + C$

121. $6x + 3\sin\left(2x - \frac{\pi}{3}\right) + C$ Other forms of this answer possible.

122. $^-4\cos\left(2x + \frac{2\pi}{5}\right) + C$ Other forms of this answer possible

123. $2x^2 + 6x - \frac{3}{2}\sin 4x + C$

124. $6x - 4\sin 2x + \frac{1}{2}\sin 4x + C$

Page 190

125. $(2x+3)^6 + C$

126. $\frac{^-6}{(x-2)^5} + C$

127. $3(x-6)^4 + C$

128. $\frac{^-8}{(x+2)^3} + C$ OR
$^-8(x+2)^{-3} + C$

Page 191

129. $5(x+3) - 9\ln|x+3| + C$

130. $\frac{2}{5}(x+2)^{5/2} - \frac{4}{3}(x+2)^{3/2} + C$

131. $\frac{1}{2}(x^2+4)^6 + C$

132. $8\sqrt{x+2} + C$

133. $\frac{1}{7}(x+5)^7 - \frac{5}{3}(x+5)^6 + 5(x+5)^5 + C$

134. $\int (2u+5)u^5\,du$
$= \int 2u^6 + 5u^5\,du$
$= \frac{2}{7}(x-2)^7 + \frac{5}{6}(x-2)^6 + C$

135. $\ln|x-3| - \frac{3}{x-3} + C$

136. $\frac{^-1}{2(x^2+4x+5)^2} + C$

Page 192

137. $25(x+2)^{6/5} + C$

138. $\frac{1}{8}(x^2+5)^4 + C$

139. $\ln|\ln|x|| + C$

140. $\ln|e^x - 2| + C$

141. $\frac{1}{3}(2x-1)^{3/2} + (2x-1)^{1/2} + C$
$= \frac{1}{3}\sqrt{(2x-1)^3} + \sqrt{2x-1} + C$

142. $\frac{1}{3}\sqrt{(2x-1)^3} + \sqrt{2x-1} + C$
$= \frac{2}{3}(x+1)\sqrt{2x-1} + C$

143. $3e^{x^2} + C$

144. $3e^{x^2} + C$

Page 194

145. $\left[2x^3 - x^2 + x\right]_1^2 = 12$

146. $\left[\frac{1}{3}x^3 - 9x\right]_{-2}^{-1} = -6.667$ (4 sf)

147. $\left[\frac{2}{3}x^{1.5} + 4x^{0.5}\right]_1^4 = 8.667$ (4 sf)

148. $\left[8\ln|x| + \frac{2}{3}x^{1.5}\right]_1^5 = 19.66$ (4 sf)

149. $\left[2\ln|x| + 3x\right]_{0.112}^{1.245} = 8.216$ (4 sf)

150. $\left[\frac{1}{6}e^{3x} + x\right]_{-2}^{-1} = 1.008$ (4 sf)

151. $\left[\ln|e^x + 1|\right]_k^4 = 3.891$, $k = -2$

152. $\left[\frac{1}{2}\tan 2x\right]_0^k = 0.5$, $k = 0.3927$

Page 195

153. $\left[5\sec 2x\right]_{0.12}^{0.71} = 28.14$ (4 sf)

154. $\left[\frac{-11}{2}\ln|2x+3| + 3x\right]_{-1}^{5}$
$= 3.893$ (4 sf)

155. $\left[\frac{1}{2}\ln|2x^2 + 2x|\right]_1^3$
$= 0.8959$ (4 sf)

156. $\left[-2\cot(0.5x)\right]_{\pi/2}^{3\pi/2} = 4$

157. $\left[\frac{-1}{2}\ln|1-2x|\right]_{-3}^{0.25} = 1.320$ (4 sf)

158. $\left[\sqrt{2x}\right]_k^6 = 3.017$, $k = 0.1$

159. $\left[-e^{-x} + e^x\right]_{1.41}^{2.73} = 11.42$ (4 sf)

160. $\left[\frac{2}{3}(x-3)^{1.5}\right]_4^k = 6.787$, $k = 8$

Page 200

161. a) $\int_a^c f(x)\ dx = -K + K$
$= 0$

b) Area $= 2K$

c) Area $= 5 \times (c - a) + 0$
Area $= 5 \times (c - a)$

162. a) $\text{Area}_{a \text{ to } c} = 3.6$

b) $\int_a^c h(x)\ dx = 3.2 + -0.4$
$= 2.8$

c) $\int_a^c j(x)\ dx = 2.8 + 2(c - a)$

Page 201

163. $\left[\frac{1}{4}x^4 - \frac{8}{3}x^3 + 10x^2 - 14x\right]_2^5$
Area = 8.25 units^2

164. $\left[6x^2 - x^3\right]_1^3$
Area = 22 units^2

165. $\left[-\cos x\right]_{\pi/4}^{3\pi/4}$
Area = 1.414 units^2 (4 sf)

Page 202

166. $\left[\frac{1}{3}x^3 - 3x^2 + 8x\right]_2^4$
Area $= 1\frac{1}{3}$ units^2

167. $\left|\left[\frac{x^3}{3} - 3x^2 + 8x\right]_3^4\right| +$
$\left[\frac{x^3}{3} - 3x^2 + 8x\right]_4^5$
Area $= \frac{2}{3} + \frac{4}{3} = 2$ units^2

168. $\left[3\sin(2x)\right]_{\pi/4}^{3\pi/4}$
Area = 6 units^2

169. $\left[0.25e^{2x} - \frac{5}{2}x^2 - 5x\right]_{-0.5}^{1}$
Area = 7.620 units^2 (4 sf)

Page 203

170. $\left|\left[-2\cos(2x) - 8\sin x\right]_{-\pi/2}^{\pi/2}\right|$
Area = 16 units^2

171. $\left[2x^2 - \frac{1}{3}x^3\right]_0^4$
Area $= \frac{32}{3}\left(10\frac{2}{3}\right)$ units^2

172. $\left|\left[\frac{1}{4}x^4 - 2x^3 + x^2 + 3x\right]_2^4\right|$
Area = 34 units^2

Page 204

173. $\left[20(2x-1)^{1/2}\right]_5^k = 40$, $k = 13$

174. $\left[\ln|x^3 - 1|\right]_2^k = 4.29$, $k = 8$

175. $\left[\frac{e^{0.5x}}{5}\right]_0^k = 14.7413$, $k = 8.627$

Page 205

176. $\left[\frac{1}{4}x^4 - \frac{1}{3}x^3 - x^2\right]_{-1}^{0} +$
$\left|\left[\frac{1}{4}x^4 - \frac{1}{3}x^3 - x^2\right]_0^2\right|$
Area $= 3.083\left(3\frac{1}{12}\right)$ units^2

Page 205 cont...

177. $\int_{-3}^{0} x^2 + 2.5x + 3\ dx$
$= \left[\frac{1}{3}x^3 + \frac{5}{4}x^2 + 3x\right]_{-3}^{0}$
Area = 6.75 units^2

178. $\int_{-1}^{1} 2.5 - 2.5x^2\ dx$
$= \left[2.5x - \frac{5}{6}x^3\right]_{-1}^{1}$
Area = 3.333 units^2 (4 sf)

Page 206

179. $\left[\frac{1}{4}x^4 - \frac{10}{3}x^3 + \frac{29}{2}x^2 - 20x\right]_1^4$
Area = 11.25 units^2

180. $\left[\frac{1}{4}x^4 - 3x^3 + 12x^2 - 16x\right]_1^4$
Area = 6.75 units^2

181. $\left|\left[4x - \frac{19}{2}\ln|2x+3|\right]_0^{0.875}\right| +$
$\left[4x - \frac{19}{2}\ln|2x+3|\right]_{0.875}^{8}$
Area = 16.20 units^2 (4 sf)

Page 207

182. $\int_{-1}^{0} f(x) - g(x)dx + \int_{0}^{4} g(x) - f(x)dx$
Integral =
$\left[\frac{1}{4}x^4 - \frac{4}{3}x^3 + \frac{1}{2}x^2 - 6x + 12\ln(x+2)\right]$
Area = 24.38 units^2 (4 sf)

183. $\int_{-5\pi/6}^{-\pi/6} g(x) - f(x)dx + \int_{-\pi/6}^{\pi/2} f(x) - g(x)dx$

$I = \left[\frac{-1}{2}\sin(2x) + \cos x\right]$
Area = 3.8971 units^2 (4 sf)

Page 208

184. $2\left[\frac{(x^2-4)^6}{12}\right]_{-2}^{0}$
Area = 682.7 units^2 (4 sf)

185. $2\left[-2\cos(x^2)\right]_0^{1.772}$
Area = 8.000 units^2 (4 sf)

Page 211

186. a) $t = 2 \quad s = -3$ m
$t = 4 \quad s = 9$ m
$t = 8 \quad s = 105$ m
b) 18 m/s
c) -12 m/s
d) 6 m/s^2
e) $t = 2$ seconds

187. a) $t = \pm 1.225$ s (4 sf)
b) $s = \frac{4}{3}t^3 - 6t + 2$
c) 24 m/s^2
d) $s(2) = 0.6667$ m (4 sf)
e) 6 m/s

188. a) $a = -27$ m/s^2
b) Slowing down.
c) $v = t^3 - 6t^2 - 15t + 100$ m/s
d) $t = 5$ s (ignore $t = -4$)
e) $s = \frac{1}{4}t^4 - 2t^3 - \frac{15}{2}t^2 + 100t$
$s(5) = 218.75$ m

Page 212

189. a) -1 m
b) $\frac{t-3}{t+3} = \frac{t+3-6}{t+3}$
$s = 1 - \frac{6}{t+3}$
$v = \frac{6}{(t+3)^2}$
$a = \frac{-12}{(t+3)^3}$
c) No, because $v \neq 0$ as 6 and $(t + 3)^2$ must be positive.

190. a) -1 m/s
b) $t = 0.5$ secs
c) $s = 2t - 3 \ln|t + 1|$
d) 1.841 m (4 sf)
e) $a = \frac{3}{(t+1)^2}$

191. a) $v = 81$ m/s
b) $v = 9$ m/s
c) $a = -36$ m/s^2
d) $t = 1.5$ s
e) $s = 12t^3 - 54t^2 + 81t - 30$
f) $s(1.5) = 10.5$ metres

Page 213

192. a) 49 m/s
b) $t = 5$ secs
c) $s = 49t - 4.9t^2$
d) $t = 10$ secs
e) -9.8 m/s^2
Constant deceleration (due to gravity).

193. a) $v = -9$ m/s.
Moving backwards.
b) $a = 5$ m/s^2
c) $s = 3t + \frac{5}{2}t^2 - \frac{2}{3}t^3$
d) $s = 10.67$ m (4 sf)
e) $t = 3$ and -0.5 secs

194. a) $a(0) = 5$ m/s^2
$a(2) = 1.363$ m/s^2 (4 sf)
b) $v = 5t + 4\cos t + 1$
c) $v = 9.335$ m/s (4 sf)
d) $s = 2.5t^2 + 4\sin t + t + 3$ m
e) $s = 18.64$ m

Page 215

195. $A = 8$ units^2
196. $A = 1.577$ units^2 (4 sf)
197. $A = 1.000$ units^2 (4 sf)
198. $A = 51.71$ units^2 (4 sf)

Page 218

199. $A = 4.761$ units^2 (4 sf)
200. $A = 0.7837$ units^2 (4 sf)
201. $A = 1.334$ units^2 (4 sf)
202. $A = 0.8972$ units^2 (4 sf)

Page 219

203. $A = 14.88$ units^2 (4 sf)
204. $A = 6.83$ units^2 (4 sf)
205. $A = 0.6287$ units^2 (4 sf)
206. $A = 1.486$ units^2 (4 sf)

Page 222

207. Estimate = 21.457 (4 sf)
208. $A = 5.780$ units^2 (4 sf)
209. Estimate = 3.544 (4 sf)
210. Estimate = 4.420 (4 sf)

Page 223

211. $V = 3.69395k^2$
$k = 1.802$ (4 sf)

212. Int. = 24.355
Mean = 4.060 (4 sf)
Height of a rectangle with the same width and area as the function.

Page 225

213. $f(x) = x^4 - 3x^2 + 2x + C$
214. $y = 3x^4 - 4x^3 + 6x^2 - 12x + C$
215. $f(x) = \frac{1}{2}x^4 + 8x + \frac{2}{x} + C$
216. $f(x) = \frac{x^6}{2} - \frac{2x^5}{5} - \frac{1}{x} + C$
217. $y = \frac{-1}{2}\cos(2x) + 8\sin(x) + C$
218. $y = 2e^{2x} + \frac{1}{3}\cos 6x + 3\ln|x| + C$
219. $y = \frac{5x^4}{4} - \frac{4\sqrt{x^3}}{3} - \frac{1}{2x^2} + C$
220. $y = 4\tan x + \frac{1}{x^3} + \frac{2\sqrt{x^3}}{3} + C$
221. $f(x) = 2\ln|x| + 3x + C$
222. $f(x) = 4e^{0.5x} + 4x + C$

Page 227

223. $f(x) = 12x + 3x^2 - 3x^3 - 12$
224. $y = \frac{2}{3}x^3 - \frac{1}{2}x^2 - x + 4\frac{5}{6}$
225. $f(x) = \frac{4}{3}x^3 - 16x - 20$
226. $y = \frac{-1}{2}\cos(2x) + 1$

Page 228

227. $f(x) = 3\ln|x| - 2x + 4$
228. $y = \frac{x^4}{4} + 3x^3 + \frac{27x^2}{2} + \frac{3x}{4} - 4$
229. $y = 4x - \frac{1}{2}\cos 2x$
230. $f(x) = 3\ln|x| + \frac{2\sqrt{x^3}}{3} - 1$
231. $f(x) = 2\ln|x^2 + 2x + 1| + 5$
232. $f(x) = \frac{e^{4x}}{4} + 5x + 4$
233. $f(x) = 2x + 4\ln|3x - 5| - 12$
234. $y = 3\ln|3x + 1| + x^2 - x - 3$

Page 230

235. a) $f'(x) = 5x^4 + 6x^{-2} + 4x + C$
$f(x) = x^5 - 6x^{-1} + 2x^2 + Cx + K$
b) $f(x) = x^5 - 6x^{-1} + 2x^2 + x + 13$

236. a) $f'(x) = 9x^2 - 6x^{-1} + C$
$f(x) = 3x^3 - 6\ln|x| + Cx + K$
b) $f(x) = 3x^3 - 6\ln|x| + 2x - 6$

Page 231

237. a) $f'(x) = 3x^2 + 4x^{-3} + 6x + C$

$f(x) = x^3 - 2x^{-2} + 3x^2 + Cx + K$

b) $f(x) = x^3 - 2x^{-2} + 3x^2 + 2x + 1$

238. a) $f'(x) = {}^-8\cos(4x) + 8x + C$

$f(x) = {}^-2\sin(4x) + 4x^2 + Cx + K$

b) $f(x) = {}^-2\sin(4x) + 4x^2 + 1$

239. $f'(x) = \frac{1}{2}e^{2x} + (2 - \frac{1}{2}e^2)$

$f(x) = \frac{1}{4}e^{2x} + (2 - \frac{e^2}{2})x + \frac{7}{4}$

240. $\frac{d^2y}{dx^2} = {}^-2x\cos x - 4\sin x$

Substituting in $y + \frac{d^2y}{dx^2}$

$2x\cos x + ({}^-2x\cos x - 4\sin x)$

$= {}^-4\sin x$

241. $\frac{dy}{dx} = (2x + 1)\,e^{2x}$

$\frac{d^2y}{dx^2} = (4x + 4)e^{2x}$

Substituting

$(4x + 4)e^{2x} - 4(2x + 1)e^{2x} + 4(xe^{2x}) = 0$

242. $f'(x) = 5x^4 - 20x^3 + 30x^2 - 20x + 2$

$f(x) = x^5 - 5x^4 + 10x^3 - 10x^2 + 2x + 7$

Page 232

243. a) $\frac{dy}{dx} = 3(3 + x)^4 + C$

$y = \frac{3}{5}(3 + x)^5 + Cx + K$

b) $y = \frac{3}{5}(3 + x)^5 - 240x + 5$

244. a) $f''(x) = 12 - 12\cos(4x)$

$f'(x) = 12x - 3\sin(4x) + C$

$f(x) = 6x^2 + \frac{3}{4}\cos(4x) + Cx + K$

b) $f(x) = 6x^2 + \frac{3}{4}\cos(4x) + 3x + \frac{5}{4}$

245. $f(x) = \ln|\cos x| + x + 1$

246. $\frac{dy}{dx} = 2e^{4x} + 8xe^{4x}$

$\frac{d^2y}{dx^2} = 8e^{4x} + 8e^{4x} + 32xe^{4x}$

$\frac{d^2y}{dx^2} = 4(2e^{4x} + 8xe^{4x} + 2e^{4x})$

$\frac{d^2y}{dx^2} = 4\left(\frac{dy}{dx} + \frac{2xe^{4x}}{x}\right)$

$\frac{d^2y}{dx^2} = 4\left(\frac{dy}{dx} + \frac{y}{x}\right)$

Page 235

247. $\frac{y^3}{3} = \frac{x^4}{4} + C$

248. $\ln|y| = 2x^2 + C$

This can also be rewritten as $y = ke^{2x^2}$

249. $y^2 + y = \frac{x^2}{2} + C$

250. $\frac{{}^-1}{y} = x^3 + C$

251. $\frac{y^2}{2} - \frac{y^3}{3} = \frac{x^2}{2} + C$

$3y^2 - 2y^3 = 3x^2 + K$

252. $\ln|y| = \frac{x^2}{2} + x + C$ which can be rewritten as

$y = k\,e^{0.5x^2 + x}$

Page 236

253. $\ln|y| = {}^-2\cos(0.5x)$

$y = e^{-2\cos(0.5x)}$

254. $\sin y = 6\sin 2x - 6$

255. $\ln|y| = x^3 + 15x + \ln|3|$

$y = 3e^{(x^2 + 15x)}$

256. $e^y = 6e^{2x} - 5$

Page 237

257. $y = {}^-\ln|55 - 2x^3|$

258. $\ln|2y - 1| = 2x^2 + 2x^3 + C$

$\ln|2y - 1| = 2x^2 + 2x^3 - 70.90$

259. $\frac{1}{2}\ln|1 - y^2| = \frac{1}{2}\ln\,k|1 - x^2|$

$1 - y^2 = \frac{8}{3}(1 - x^2)$

260. $\frac{y^3}{3} = (1 + x^2)^3 + 1$

$\frac{y^3}{3} = x^6 + 3x^4 + 3x^2 + 2$

Page 240

261. 7.0 years

262. 11.2 years

263. \$244 000 (3 sf)

264. 6.9% (1 dp)

265. 6.9 year (7 years)

266. 89.4 years (90 years)

Page 241

267. a) 16.2% per year

b) 9.9 years (10 years)

268. a) $\frac{dP}{dt} = kP,$

$\int \frac{1}{P}\,dP = \int k\,dt$

$\ln P = kt + c$

$P = e^{kt + c}$

$P = e^c \times e^{kt}$

$P = P_0e^{kt}$

b) $P = 725\,000\,e^{kt}$

$1\,055\,000 = 725\,000\,e^{5k}$

$k = 0.0750$

$P = 725\,000\,e^{10 \times 0.075}$

$P = 1\,535\,000$

c)

$4\,000\,000 = 725\,000\,e^{0.075\,t}$

$e^{0.075\,t} = 5.5172$

$t = 23$ years (0 dp)

269. a) \$169 000 (3 sf)

b) 23 years (0 dp)

270. Dropping at 13.9% per year

Page 244

271. a) $P = P_0e^{kt}$ t in days

$k = 0.0909$

$P = 12\,e^{0.0909\,t}$

b) t = 7.3 weeks (51 days)

272. a) T = 14° C

b) t = 274 minutes

273. r = ⁻0.1438 (4 sf)

t = 22.65 minutes

274. r = 0.026 66

t = 45.2 years

Page 245

275. P(10) = 190 (0 dp)

276. a) P(10) = \$7770 (3 sf)

b) t = 8 years 8 months

277. $T - T_a = Ce^{kt}$

$T = T_a + Ce^{kt}$

$18 = {}^-27 + C \quad (t = 0)$

$C = 45$

$^-15 = {}^-27 + 45e^{10k} \quad (t = 10)$

$k = {}^-0.1322$

$T(5.5) = {}^-27 + 45e^{-0.1322 \times 5.5}$

$T = {}^-5.25°C$

Page 245 cont...

278. k = starting No. mites

$k - 124 = k\,e^{-0.004\,32 \times 15}$

$k(1 - e^{-0.0648}) = 124$

$k = 1976$

$= 1980$ (3 sf)

Remain = 1852

$= 1850$ (3 sf)

Page 246

279. a) $\int \frac{1}{(500-x)}\,dx = \int k\,dt$

$-\log_e |500 - x| = kt + c_1$

$\log_e |500 - x| = -kt + c_2$

$500 - x = e^{-kt + c_2}$

$x = 500 - C\,e^{-kt}$

Substitute t = 0 and x = 0

$x = 500 - 500\,e^{-kt}$

b) t = 60 x = 250

k = 0.011 55

$x = 500(1 - e^{-0.011\,55t})$

c) $495 = 500(1 - e^{-0.011\,55t})$

t = 399 s (3 sf)

or t = 397 s if k = 0.0116 (3 sf)

280. a) $\frac{dP}{dt} = -6.52(t - 5)^2 - 1216$

$\frac{dP}{dt} = -6.52t^2 + 65.2t - 1379$

$P = -2.17t^3 + 32.6t^2 - 1379t + 15200$

b) at t = 10, P = 2497

c) $\frac{dP}{dt} = -0.187P$

$P = P_o e^{-0.187t}$

$P = 152\,00\,e^{-0.187t}$

at t = 10, P = 2343

Pages 247 – 252

Practice External Assessment – Integration

In the external examinations NZQA uses a different approach to marking based on understanding (u), relational thinking (r) and abstract thinking (t). They then allocate marks to these concepts and add them up to decide upon the overall grade. This approach is not as easy for students to self mark as the NuLake approach but the results should be broadly similar.

Question One

(a) $I = \int \frac{x^2 - 8x + 16}{x}\,dx$

$I = \int x - 8 + \frac{16}{x}\,dx$

$I = \frac{1}{2}x^2 - 8x + 16\ln x + C$ **A**

(b) Simpson's rule

$I = \frac{0.5}{3}\left[3 + 4(1.3919) + 2(.6111)\right]$

$= 1.632$ (3 dp) **A**

All working must be shown.

(c) $\left[\ln(x-4)^2\right]_5^k = 3.219$

$k^2 - 8k - 9 = 0$

$(k - 9)(k + 1) = 0$

$k = 9$ only **M**

(d) i) $s = \frac{-30}{(t+3)^2} + C$ When t = 0, s = 0

$C = \frac{30}{9}$ $s = \frac{-30}{(t+3)^2} + \frac{30}{9}$

When t = 2, s = 2.133 m **A**

ii) s tends to $\frac{30}{9}$ and v to 0. **M**

(e) $\frac{dV}{dt} = -kV$

$\frac{1}{V}\frac{dV}{dt} = -k$

$\int \frac{1}{V}\,dV = \int -k\,dt$

$\ln v = -kt + C$

t = 0, V = 1000 so C = ln 1000

$\ln V = \ln 1000 - kt$

$\ln(\frac{V}{1000}) = -kt$

$V = 1000e^{-kt}$

$V(30) = 850$ so $850 = 1000e^{-30k}$

$k = 0.005417$

$200 = 1000e^{-0.005417t}$

$t = 297.1$ hours **E**

Question Two

(a) $\frac{2x^{3/2}}{3} - \frac{1}{x} + C$ **A**

(b) $38000 = 65000e^{-4i}$

$i = 0.1342\ (13.4\%)$ **A**

(c) $\int \frac{1}{y}dy = \int 2(x^2+2)dx$

$\ln y = \frac{2x^3}{3} + 4x + C$ **A**

(0,2) $\ln y = \frac{2x^3}{3} + 4x + \ln 2$

$y = 2e^{\frac{2x^3}{3}+4x}$ **M**

(d) Intersections at x = 0.5, 2.5 and 6.5 units.

$A_1 = \left[\frac{-3}{\pi}\cos\left(\frac{\pi x}{3}\right) - 0.5x\right]_{0.5}^{2.5}$

$A_2 = -\left[\frac{-3}{\pi}\cos\left(\frac{\pi x}{3}\right) - 0.5x\right]_{2.5}^{6.5}$

Area $= 4.3080$ (4 dp) **M**

(e) If $\alpha > 0$, $y = C(x-k)(x+k)$ as intercept $(0, \alpha)$

$\alpha = C(-k)(k)$

$C = \frac{\alpha}{-k^2}$ so $y = \frac{\alpha}{-k^2}(x^2 - k^2)$

$8 = 2\int_0^k \frac{a}{-k^2}(x^2 - k^2)\,dx$

$8 = \frac{-2\alpha}{k^2}\left[\frac{x^3}{3} - k^2x\right]_0^k$

$\alpha = \frac{6}{k}$ **E**

Question Three

(a) $\frac{1}{4}\cot(4x) + 5x + C$ **A**

(b) $\int_P^R f(x)dx = 4.7$ **A**

(c) $v(t) = -9.8t + t^{1.5} + 0.12t^2 + C$

$v(0) = -4.5$ so $C = -4.5$

$v(t) = -9.8t + t^{1.5} + 0.12t^2 - 4.5$ **A**

$s(t) = -4.9t^2 + 0.4t^{2.5} + 0.04t^3 - 4.5t + K$

$s(0) = 45$ so $K = 45$

Distance at 2 seconds = 19 m. **M**

(d) f(x) – g(x) is positive from x = 1 to 2.9009 while g(x) – f(x) is positive from x = 2.9009 to 3.9389.

$A = \int_1^{2.9009} f(x) - g(x)\,dx - \int_{2.9009}^{3.9389} g(x) - f(x)\,dx$

$= \left[\frac{1}{4}x^4 - \frac{8}{3}x^3 + \frac{19}{2}x - 10x - 10\ln[x+1] - \frac{3}{x}\right]_1^{2.9009}$

$- \left[\frac{1}{4}x^4 - \frac{8}{3}x^3 + \frac{19}{2}x - 10x - 10\ln[x+1] - \frac{3}{x}\right]_{2.9009}^{3.9389}$

$= 2.15565$ **M**

(e) $v(t) = 400 \sin kt$ so $(v(t))^2 = 400^2 \sin^2 kt$

$I = \int 400^2 \sin^2 kt\ dt$

$= 400^2 \int 0.5 - 0.5\cos\ 2kt\ dt$

$= \frac{400^2}{2}\left(t - \frac{\sin 2kt}{2k}\right)_0^{2\pi/k}$

$= \frac{2\pi 400^2}{2k}$

$RMS = \sqrt{\frac{\int_0^T v^2(t)dt}{T}} = \sqrt{\frac{\frac{\pi 400^2}{k}}{\frac{2\pi}{k}}}$

$RMS = \frac{400}{\sqrt{2}}$ **E**

Sufficiency. For each question award yourself a score out of 8 using this table. Add the three scores for a score out of 24 and compare to the cut scores. All answers must include integrals where appropriate.

Quest.	N0	N1	N2	A3	A4	M5	M6	E7	E8
ONE	No integ's. correct.	1 Integ. with error.	1 A or 1 integ. correct.	2A or 2 integ's. correct.	3A or 3 integ's. correct.	1M + 1M minor error.	2M incldes 2 integ's.	1E Equation for V.	1E all correct.
TWO	No integ's. correct.	1 Integ. with error.	1 A or 1 integ. correct.	2A or 2 integ's. correct.	3A or 3 integ's. correct.	1M + 1M minor error.	2M all correct.	1E integ. correct.	1E integ + α correct.
THREE	No integ's. correct.	1 Integ. with error.	1 A or 1 integ. correct.	2A or 2 integ's. correct.	3A or 3 integ's. correct.	1M + 1M minor error.	2M all correct.	Integ. $v^2(t)$ correct.	1E all correct.

Cut Scores

Not Achieved	Achievement	Achievement with Merit	Achievement with Excellence	
0 – 6	7 – 13	14 – 20	21 – 24	