

## Answers

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1. This is an experiment. Not randomised and poorly constructed as the content she teaches is different.  
Explanatory variable is the amount of homework and the response variable is class performance in a test.
2. This a survey. His first 30 should be roughly random but his sample size is small.  
Explanatory variable is time they get up in the morning and the response variable paid working hours. Could be the other way around, it depends on which you think causes what.
3. This is a randomised experiment. Depending upon the number of players in his squad he may get a good result. He should make sure his backs and forwards are evenly distributed.  
Explanatory variable is the method of kicking the ball and the response variable is the distance it travels.
4. This is a survey as the behaviour has already occurred and he is looking at the results.  
Explanatory variable is the amount of binge drinking and the response variable is school academic results.

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5. Population: All members of the social networking site.  
Sample Frame: Members who log in on Thursday 6th December.  
Sample method: Cluster sample of one day's use.  
Potential errors: Potential high no-response rate as many members may not be concerned about the privacy policy. Those that do log in may have strong opinions so it almost becomes a self-selected survey. Workers may not log in as much on a work day.
6. Population: Electors in her electorate.  
Sample Frame: Electors whose family name starts with the letter R.  
Sample method: Cluster sample of family names starting with R.  
Potential errors: Some minority groups may be under represented (e.g. Asia electors) and some families may be over represented (e.g. the Ranganui families).
7. Population: Customers of the internet service provider.  
Sample Frame: Customers of the internet service provider.  
Sample method: Random sample. Potential errors: The no-response rate probably means they missed their business customers and busy customers which may be a problem.

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8. Population: Students in year 13.  
Sample Frame: Students in two Year 13 classes.  
Sample method: Cluster sample.  
Potential errors: If the classes were in E block for one subject they are unlikely to be representative of the whole population. The classes are likely to include Year 12 students and not have Year 13 students repeating Year 12 classes.

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9. Population: All customers of the supermarket.  
Sample Frame: Customers who are members of the loyalty scheme.  
Sample method: Random sample.  
Potential errors: If they want to attract new customers they will not be loyalty card holders. Either way the sample frame does not match the population.
10. Population: Teenage children of members.  
Sample Frame: Members with families with teenage children.  
Sample method: Physical random sample of members with teenagers.  
Potential errors: Families may have more than one teenager and if that is the case how do they select the respondent? Also in such a family each teenager has a reduced probability of being selected compared to a family with one teenager.
11. Population: All drivers who use the roundabout.  
Sample Frame: Drivers who are near the roundabout Thursday morning.  
Sample method: Systematic sample.  
Potential errors: Good representation of the sample frame although using a police officer may limit what people say. Also doing the survey on a work day morning cuts out many workers and may not get opinions of people using the roundabout at night.
12. Population: Local residents.  
Sample Frame: People on the local parliament General and Maori electoral rolls.  
Sample method: Random off two rolls.  
Potential errors: If the intention is to get the opinion of all local residents then the electoral rolls are unlikely to be a good match as the Maori electorates cover a huge area. Also if they mix the opinions of the 60 electors then they will over represent Maori opinion.  
If they keep the opinions separate then they may get a better representation of New Zealand's bicultural community.

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13. Advantages: Ease of use. The sample frame is not limited to listed numbers which often excludes police, prison officers, people avoiding debt collectors etc. Purely random sample.

Disadvantages: Need to filter out businesses from the calls made. Hard to limit calls to one geographic area and would have to ask questions to further filter respondents. If one adult lives at the address they will have a higher probability of being selected than living at an address with two or more adults. Limited to landlines which is a problem as many people under 25 have only mobile phones. Other points are possible but they must explain why the method is likely to misrepresent the population in some way.

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14. Advantages: A letter informing respondents prior to the visit is good. The meshblocks have credibility in that they are used by Statistics New Zealand. Keeping blocks of respondents in close proximity reduces cost and time between interviews. Systematic sampling of randomly selected meshblocks should be good if enough meshblocks are selected.

Disadvantages: The cost of individual interviews throughout the country will be expensive. Areas that are over or under represented will need weighting in the results. Extending information gathered for transport over two days and extrapolating this to travel over a year will increase the sampling error. Also conclusions about single cities means that enough meshblocks in each city will need to be selected to represent all the opinions within that city.

Other points are possible.

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15. Advantages: Panel sampling is very fast and efficient. A panel is usually already established and motivated to respond (by rewards).

Disadvantages: Adjusting a panel so its demographic profile fits the profile of the population being sampled does not mean the panel is a representative sample of the population.

The panel is computer literate and will access the internet regularly but a lot of the population whose opinions they represent will not be as technology savvy. 91% of New Zealand may have the internet but the opinions of the other 9% are likely to be very different. Using a panel up to two times a month means they become part of the process and may preform opinions just in case they are contacted.

Other points are possible.

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16. Advantages: Reduces the effect of the interviewer in affecting the answers of the respondents and ensures a consistency of approach. Using listed numbers will enable most business numbers to be filtered out in advance. (Note: It is unclear from the Colmar-Brunton website whether they use listed numbers or RDD).

Disadvantages: Using random numbers from a listed directory has the problem that police, prison officers, people avoiding debt collectors etc. are excluded. Also limiting calls to landlines risks removing many people under 25 from the sample frame. Two weeks is a long time for a survey as many things may happen during the information collection process. Single adults with one landline have a higher chance of being selected than adults in multi-adult homes.

Other points are possible but they must explain why the method is likely to misrepresent the population in some way.

17. Advantages: Having specialist panels that can be adjusted so their demographics approximate the population will appeal to advertisers who want to target a special group. Rewards and the fact they have agreed to be on a panel will help the response rate. The SayWhat Youth Panel is likely to appeal to advertisers targeting young people.

Disadvantages: Just because the panel has similar demographics to the population does not mean they are representative of the population. Panel members used repeatedly may alter their opinions based on previous surveys or they become desensitised to the experience and give similar opinions to previous surveys.

Other points are possible but they must explain why the method is likely to misrepresent the population in some way.

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18. The first part of the question will make the respondent keen to answer yes otherwise they have missed the most famous movie. Have you seen the New Zealand produced movie 'Lord of the Rings'?
19. Doubled barrelled question.  
In your opinion is Dan Carter a great first five?  
In your opinion is Dan Carter a great goal kicker?
20. It is illegal so the respondent may feel obliged to lie. Also over what time period, 1 month or 20 years? Even though it is against the law some people feel the need to answer a mobile phone while driving. Have you answered a mobile phone while driving in the last month?

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21. Too limited a choice. What happens if they have two parents and they live alternatively with one or the other. They may live with two adults, one of whom is a parent.

Select the best description for each of the adults you normally live with

<input type="checkbox"/> Mother	<input type="checkbox"/> Father
<input type="checkbox"/> Step parent	<input type="checkbox"/> Guardian
<input type="checkbox"/> Other (specify) _____	

22. Double negative. Respondents will be confused as to whether Agree is for or against.  
Do you agree spending on education should be reduced?  
a) Agree      b) Disagree      c) Undecided
23. Colloquial term which may or may not be known.  
Cannabis is also called 'grass' and 'pot'. Do you think it should be legal for people to buy and sell cannabis?  
a) Yes      b) No      c) Undecided
24. Leading question. Should the age for National Superannuation be kept at 65 or increased?  
a) Kept at 65      b) Increased      c) Undecided
25. Colloquial term binge drinking which may not be known.  
Binge drinking is where people drink a large amount of alcohol in a short period of time to get drunk. Would binge drinking be reduced if there was a higher purchasing age for bottle stores, dairies and supermarkets?

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26. Colmar - Brunton uses computer-aided interviewing (CATI) of random household telephone numbers (landlines), typically sampled in proportion to geographical regions with a rural/urban split (usually called a stratified random sample). A random eligible voter is usually found by asking which of the household's eligible voters had the most recent birthday, and talking to that person. But the fact that not all households have landlines is an increasing concern with CATI interviewing.  
The poll is of eligible voters. The party vote question is filtered by those claiming they would be 'quite likely' or 'very likely' to vote.  
If an eligible voter refuses to answer questions, they become a 'non-respondent' and attempts should be made to re-weight the data so that this non-response effect is diminished. The catch is that the data is adjusted on the assumption that the respondents selected represented the opinion of a non-respondent on whom, by definition, we have no information on.

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27. Horizon polls are on-line surveys using their established panel. People are invited to join the Horizon poll national online research panel based on the profile of the population at the 2006 census. Large sample size 1800+ (to provide sufficient respondents from minor parties). The results are weighted by age, gender, ethnicity, region, personal income and party vote in the previous election.

Horizon polls include both decided voters and undecided voters leaning towards a preference called a Net Potential Vote poll.

28. DigiPoll uses RDD to select a random sample of 750 registered voters. With many new telephone providers it has become increasingly difficult to produce accurate samples from listed phone numbers. DigiPoll overcomes this problem by using Random Digit Dialing and having continuously updated "telephone maps" of populations. Accurate maps are also vital for representing ethnic minorities and other hard to reach groups.

Weighting survey responses is designed to remove bias from a survey sample and make the results better project the target population. DigiPoll has formulated a weighting methodology that is applied to the raw data in order to reflect the demographic composition of the sample, which has been proven to be successful time and time again.

Digipoll use CATI methodology to structure the interview. Having professional telephone operators working in a comfortable atmosphere with the best software and hardware is far superior to calling from home. It allows for ongoing supervision and monitoring by trained and experienced supervisors who can also assist interviewers and support them when needed.

29. Methodology: Reid Research generate a random nationally representative sample from which they apply strict area, age and gender interlocking quotas. They use CATI and online methods to collect data from 1000 respondents (750 by phone, 250 online) giving a margin of error of  $\pm 3.2\%$ . During the questionnaire they ask respondents if a general election was held yesterday which political party would they have voted for. They also ask if they voted in the previous election and if so which political party they voted for.

From the data collected Reid Research can analyze the results in various ways including looking at results of those who voted at the previous election.

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30. a) Randomised experiment.  
 b) Explanatory variable: Nicotine patches.  
 c) Response variable: Smoking  
 d) Placebo: Patches with no Nicotine.  
 e) Blind: Double blind experiment.  
 f) Causal: Yes causal relationship shown.  
 g) Confounding variable: None.
31. a) Observational study  
 b) Explanatory variable: Parents' average height.  
 c) Response variable: Student's height.  
 d) Placebo: None  
 e) Blind: None  
 f) Causal: Not proven.  
 g) Confounding variable: Food and income.

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32. a) Randomised experiment.  
 b) Explanatory variable: Chromium supplements.  
 c) Response variable: Strength.  
 d) Placebo: Fake substitute.  
 e) Blind: Single blind.  
 f) Causal: Not proven.  
 g) Confounding variable: The daily weight training.
33. a) Observational study  
 b) Explanatory variable: Drink taken to exam.  
 c) Response variable: Examination mark.  
 d) Placebo: None  
 e) Blind: None  
 f) Causal: Not proven.  
 g) Confounding variable: Motivation of the student.

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34. a) Randomised experiment.  
 b) Explanatory variable: Practiced Tai Chi  
 c) Response variable: Brain volume and cognitive ability.  
 d) Placebo: None  
 e) Blind: No  
 f) Causal: Not proven.  
 g) Confounding variable: Any activity by the seniors.
35. a) Randomised experiment.  
 b) Explanatory variable: Motivational interview.  
 c) Response variable: Class participation, math grades and academic results.  
 d) Placebo: None  
 e) Blind: No  
 f) Causal: Not proven.  
 g) Confounding variable: Involvement in any sort of a trial.

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36. a) Randomised experiment.  
 b) Explanatory variable: Caffeine  
 c) Response variable: Reaction times and irritability.  
 d) Placebo: Coffee with no caffeine added.  
 e) Blind: Single blind  
 f) Causal: That withdrawal of caffeine leads to irritability.  
 g) Confounding variable: None
37. a) Randomised experiment.  
 b) Explanatory variable: Omega-3 fatty acids.  
 c) Response variable: Psychiatric symptoms.  
 d) Placebo: Yes but not stated.  
 e) Blind: Double blind.  
 f) Causal: Not proven as ongoing study but substantial improvement.  
 g) Confounding variable: None.

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38. a) Experiment.  
 b) Explanatory variable: Resveratrol.  
 c) Response variable: Heart function and muscle strength.  
 d) Placebo: None.  
 e) Blind: Single blind (rats had no idea) or not blind.  
 f) Causal: No figures but talks of potential so no.  
 g) Confounding variable: Food or exercise testing.
39. a) Randomised experiment.  
 b) Explanatory variable: Resveratrol.  
 c) Response variable: Heart function and longevity.  
 d) Placebo: Sugar pill.  
 e) Blind: At least single blind.  
 f) Causal: None.  
 g) Confounding variable: None.

**Page 40 (RoT stands for Rule of Thumb)**

40. a) Support 56.7%  
 Margin of Error =  $\pm 9.1\%$   
 b) Support from 47.6% to 65.8% so cannot conclude that a majority support a change as the lower limit of the interval is less than 50%.
41. a) Support 62.7%  
 Margin of Error =  $\pm 11.5\%$   
 b) Support from 51.2% to 74.2% so yes a majority of customers support the application for a liquor licence as the lower limit of the interval is greater than 50%.
42. a) 69 people.  
 b) 24 people.

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43. a) Margin of Error =  $\pm 4.5\%$   
 b) Cannot conclude he is real as the interval 47.5% to 56.5% has a lower limit less than 50%. Can conclude that less than half believe Santa Claus is a Democrat because the upper limit of the interval 39.5% to 48.5% is less than 50%.

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44. a) Margin of Error =  $\pm 2.8\%$   
 Confidence interval 88.2% to 93.8%.  
 b) As the online poll percentage (0.91) is outside the range 0.3 to 0.7 the margin of error will be smaller than that calculated using the RoT in part a).
45. a) Margin of Error =  $\pm 3.1\%$   
 b) Confidence interval 47.9% to 54.1%.
46. a) Margin of Error =  $\pm 2.2\%$   
 b) 1189 people in this group without a tattoo. Margin of Error =  $\pm 2.9\%$   
 Confidence interval 42.1% to 47.9%. No the majority of non-respondents do not feel a tattoo makes a person less attractive as the upper limit of the interval is less than 50%.
47. a) Margin of Error =  $\pm 3.2\%$   
 b) Yes majority oppose keeping the drinking age at 18 years as between 53.8% to 60.2% say they were wrong and the lower limit of the interval is greater than 50%.
48. a) Margin of Error =  $\pm 3.2\%$   
 b) Confidence interval 57.8% to 64.2%.  
 Yes majority believe in evolution as the lower limit of the interval is above 50%.
49. A sample size of 625.

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50. a) Margin of Error =  $\pm 2.9\%$   
 b) MoE (difference) =  $\pm 5.8\%$  (2 x MoE)  
 95% CI for difference =  $8.7\% \pm 5.8\%$   
 = [2.9%, 14.5%]
51. a) Margin of Error =  $\pm 2.27\%$   
 b) MoE (difference) =  $\pm 4.5\%$  (2 x MoE)  
 95% CI for difference =  $3\% \pm 4.5\%$   
 = [-1.5%, 7.5%].  
 Because the lower limit of the confidence interval is below 0 we cannot claim there are more strong supporters in the population than opponents.

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52. a) 1111 people.  
 b) MoE (difference) =  $\pm 6.0\%$  (2 x MoE)  
 95% CI for difference =  $15.0\% \pm 6.0\%$   
 = [9%, 21%].  
 Because the the lower limit of the confidence interval is above 0 we can state that there is a statistical difference in support for banning semi-automatic guns.

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53. Margin of Error = 1.8%  
 MoE of difference =  $\pm 2.7\%$ . (1.5 x Av MoE)  
 95% CI for difference =  $6\% \pm 2.7\%$   
 = [3.3%, 8.7%].  
 Support has fallen by between 3.3% and 8.7% as the lower limit of the confidence interval is greater than 0.
54. a) MoE of difference =  $\pm 6.8\%$ . (1.5 x Av MoE)  
 b) 95% CI for difference =  $23\% \pm 6.8\%$   
 = [16.2%, 29.8%].  
 Yes support has increased for changing the flag as the lower limit of the confidence interval is greater than 0.
55. a) MoE of difference =  $\pm 8.3\%$ . (1.5 x Av MoE)  
 b) 95% CI for difference =  $7\% \pm 8.3\%$   
 = [-1.3%, 15.3%].  
 Cannot conclude that there is a difference between the local community and the rest of New Zealand as the lower limit of the confidence interval is less than 0.
56. a) MoE of difference =  $\pm 5.8\%$ . (1.5 x Av MoE)  
 b) 95% CI for difference =  $8\% \pm 5.8\%$   
 = [2.2%, 13.8%].  
 Can conclude that there is an increase in the percentage of Americans who believe in global warming as the lower limit of the confidence interval is greater than 0.

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57. MoE (difference) =  $\pm 8.9\%$  (2 x MoE)  
 95% CI for difference =  $7.0\% \pm 8.9\%$   
 = [-1.9%, 15.9%].  
 Cannot conclude that more New Zealanders oppose than support mining in World Heritage sites as the lower limit of the confidence interval is less than 0, i.e. support could range in favour of mining by up to 1.9% and opposing mining up 15.9%.
58. MoE (difference) =  $\pm 6.1\%$  (2 x MoE)  
 95% CI for difference =  $14.0\% \pm 6.1\%$   
 = [7.9%, 20.1%].  
 Can conclude that the majority of people in Germany support the view as the lower limit of the confidence interval is greater than 0.
59. a) MoE of difference =  $\pm 3.9\%$ . (1.5 x Av MoE)  
 b) 95% CI for difference =  $5\% \pm 3.9\%$   
 = [1.1%, 8.9%].  
 Can conclude that there is an increase in the percentage who want farmers to be taxed for taking water from the environment as the lower limit of the confidence interval is greater than 0.
60. a) MoE of difference =  $\pm 5.2\%$ . (2 x MoE)  
 b) 95% CI for difference =  $17\% \pm 5.2\%$   
 = [11.8%, 22.2%].  
 Can conclude that there are more supporters

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than opponents for abortion in the population as the lower limit of the confidence interval is greater than 0.

61. MoE of difference =  $\pm 6.7\%$ . ( $1.5 \times \text{Av MoE}$ )

95% CI for difference =  $10\% \pm 6.7\%$   
= [3.3%, 16.7%].

Can conclude that females were more likely to agree with the statement than males as the lower limit of the confidence interval is greater than 0.

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62. a) Population of New Zealand aged 18 years and over. UMR would use the demographics of New Zealand to match their online sample to best represent the population of New Zealand.

b) This is because most of the people on Facebook are on other social media sites. With 96% on Facebook or no social media it means that almost all the other groups are on Facebook as well.

c) The quoted margin of error for the poll is  $\pm 3.1\%$ , but since the proportion on Pinterest is small, 8%, the actual margin of error would be somewhat smaller, around  $\pm 1.7\%$ . Note. Students need to be aware that when using the 'rule of thumb' for larger or smaller proportions the actual margin of error is smaller than that quoted for the poll.

- d) Quoted margin of error is  $\pm 3.1\%$  so 95% confidence interval of the percentage of population on Facebook is 72.9% to 79.1%.

In the context of the poll we would be 95% confident that the interval [72.9%, 79.1%] includes the percentage of New Zealanders aged 18+ that were on Facebook in 2012.

Note. As the proportion on Facebook is large, 76%, the actual margin of error would be somewhat smaller, around  $\pm 2.7\%$ , so the confidence interval would be a little smaller than that quoted above. Note. Students need to be aware that when using the 'rule of thumb' for larger or smaller proportions the actual margin of error is smaller than that quoted for the poll.

- e) MoE of difference =  $\pm 4.7\%$ . ( $1.5 \times \text{Av MoE}$ )  
95% CI for difference =  $7\% \pm 4.7\%$   
= [2.3%, 11.7%].

Can conclude that the percentage of New Zealand Facebook users aged 18 years and over has increased from 2011 to 2012 as the lower limit of the confidence interval is greater than 0.

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63. a) All of New Zealand in all age groups.  
b) Lack of landlines means it is hard to adequately sample groups who use mobiles instead of landlines. A major concern is they are looking at small parts of the sample (16 - 17 year olds) which collectively are about 3% of the population and should be 3% of the sample.

c) 3% of 2000 is 60.  
Expect 30 girls and 30 boys.

- d) Sample size for girls aged 16 – 17 is 30.  
Sample size for boys aged 16 – 17 is 30.

MoE girls =  $\pm 18.3\%$

MoE boys =  $\pm 18.3\%$

MoE of diff. =  $\pm 27.5\%$ . ( $1.5 \times \text{Av MoE}$ )

95% CI for difference =  $3\% \pm 27.5\%$   
= [-24.5%, 30.5%].

Cannot conclude that girls in the 16 - 17 age group are more likely to binge drink than boys aged 16 – 17 as the lower limit of the confidence interval is less than 0. Sample size is too small.

Note. Students need to be aware that when using the 'rule of thumb' for larger or smaller proportions the actual margin of error is smaller than that quoted for the survey.

- e) A drinking session implies that it is long and many drinks are consumed. A respondent may often have one drink which they do not consider as a drinking session but in another session have eight or more drinks. The term 'typical drinking session' could be misleading. We would need to see the original survey to see whether the term 'typical drinking session' was defined and if it was misleading.  
f) Very little. Just the statement that in other age groups (18 upwards) boys drink more than girls and the assumption that girls associate with older boys.  
g) It does not mention the single age group and implies that overall they have taken the lead which is not the case.

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## Practice External Assessment – Question 1

Q	Evidence	Achievement	Merit	Excellence
1 a)	<p>Yes with justification.</p> <ul style="list-style-type: none"> <li>The in-depth interview did not attempt to be a representative poll.</li> <li>It was based on a different population (immigrant communities).</li> <li>In face-to-face interviews the interviewees may have given answers that they thought were wanted.</li> </ul>	Yes and 1 good point.	Yes and any 2 good points.	Yes and at least 3 good points.
b)	<p>MoE = <math>\pm 3.7\%</math>  Confidence interval <math>11\% \pm 3.7\%</math>  = 7.3% to 14.7% [7.3%, 14.7%]  95% confident that the interval 7.3% to 14.7% includes the percentage of New Zealanders 15 and over who would choose to live in Australia.</p> <p>Note. Students need to be aware that when using the 'rule of thumb' for larger or smaller proportions, 0.11 in this case, the actual margin of error would be smaller than that quoted for the poll.</p>	MoE and CI calculated.	Correct interpretation of CI.	Comment about margin of error being smaller for larger and smaller proportions.
c)	<p>MoE<sub>&lt;40</sub> = <math>\pm 5.5\%</math> from n = 330  MoE<sub>&gt;40</sub> = <math>\pm 4.9\%</math> from n = 420  MoE of difference = <math>\pm 7.8\%</math>. (1.5 x Av MoE)  95% CI for difference = <math>14\% \pm 7.8\%</math>  = [6.2%, 21.8%].  Can conclude that more people 40 and over preferred living in New Zealand than those under 40 as the lower limit of the confidence interval is greater than 0.</p>	Calculates both MoEs.	Calculates MoE of difference and CI.	Appropriate conclusion drawn from CI.
d)	<p>Random digit dialing has the advantage that it includes unlisted numbers, changed or new phones that would be missed if the numbers were selected from a phone book. In populations where there is a high telephone-ownership rate, it can be a cost efficient way to get complete coverage of a geographic area. An important consideration in random digit dialing surveys is bias introduced by non-responders. Non-response bias can be a problem if responders differ from non-responders for the measured variables.</p>	Identifies 1 valid advantage and/or disadvantage.	Identifies 2 valid advantages and/or disadvantages.	Identifies 3 valid advantages and/or disadvantages.
e)	<ul style="list-style-type: none"> <li>Different populations. 750, 15+ is different from a census of all NZ. May be more Asians under 15.</li> <li>Non-sampling error. Possibly Asian respondents were reluctant to be in the poll or answer questions over the phone.</li> <li>Sampling errors could also account for the difference. In any given sample, some attributes of interest may be overrepresented and some underrepresented. It is the nature of a sample.</li> </ul>	One valid point with justification.	Two valid points with justification.	Three valid points with justification.

## Pages 57 – 58 Practice External Assessment – Question 2

Q	Evidence	Achievement	Merit	Excellence
2 a)	<p>MOE = <math>\pm 3.7\%</math></p> <p>The margin of error gives the random sampling error in a survey result. It expresses the maximum expected difference between the true population parameter and a sample estimate of that parameter. It takes into account the variation in survey percentages due to sampling. In the context of this survey we would be 95% confident that the interval [13.3%, 20.7%] includes the percentage of New Zealanders aged 18 and over that were not concerned over individual privacy and personal information.</p> <p>Note. Students need to be aware that when using the 'rule of thumb' for larger or smaller proportions, 0.17 in this case, the actual margin of error would be smaller than that quoted for the poll.</p>	Calculates MoE.	Calculates MoE. and explains what MoE is.	Calculates MoE. and explains what MoE is in context. Comment about margin of error being smaller for larger and smaller proportions.
b) i)	<p>MoE<sub>2012</sub> = <math>\pm 3.7\%</math></p> <p>MoE<sub>2010</sub> = <math>\pm 3.7\%</math></p> <p>MoE of difference = <math>\pm 5.6\%</math>. (1.5 x Av MoE)</p> <p>95% CI for difference = <math>10\% \pm 5.6\%</math> = [4.4%, 15.6%].</p> <p>Can conclude that the level of concern over individual privacy and personal information has increased from 2010 to 2012 as the lower limit of the confidence interval is greater than 0.</p>	MoE of difference calculated.	MoE of difference calculated and CI.	MoE of difference calculated and CI. Correct interpretation of the interval.
ii)	<p>The ACC release of private data on 15th March, one week prior to poll starting 21st March. It would have highlighted the issue and it would be fresh in the minds of those surveyed.</p>	Mentions release of private data.	Mentions release of private data and explains why release makes poll results suspect.	
c)	<p>MoE of difference = <math>\pm 9.9\%</math>. (2 x MoE)</p> <p>95% CI for difference = <math>12\% \pm 9.9\%</math> = [2.1%, 21.9%].</p> <p>Can conclude that a greater percentage of Facebook users believe sites are mainly private spaces rather than public as the lower limit of the confidence interval is greater than 0.</p>	MoE of difference calculated.	MoE of difference calculated and CI.	MoE of difference calculated and CI. Correct interpretation of the interval.
d)	<p>MoE<sub>&lt;\$15 000</sub> = <math>\pm 7.1\%</math></p> <p>MoE<sub>&gt;\$70 000</sub> = <math>\pm 10.5\%</math></p> <p>MoE of difference = <math>\pm 13.2\%</math>. (1.5 x Av MoE)</p> <p>95% CI for difference = <math>26\% \pm 13.2\%</math> = [12.8%, 39.2%].</p> <p>Can conclude that those on a lower income (&lt; \$15 000) have greater concerns over privacy than those on larger incomes (&gt; \$70 000) as the lower limit of the confidence interval is greater than 0.</p>	MoE of difference calculated.	MoE of difference calculated and CI.	MoE of difference calculated and CI. Correct interpretation of the interval.



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Q	Evidence	Achievement	Merit	Excellence
2 e)	A telephone poll of all New Zealanders 18+ cuts out the mainly young adults who use exclusively mobiles and Skype. A higher proportion of these will be on Facebook.	Under represent and any one explained point.		

## Pages 59 – 60 Practice External Assessment – Question 3

Q	Evidence	Achievement	Merit	Excellence
3 a)	The study was an observational study done on one group for one year.	Uses the term observational.		
b)	Not statistically significant means that there was no benefit from offering a reward for passing credits. The experiment showed that although the probability of passing increased it was not large enough to identify it as being statistically significant, that is, it would have been possible to get a similar or better result without offering a reward, by chance alone, that is, more than 5% of the time.	Identifies no benefit from offering a reward.	Identifies no benefit from offering a reward. Could expect such results by chance alone.	Identifies no benefit from offering a reward. Could expect such results by chance alone. Could have occurred more than 5% of the time.
c)	That by chance alone you would expect to get the results under 5% of the time.	Expect result under 5%.		
d) i)	<ul style="list-style-type: none"> <li>By interviewing each student and removing from the study those whose parents had substituted a reward.</li> <li>By blinding the lecturers to which participants were in the study.</li> </ul>	One acceptable response to the possible confounding factors for the study.	Two acceptable responses to the possible confounding factors for the study.	
ii)	<ul style="list-style-type: none"> <li>1999/2000 The one exam (econometrics) could have been easier.</li> <li>Differences in lectures one teaching econometrics better than another etc.</li> </ul>	One acceptable response to the possible confounding factors for 1999/2000 study on econometrics.	Two acceptable responses to the possible confounding factors for 1999/2000 study on econometrics.	
e)	It was not causal as the results came after 3 years. If the experiment had returned a significant result on the original premise, large reward, median reward no reward after one year then it would have been considered causal.		It was not causal as the results came after three years and on a different premise.	Causal if randomised experiment and conditions unchanged.

## Practice External Assessment – Statistical Reports

In the external examinations NZQA uses a different approach to marking based on understanding (u), relational thinking (r) and abstract thinking (t). They then allocate marks to these concepts and add them up to decide upon the overall grade. This approach is not as easy for students to self mark as the NuLake approach, but the results should be broadly similar.

**Sufficiency.** For each question award yourself a score out of 8 using this table. Add the three scores for a score out of 24 and compare to the cut scores. All answers must include evidence / justification where appropriate.

Quest.	N0	N1	N2	A3	A4	M5	M6	E7	E8
ONE	No correct MoE.	1 correct MoE.	1 A	2A or equiv.	3A or equiv.	1M + 1M minor error	2M + 1M minor error	1E + 1E minor error	3E all correct
TWO	No correct MoE.	1 correct MoE.	1 A	2A or equiv.	3A or equiv.	1M + 1M minor error	2M + 1M minor error	1E + 1E minor error	2E all correct
THREE	No correct MoE.	1 correct MoE.	1 A	2A or equiv.	3A or equiv.	1M + 1M minor error	2M	1E Minor error	2E all correct
Cut Scores									
	<b>Not Achieved</b>	<b>Achievement</b>	<b>Achievement with Merit</b>	<b>Achievement with Excellence</b>					
	0 – 6	7 – 13	14 – 20	21 – 24					

**Answers**

**Page 74**

1. a)

	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	8	10	12
3	3	6	9	12	15	18
4	4	8	12	16	20	24
5	5	10	15	20	25	30
6	6	12	18	24	30	36

- b)  $P(\text{prod.} = 6) = \frac{4}{36} \left( \frac{1}{9} \right)$   
 c)  $P(\text{prod.} = 12) = \frac{4}{36} \left( \frac{1}{9} \right)$   
 d)  $P(\text{prod. is prime}) = \frac{6}{36} \left( \frac{1}{6} \right)$   
 e)  $P(\text{prod.} = 14) = 0$
2. a)  $P(\text{club}) = \frac{13}{54}$   
 b)  $P(6 \text{ of spades}) = \frac{1}{54}$   
 c)  $P(\text{red ace}) = \frac{2}{54} \left( \frac{1}{27} \right)$   
 d)  $P(\text{not joker}) = \frac{52}{54} \left( \frac{26}{27} \right)$   
 e)  $P(\text{second ace}) = \frac{3}{53}$
3. a)  $P(2\text{nd red}) = \frac{3}{9} \left( \frac{1}{3} \right)$   
 b)  $P(3\text{rd blue}) = \frac{6}{8} \left( \frac{3}{4} \right)$
4. a)  $P(\text{both}) = 0.004\ 875$   
 b)  $P(A \text{ and not } B) = 0.070\ 125$   
 c)  $P(\text{neither}) = 0.864\ 875$

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5. a)  $P(\text{both miss}) = 0.15 \times 0.20 = 0.03$   
 b)  $P(\text{one late}) = 0.15 \times 0.80 + 0.20 \times 0.85 = 0.29$   
 c)  $P(\text{neither miss}) = 0.85 \times 0.80 = 0.68$
6. a)  $P(\text{GGR}) + P(\text{GRG}) + P(\text{RGG}) = (0.6 \times 0.6 \times 0.4) \times 3 = 0.432$   
 b)  $P(\text{GGR}) + P(\text{GRG}) + P(\text{RGG}) = (0.6 \times 0.5555 \times 0.5) \times 3 = 0.5$

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7. a)  $P(\text{all red}) = 0.05522$   
 b)  $P(2B \text{ and } 2R) = 0.3902$
8. a)  $P(\text{blue from 1st garden}) = 0.3333 \times 0.25 = 0.08333$   
 b)  $P(\text{not blue from 3rd garden}) = 1 - (0.3333 \times 0.5) = 0.8333$   
 c)  $P(\text{blue flower}) = (0.3 \times 0.25) + (0.3 \times 0.3) + (0.3 \times 0.5) = 0.3611$
9. a)  $P(\text{both chrome}) = 0.06667$   
 b)  $P(\text{same}) = 0.06667 + 0.02222 + 0.2222 = 0.3111$   
 c)  $P(\text{different}) = 1 - 0.3111 = 0.6889$

**Page 76**

10. a)  $P(\text{solved}) = 1 - P(\text{not solved}) = 1 - (0.8 \times 0.75 \times 0.6) = 0.64$   
 b) Assumed the result of each student is independent and the performance of past problems give an accurate indication of performance on future problems. Neither assumption is likely to be valid. The problem could be easy and if one gets it the other is also likely to get it. Problems vary a lot so past performance is not likely to predict to 2 significant figures performance in the future.
11. a)  $P(> 1) = 1 - (P(0) + P(1)) = 1 - (0.9^{10} + 10 \times 0.9^9 \times 0.1) = 0.264 \quad (3 \text{ sf})$   
 b) Independence is assumed and if the 10% of faults are random and not in clusters then the assumption is valid.
12. a)  $P(\text{wins } 0) = 0.04$   
 b)  $P(\text{wins } 1) = 0.26$   
 c)  $P(\text{wins } 2, 3) = 0.7$   
 d) Independence is assumed. If winning or losing the first race changes the likelihood then it is not valid. Also past performance is not a valid predictor of future performance because competitors change, so it is not likely to be correct.

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13. a)  $P(\text{green \& 1st jar}) = 0.3 \times 0.2 = 0.06667$   
 b)  $P(\text{green lolly}) = (0.3 \times 0.2) + (0.3 \times 0.3) + (0.3 \times 0.5) = 0.3444$   
 c)  $P(\text{green}) = 0.3$   
 d) Possibly not, as a child may be inclined to go for the jar with more lollies in it or the closest jar.

**Page 77**

14. a)  $P(\text{one successful}) = (0.5 \times 0.6666 \times 0.75) + (0.3333 \times 0.5 \times 0.75) + (0.25 \times 0.5 \times 0.6666) = 0.4583$   
 b)  $P(\text{both ring and work at B}) = 0.45 \times 0.1 = 0.045$   
 c)  $P(\text{both ring and not work}) = 0.55 \times 0.3 + 0.45 \times 0.9 = 0.57$   
 d)  $P(A) = 0.0305, P(B) = 0.35$   
 $P(A) \times P(B) = 0.01068$   
 $P(A \cap B) = 0.35 \times 0.05 = 0.0175$   
 Not independent.
15. a) Tossing will continue until someone tosses a head. If we let the probability that Cloris will win be  $p$  then the chance of Brian winning is twice as likely  $2p$ . Similarly the chance of Ana winning is twice as high as Brian so  $4p$ . As  $p + 2p + 4p = 1$  we have  

$$P(C) = p = \frac{1}{7}$$
 This gives the probability of Ana winning as  

$$P(A) = 4p = \frac{4}{7}$$
- b) That the coin is not biased or asymmetrical which causes the true probability to vary from 0.5.  
 That the height it is tossed is random for each participant. Each participant is honest in reporting the outcome.

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16. a)  $P(\text{not } W \text{ and not } W)$   
 $= 0.65 \times 0.65$   
 $= 0.4225$
- b)  $P(\geq 1 \text{ white}) = 1 - 0.4225$   
 $= 0.5775$
- c) That the distribution of white Camrys is even across NZ.  
 That the proportion of white Camrys is the same each year as older cars are scrapped.  
 That seeing one white Camry is independent of seeing another white Camry.

17. a)  $P(\geq 1 \text{ fails}) = 1 - 0.96 \times 0.96$   
 $= 0.0784$
- b)  $P(\text{both fail}) = 0.0016$
- c) That the memory modules are correctly installed and not mistreated.  
 That the computer is used the same amount and not subject to a harsh environment.  
 That one module failing is independent of the another module failing (no power surges etc.).

18. a)  $P(\text{all}) = 0.133^4$   
 $= 0.000313$
- b)  $P(\text{at least one}) = 1 - 0.867^4$   
 $= 0.4350$
- c) That the researcher is calling mobile phones as many in this age group only have mobile phones.  
 That the time of day does not affect the likelihood of the phone being answered.  
 That members of this group have the same likelihood of answering calls when they do not know the caller.

19. a)  $P(2 \text{ no fault}) = 0.88^2$   
 $= 0.7744$
- b)  $P(3 \text{ faults}) = 0.12^3$   
 $= 0.0017$
- c) That the rate of fault does not depend upon the time of the year (e.g. winter).  
 That the probability of a fault does not depend upon how many days there are in a month.

Page 78 Q 19 c) cont...

19. c) That the rate of faults does not vary throughout the country. That faults are independent of one another.

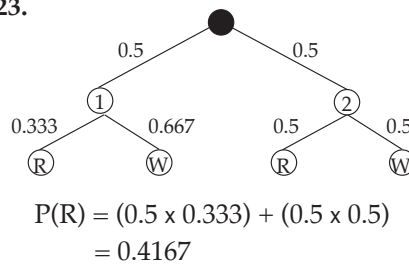
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20.  $P(3 \text{ draws required}) = 0.3333$

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21. a)  $P(T \text{ and } 2) = 0.08333$   
 b)  $P(\text{coin tossed twice}) = 0.5$
22. a)  $P(\text{passes all}) = 0.6 \times 0.7 \times 0.8$   
 $= 0.336$   
 b)  $P(\text{passes one}) = 0.6 \times 0.3 \times 0.4$   
 $+ 0.4 \times 0.5 \times 0.4 + 0.4 \times 0.5 \times 0.4$   
 $= 0.232$

23.

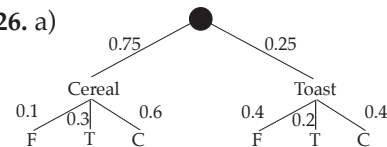


24. a)  $P(\text{sum} < 7) = 0.6$   
 b)  $P(\text{sum} \geq 5) = 0.8$

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25. a)
- b)  $P(\text{uninjured})$   
 $= 0.6 + (0.4 \times 0.35 \times 0.98)$   
 $+ (0.4 \times 0.65 \times 0.85)$   
 $= 0.9582$
- c)  $P(\text{injured}) = 0.0418$   
 $P(\text{inj. \& no helm.}) = 0.0390$   
 $P(\text{No helm. of inj.}) = 0.9330$

26. a)



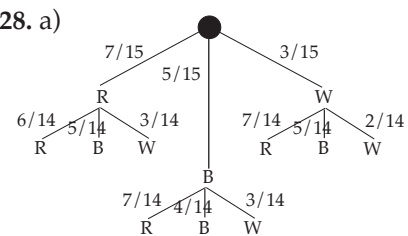
- b)  $P(\text{cereal and coffee})$   
 $= 0.75 \times 0.6$   
 $= 0.45$
- c)  $P(\text{fruit juice}) = (0.75 \times 0.1) + (0.25 \times 0.4)$   
 $= 0.175$
- d) Least popular is tea and toast with probability 0.05.

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27. a)
- b)  $P(\text{positive}) = (0.04 \times 0.99) + (0.96 \times 0.02)$   
 $= 0.0588$

- c)  $P(\text{disease given positive})$   
 $= \frac{0.0396}{0.0588}$   
 $= 0.6735$
- d)  $P(\text{not disease given negative})$   
 $= \frac{0.9408}{0.9412}$   
 $= 0.9996$

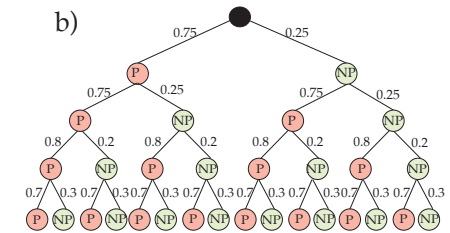
28. a)



- b)  $P(\text{RW in any order}) = 0.2$
- c)  $P(\text{different colours})$   
 $= 1 - P(\text{same})$   
 $= 0.6762$
- d)  $P(\text{one white marble})$   
 $= P(\text{WR, RW, WB, BW, WW})$   
 $= 0.3714$
- e) Two red marbles.

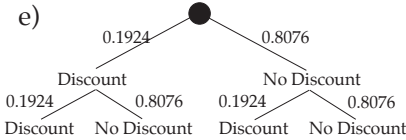
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29. a)  $P(4 \text{ aces}) = 0.00000625$



- c)  $P(\text{one picture card})$   
 $= 0.04625$
- d) If played 160 000 times ( $20^4$ ) it would earn \$320 000 and expect to return  
 $= \$100\ 000 + 240 \times \$500 + 7400 \times \$2$   
 $= \$234\ 800$   
 Profit for club per play  
 $= \$85\ 200 \div 160\ 000$   
 $= \$0.5325 \text{ or } 53.25\text{c}$

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30. a) Small triangle  
 $= 0.5 \times 20 \times 20 \times \sin 60^\circ$   
 $= 173.2 \text{ cm}^2$   
 Large triangle  
 $= 0.5 \times 40 \times 40 \times \sin 60^\circ$   
 $= 692.8 \text{ cm}^2$
- b)  $P(15\% \text{ discount}) = 0.1443$   
 c)  $P(50\% \text{ discount}) = 0.0481$   
 d)  $P(\text{no discount}) = 0.8076$   
 e)   
 f)  $P(\text{one discount}) = 0.3108$   
 g)  $P(\text{at least 1 discount}) = 0.3479$   
 h)  $P(50\% \text{ on at least one throw}) = 0.0939$   
 i) It may depend upon the experience of the customer at throwing darts. It could be that less experienced customers throw weakly so the dart tends to land in the bottom of the area. The results then would be worse than random.  
 Investigate by running an exercise with about 30 randomly selected blindfolded individuals each of which is limited to one throw in the square. Then compare the experimental results to the theoretical result.

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31. a) Sample space  
 BBB, BBG, BGB, BGG, GBB, GBG, GGB, GGG,
- b)  $P(\text{same sex}) = \frac{2}{8}$   
 $= 0.25$
- c)  $P(3B | \geq 2B) = \frac{1}{4}$   
 $= 0.25$
32. a)  $0.05 + 0.03 + 0.105 = 0.185$   
 b)  $P(N | F) = \frac{0.105}{0.185}$   
 $= 0.568 \text{ (3 sf)}$

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33. a)  $P(\text{exercises regularly}) = 0.375$   
 b)  $P(50 \text{ or over}) = 0.525$   
 c)  $P(\text{not exercise} | < 50) = 0.4211$   
 d)  $P(\text{exercises} | < 50) = 0.5789$   
 e)  $P(< 50 \text{ or no exercise}) = 0.9$   
 f)  $P(\geq 50 | \text{no exercise}) = 0.68$

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34. a)  $P(B) = 0.6$   
 b)  $P(G) = 0.4$   
 c)  $P(G \cap S) = 0.1333$   
 d)  $P(S | G) = 0.3333$   
 e)  $P(B | S') = 0.4286$   
 f)  $P(S \cup B) = 0.7333$
35. a)  $P(\text{credit card}) = 0.2364$   
 b)  $P(\text{credit card} | \text{Thu}) = 0.3333$   
 c)  $P(\text{credit card and Thu}) = 0.1455$   
 d)  $P(\text{it was on Friday}) = 0.5636$   
 e)  $P(\text{Thu} | \text{credit card}) = 0.6154$   
 f)  $P(\text{not purchased with EFT on Thursday}) = 0.7091$
36. a)  $P(\text{roll} | \text{pie}) = 0.6667$   
 b)  $P(\text{pie} | \text{roll}) = 0.32$   
 c)  $P(\text{pie or roll}) = 0.29$

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37. a)  $P(2\text{nd red given } 1\text{st black}) = 0.5098$   
 b)  $P(2\text{nd ace given } 1\text{st not ace}) = 0.0784$
38.  $P(\text{one putt} | 1\text{st on green}) = 0.25$
39. a)  $P(M | \text{NCEA } 2) = 0.4898$   
 b)  $P(\text{NCEA } 3 | M) = 0.2512$   
 c)  $P(F | \text{NCEA } 1) = 0.5514$

40.  $P(\text{buys lunch} | \text{bus}) = 0.625$
41. a)  $P(A \cap B) = 0.27$   
 b)  $P(C | B) = 0.35$   
 c)  $P(A | B) = 0.3375$
42. a)  $P(\text{tails once} | \text{all same}) = 0.5$   
 b)  $P(\text{tails once} | 3\text{rd heads}) = 0.75$
43.  $P(2\text{nd red} | \text{first red}) = 0.625$

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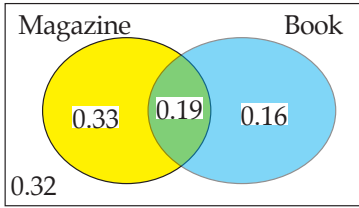
44. a)  $P(\text{same} | \text{at least one is } 3) = 0.0909$   
 b)  $P(\text{prod} > 20 | \text{one number } 5) = 0.2$
45. a)  $P(\text{first is chocolate}) = \frac{x}{x+y}$   
 b)  $P(\text{first two chocolate}) = \frac{x}{x+y} \times \frac{x-1}{x+y-1}$   
 c)  $P(1\text{st chocolate } 2\text{nd caramel}) = \frac{x}{x+y} \times \frac{y}{x+y-1}$   
 d)  $P(2\text{nd caramel} | 1\text{st chocolate}) = \frac{\frac{x}{x+y} \times \frac{y}{x+y-1}}{\frac{x}{x+y}} = \frac{y}{x+y-1}$   
 e)  $P(\text{two or more caramel}) = \frac{y(y-1)(y-2) + 3xy(y-1)}{(x+y)(x+y-1)(x+y-2)}$   
 f)  $P(\text{most caramel} | \text{first choc}) = \frac{xy(y-1)}{(x+y)(x+y-1)(x+y-2)} = \frac{\frac{x}{x+y} \times y(y-1)}{(x+y-1)(x+y-2)}$

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46. a)  $P(\text{absent and in Year } 9) = 0.0276$   
 b)  $P(\text{not absent and in Year } 9) = 0.2024$   
 c)  $P(\text{absent or in Year } 9) = 0.2750$
47. a) 15  
 b) Soccer and netball  
 c)  $P(\text{netball or baseball}) = 0.56$   
 d)  $P(\text{soccer or netball}) = 0.48$

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48. a)



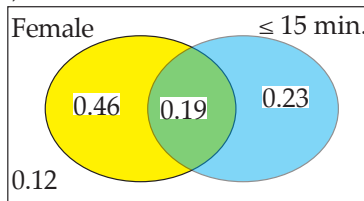
- b)  $P(\text{book and mag}) = 0.19$
- c)  $P(\text{mag and not book}) = 0.33$
- d)  $P(\text{lotto, book and mag}) = 0.1235$

- 49. a)  $P(\text{two successive tickets}) = 0.0132$
- b)  $P(1 \text{ prize on either 2 tickets}) = 0.2036$
- c)  $P(\text{no prize on three tickets}) = 0.6932$

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- 50. a)  $P(\text{at least one subject}) = 0.902$
- b)  $P(\text{only 1 of the 2 subjects}) = 0.434$
- c)  $P(\text{neither of the two subjects}) = 0.098$
- 51. a)  $P(\text{away and win}) = 0.13$
- b)  $P(\text{lose a home game}) = 0.18$
- c)  $P(\text{away and lose}) = 0.27$

52. a)



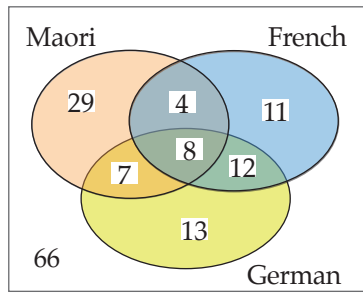
- b)  $P(\text{male and } \leq 15 \text{ min}) = 0.23$
- c)  $P(\text{male and requires } \geq 15 \text{ min}) = 0.12$
- d)  $P(\text{female or requires } \leq 15 \text{ min}) = 0.88$

53. a) 120 students

- b)  $P(\text{statistics and English}) = 0.075$
- c)  $P(\text{all three subjects}) = 0.0417$
- d)  $P(\text{English but not biology}) = 0.2417$

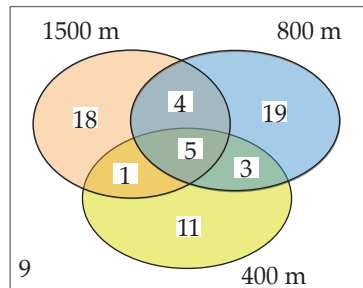
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54. a)



- b)  $P(\text{not study a language}) = 0.44$
- c)  $P(\text{at least two languages}) = 0.2067$
- d) % study language, study French = 41.7%

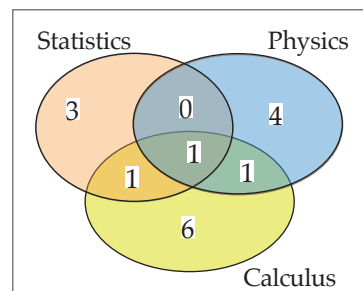
55. a)



- b) 9 athletes
- c)  $P(\text{runs in only one event}) = 0.6857$
- d)  $P(\text{runs in two events}) = 0.1143$

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56. a)



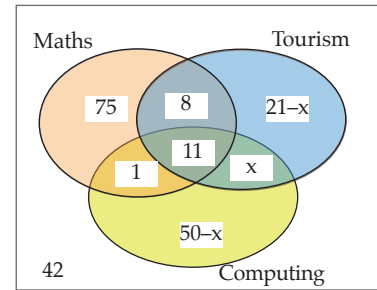
- b)  $P(\text{calc and/or stat, not phy}) = 0.625$
- c)  $P(\text{more than one subject}) = 0.1875$
- d) 81.25% teach only one subject
- e)  $P(\text{teach another subject given they teach calculus}) = 0.3333$

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- 57. a) 0.72
- b) 0.56
- c) 0.24
- d) 0.27
- e) 0.4231
- f) 0.08
- g) 0.4681
- h) 0.61

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58.



a)  $95 + (21 - x) + x + (50 - x) = 158$   
 $x = 8$

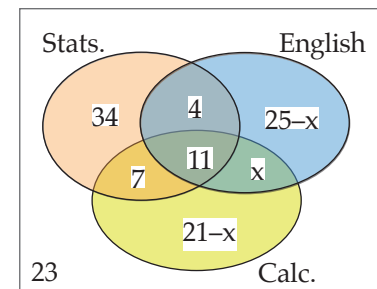
b)  $P(T \text{ or } C) = \frac{83}{200} (0.415)$

c)  $P(M \text{ and } T) = \frac{19}{200} (0.095)$

$P(M) \times P(T) = \frac{95}{200} \times \frac{40}{200}$   
 $= \frac{19}{200} (0.095)$   
 $= P(M \text{ and } T)$

Therefore independent.

59. a)



$56 + (21 - x) + x + (25 - x) = 102$   
 $x = 0$   
 $P(C \text{ and } E) = 0$

b)  $P(\text{another } | S) = \frac{22}{56} (0.3929)$

c) That students in these three classes all come from Year 13. That Year 12 students are excluded from these figures.

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60.  $4 \times 3 \times 4 = 48$   
 61.  $8 \times 7 \times 6 = 336$   
 62. a)  $5 \times 5 \times 5 = 125$   
 b)  $5 \times 4 \times 3 = 60$   
 63. a)  $4 \times 4 \times 4 = 64$   
 b)  $3 \times 4 \times 4 = 48$   
 64.  $9 \times 10 \times 10 \times 10 \times 10 = 90\,000$   
 65.  $9 \times 9 \times 8 \times 7 \times 6 = 27\,216$

## Page 104

66.  $2 \times 5 \times 4 \times 3 \times 2 \times 1 = 240$   
 67.  $1 \times 2 \times 10 = 20$   
 68.  $6 \times 5 \times 4 \times 3 \times 2 = 720$   
 69.  $120 \times 0.6 = 72$ . Only 0.6 of all the numbers will end in 3, 5 or 7 and hence be odd.  
 70.  $4 \times 3 = 12$   
 71.  $15 \times 6 \times 8 = 720$   
 72.  $2 \times 2 \times 4 = 16$   
 73.  $\frac{2 \times 3 \times 2}{3 \times 5 \times 4} = 0.2$   
 74.  $6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$   
 75.  $3 \times 2 \times 1 = 6$   
 76. 8 000 000  
 77.  $8 \times 10 \times 10 \times 5 \times 5 \times 5 \times 5 = 500\,000$   
 78.  $5 \times 5 \times 5 \times 5 \times 5 = 3125$   
 79.  $3 \times 6 \times 6 \times 6 \times 6 = 3888$

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80. a)  $P(\text{Spectator}) = \frac{20}{48}(0.4167)$   
 b)  $P(\text{Male}) = \frac{29}{48}(0.6042)$   
 c)  $P(\text{Spect.} | F) = \frac{12}{19}(0.6316)$   
 81. a)  $P(\text{Junior}) = \frac{34}{66}(0.5152)$   
 b)  $P(J | Ab) = 1$   
 c) No intersection.

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82. a) No.  $P(\text{Fossil}) \neq 1 - P(\text{Elect.})$   
 b) No.  $P(\text{Fossil}) = 0.489$   
 $P(\text{Rented}) = 0.468$   
 $P(\text{Fossil}) \times P(\text{Rented}) = 0.229$   
 $P(\text{Rent AND Fossil}) = 0.266$   
 Not equal so NOT independent.  
 c) The figures are for primary source. They may use electricity as their primary source and also use fossil fuel.  
 83. a) The figures are percentages of the 52% that use Facebook.  
 b) 52.5% of the sample of Facebook users are under 25. Not of all employees.  
 c)  $P(F. \text{ at } W) = 0.419$   
 $P(< 25) = 0.525$   
 $P(F. \text{ at } W \text{ and } < 25) = 0.233$   
 $P(F. \text{ at } W) \times P(< 25) = 0.220$   
 Not equal so NOT independent.  
 84. a) NOT independent. There is the same number in the samples from each city so number prepared should be the same to be in the same proportion. Alternatively  
 $P(W) = 0.5$   
 $P(\text{Prep.}) = 0.1845$   
 $P(W) \times P(\text{Prep.}) = 0.0923$   
 $P(W \text{ and Prep.}) = 0.1235$   
 As  $P(W) \times P(\text{Prep.})$  is NOT equal to  $P(W \text{ and Prep.})$  NOT independent.  
 b)  $P(W | \text{Prep.}) = 0.6694$   
 c)  $P(\text{Prepared}) = \frac{269\,600}{1\,800\,000} = 0.1498$

## Page 108

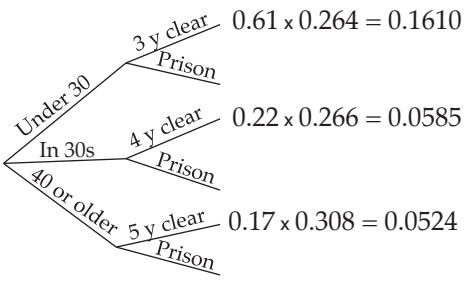
85. a)  $P(\text{wk. unemp.}) = \frac{141\,800}{2\,371\,000} = 0.060$   
 b)  $P(\text{wk. 15-24 unemp.}) = \frac{59\,500}{407\,200} = 0.146$   
 $P(\text{wk. 25-64 unemp.}) = \frac{82\,300}{1\,963\,800} = 0.042$   
 c) Likelihood Ratio = 3.48. A younger worker (15 – 24 years old) is 3.5 times as likely to be unemployed.

## Page 109

86. a) Likely ratio = 0.811  
 b) Likely ratio = 2.654  
 c) Younger people generally feel safe using the internet for transactions (73.5%) whereas older people are 0.811 times as likely to feel safe.  
 Younger people feel they experience discrimination at 2.65 times the rate of older people.  
 It is possible that younger people use the internet a lot more for online transactions and hence have gained confidence while many older people lack this experience and hence feel less safe.  
 It is likely that younger people are more socially mobile and therefore likely to come across situations where they experience discrimination. Also the proportion of minority groups in the younger population is likely to be higher. Older people are more likely to stay in their own social groups and are therefore less likely to suffer from discrimination.

Pages 109 – 114 Practice External Assessment Task – Probability

Q	Evidence	Achievement	Merit	Excellence																
1 a)	i) $P(\text{not sport}) = 1 - \frac{6}{11} \times \frac{5}{10} = \frac{8}{11} (0.727)$	Correct probability.																		
a)	ii) $P(\text{Sport 1}) = \text{1st selection OR 2nd selection}$ $= \left(\frac{1}{11} \times \frac{10}{10}\right) + \left(\frac{10}{11} \times \frac{1}{10}\right) = \frac{2}{11}$	Correct probability.																		
b)	i) <table border="1" style="margin-left: 20px;"> <thead> <tr> <th></th> <th>Female</th> <th>Male</th> <th>Totals</th> </tr> </thead> <tbody> <tr> <td>Entert.</td> <td>330</td> <td>240</td> <td>570</td> </tr> <tr> <td>Non-Ent.</td> <td>260</td> <td>210</td> <td>470</td> </tr> <tr> <td>Totals</td> <td>590</td> <td>450</td> <td>1040</td> </tr> </tbody> </table>		Female	Male	Totals	Entert.	330	240	570	Non-Ent.	260	210	470	Totals	590	450	1040	At least one correct probability. $P(E   M) = 0.5333$ $P(E   F) = 0.5593$	Calculates all probabilities and states conclusion that females more likely.	
	Female	Male	Totals																	
Entert.	330	240	570																	
Non-Ent.	260	210	470																	
Totals	590	450	1040																	
b)	ii) Conclusion not wise because: <ul style="list-style-type: none"> <li>• It was only ONE survey and there will be a variation of results from each survey.</li> <li>• The time of the survey as one gender may be more likely to not be watching TV from 7 to 9 pm.</li> <li>• The day of the survey will likely affect the result as on Wednesday there is very little sport so the probability is likely to be different on other days.</li> <li>• The survey was conducted at one time of the year and in different seasons so there could be a different proportion watching other channels.</li> <li>• etc.</li> </ul>		Gives a plausible explanation why the true probability is likely to be different from this survey.	Gives at least two plausible explanations why the true probability is likely to be different from this survey.																
c)	$P(\text{different}) = P(e, d, s) + P(e, s, d) + P(d, e, s)$ $+ P(d, s, e) + P(s, e, d) + P(s, d, e)$ $= \frac{35}{136} (0.2574)$	$P(e, d, s)$ $= 0.0429$	Correct probability.																	
2 a)	i) $P(\text{Everynight}   \text{at least weekly}) = \frac{200}{371} (0.539)$	Correct probability.																		
a)	ii) $P(\text{TV and 1 to 3.5 h}) = 0.539 \times 0.21 = 0.113$ Assuming that the events of watching TV each night (frequency and length) are independent of each other. The frequency of watching television is likely to be related to how long a person watches television for, so unlikely to be independent.	Correct probability.	Correct probability and recognises that independence must be assumed.	Correct probability and correct statement as to why independence may not be valid.																
a)	iii) <table border="1" style="margin-left: 20px;"> <thead> <tr> <th>%</th> <th>Female</th> <th>Male</th> <th>Totals</th> </tr> </thead> <tbody> <tr> <td>&lt; 1 h.</td> <td>8</td> <td>3</td> <td>11</td> </tr> <tr> <td>≥ 1 h.</td> <td>49</td> <td>40</td> <td>89</td> </tr> <tr> <td>Totals</td> <td>57</td> <td>43</td> <td>100</td> </tr> </tbody> </table>	%	Female	Male	Totals	< 1 h.	8	3	11	≥ 1 h.	49	40	89	Totals	57	43	100	Table correct OR Uses 417 instead of 400.	Correct Number $= 0.03 \times 400$ $= 12$	
%	Female	Male	Totals																	
< 1 h.	8	3	11																	
≥ 1 h.	49	40	89																	
Totals	57	43	100																	
b)	i)	Correctly draws Venn diagram with at most one error. 10 viewers are both Sky and Freeview and not TV One.	Correctly draws Venn diagram and calculates probability. $P(\text{Sky and Free NOT TV One}) = 0.1$																	
b)	ii) Both 12% but based on <ul style="list-style-type: none"> <li>• One survey only.</li> <li>• Timing of survey will affect how many watch TV One.</li> <li>• Over time the number streaming will change.</li> <li>• Some people will stream to a digital device not TV.</li> </ul>		Gives a plausible explanation why the conclusion is likely invalid.	Gives at least two plausible explanations why conclusion likely invalid.																

Q	Evidence	Achievement	Merit	Excellence
3 a)	i) $P(\text{parole and prison}) = 0.3576 \times 0.49 = 0.175$	Correct probability.		
a)	ii) $P(\text{Violation}) = 0.19 \times 0.3576 + 0.01 \times 0.6424 = 0.07437$ $P(\text{Early} \mid \text{Violation}) = 0.06795 \div 0.07437 = 0.9137$	At least one correct probability.	Conditional probability correct.	
a)	iii) Likely be different types of inmates or different offence committed to be considered for early release / parole etc.		One plausible explanation.	
b)	i) $P(\text{No prison} \leq 36) = 1 - 0.678 = 0.322$	Correct probability.		
b)	ii)  $P(> 3 \text{ years original sentence}) = 0.2719$	At least one correct probability. $\leq 29$ years need 3 years free. 30 - 39 years need 4 years free. 40+ years need 5 years free.	Tree diagram substantially correct.	Tree diagram and model correct. Probability correct.
b)	iii) Model is unlikely to lead to a change in behaviour as it is based on the inmates age only. You are likely to still have the same offences but spread over a different period. You could reduce prison population by reducing all sentences by 1 year but this is unlikely to reduce re-offending rates.		One plausible explanation.	

**Practice Assessment – Probability**

In the external examinations NZQA uses a different approach to marking based on understanding (u), relational thinking (r) and abstract thinking (t). They then allocate marks to these concepts and add them up to decide upon the overall grade. This approach is not as easy for students to self mark as the NuLake approach, but the results should be broadly similar.

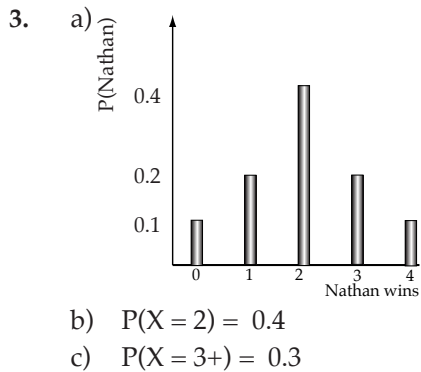
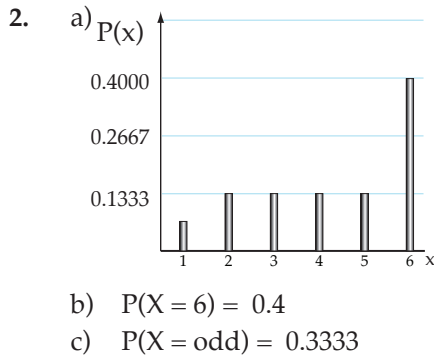
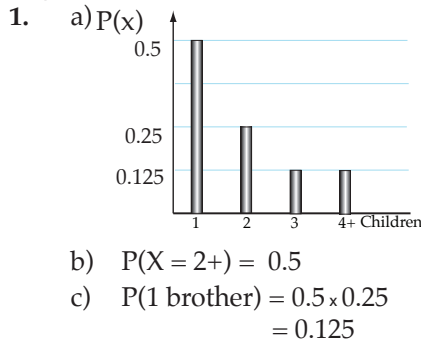
**Sufficiency.** For each question award yourself a score out of 8 using this table. Add the three scores for a score out of 24 and compare to the cut scores. All answers must include evidence / justification where appropriate.

Quest.	N0	N1	N2	A3	A4	M5	M6	E7	E8
ONE	No correct Prob.	1 correct Prob.	1 A	2A or equiv.	3A or equiv.	1M + 1M minor error.	2M	1E Minor error.	1E all correct.
TWO	No correct Prob.	1 correct Prob.	1 A	2A or equiv.	3A or equiv.	1M + 1M minor error.	2M	1E correct.	2E Minor error.
THREE	No correct Prob.	1 correct Prob.	1 A	2A or equiv.	3A or equiv.	1M + 1M minor error.	2M	1E Minor error.	1E all correct.
Cut Scores									
Not Achieved		Achievement		Achievement with Merit		Achievement with Excellence			
0 – 6		7 – 13		14 – 20		21 – 24			



Answers

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4. a) 

X	1	2	3	4
P(X = x)	0.25	0.25	0.25	0.25
- b)  $E(X) = 2.5$
5. a)  $E(X) = 1.59$  hours  
 $\text{Var}(X) = 0.637$  (3 sf)  
 b)  $P(X < 2) = 0.47$
6. a) 

X	1	2	3	4	5
P(X = x)	0.45	0.30	0.12	0.10	0.03
- b)  $P(5 \text{ days}) = 0.03$   
 c)  $E(X) = 1.96$  days
7. a) 

X	0	1	2	3
P(X = x)	p	6p	2p	p
P(X = x)	0.1	0.6	0.2	0.1
- b)  $P(3 \text{ lambs}) = 0.1$   
 c)  $E(X) = 1.3$  lambs  
 $\text{Var}(X) = 0.610$  (3 sf)

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8. 

X	-6	3	4	5	12
P(X = x)	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$
- $E(X) = 2$  squares  
 $\text{Var}(X) = 40.3$  (3 sf)
9. 

X	0	\$5	\$10
P(X = x)	0.375	0.375	0.25
- $E(\text{Winnings}) = 4.375$   
 $E(\text{Result}) = -0.625$   
 Loss of 62 or 63 cents  
 $\text{Var}(X) = 15.2$

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10.  $E(2X) = 14$   
 $\text{Var}(2X) = 2^2 \times 5.83 = 23.32$
11.  $E(X + X) = 14$   
 $\text{Var}(X + X) = 5.83 + 5.83 = 11.66$
12. a)  $E(M) = 4.2$   
 $\text{Var}(M) = 2.676^2 = 7.16$   
 $E(C) = 3.75$   
 $\text{Var}(C) = 2.095^2 = 4.39$
- b)  $P(\text{some}) = 1 - 0.03 = 0.97$
- c)  $E(\text{Tot.}) = 7.95$   
 $\text{Var}(\text{Tot.}) = 11.55$
13. a) About 10. Distribution fairly symmetrical about 10.  
 b)  $E(C) = 9.9$   
 $\text{Var}(C) = 45.1$   
 c)  $E(\$) = 123.75$   
 $\text{Var}(\$) = 7047$

14. a) Kaydee  $E(C) = 15$  (symmetrical)  
 Jenni  $E(C) = 12$  Skewed distribution.  
 b) Kaydee  $E(C) = 15$   
 $\text{Var}(C) = 30.7$   
 Jenni  $E(C) = 11.85$   
 $\text{Var}(C) = 18.4$ .  
 c)  $E(K - J) = 3.15$   
 $\text{Var}(K - J) = 49.1$

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15. a)  $P(X = 2) = 0.2791$   
 b)  $P(X < 3) = 0.8217$   
 c)  $P(X \leq 2) = 0.8217(6)$   
 d)  $P(X > 2) = 0.1783(4)$

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16. a)  $P(X = 0) = 0.0576$   
 b)  $P(X = 4) = 0.1361$   
 c)  $P(X \leq 3) = 0.8059$   
 d)  $P(X \geq 6) = 0.0113$
17. a)  $P(X = 5) = 0.0001$   
 b)  $P(X = 0) = 0.4437$   
 c)  $P(X > 3) = 0.0023$   
 d)  $P(X < 2) = 0.8352$
18. a)  $P(X = 5) = 0.0467$   
 b)  $P(X \geq 5) = 0.0580$   
 c)  $P(X = 1) = 0.1977$   
 d)  $P(X \leq 4) = 0.9420$

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19. a)  $P(X = 6) = 0.1762$   
 b)  $P(X < 5) = 0.0196$   
 c)  $P(3 \leq X \leq 7) = 0.5636(5)$   
 d)  $P(X = 9) = 0.1342$
20. a)  $P(X = 2) = 0.0229$   
 b)  $P(X < 5) = 0.2616$   
 c)  $P(X = 0) = 0.0003$   
 d)  $P(X = 8) = 0.0763$
21. a)  $P(X = 4) = 0.2322$   
 b)  $P(X \leq 3) = 0.1737$   
 c)  $P(X > 4) = 0.5941$   
 d)  $P(X = 0) = 0.0007$
22. a)  $P(X \geq 5) = 0.8552(1)$   
 b)  $P(X = 6) = 0.2731$   
 c)  $P(5 \leq X \leq 7) = 0.7121$   
 d)  $P(X > 0) = 0.9999$

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23. a) Fixed number of events. Independence assumed. Probability constant. Only two outcomes.  
 b)  $P(X \geq 2) = 1 - [P(X = 0) + P(X = 1)] = 0.4573(4)$   
 c)  $P(X \geq 2)$  and  $P(X \geq 2) = 0.2091$
24. a)  $P(X = 3) = 0.2668$   
 b)  $P(X > 2) = 1 - 0.3828 = 0.6172$   
 c)  $[P(X > 2)]^2 = 0.3809$
25. a)  $P(X \geq 5) = 0.9936$   
 b)  $[P(X \geq 5)]^5 = 0.9686$   
 c) Yes. Only two outcomes. Fixed No. of trials = 10. Independence assumed. Constant probability = 0.8. If a disease or fungi spreads from one bulb to another, results are not independent, for example.

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26. a) Fixed No. of trials = 5  
Independence assumed  
Constant probability = 0.2  
Only two outcomes.  
b) Possibly not independent.  
If they are late one day it may affect probability on following days.  
c)  $P(X = 1)^2 = 0.1678$   
d)  $P(X = 1 \text{ in two weeks}) = 0.2684$

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27. a) Fixed number of events.  
Independence of one test to the next test.  
b)  $P(X \leq 2) = 0.2616$  (7)  
c)  $P(X > 7) = 1 - P(X \leq 7) = 0.3990$   
28. a) Fixed No. of throws per game.  
Independence assumed.  
Probability constant = 0.8.  
Only two outcomes.  
b)  $P(X = 6) \times P(X = 3) = 0.02147$   
c)  $P(X = 9) = 0.2362$   
d) No for independence as one result could affect the next and no for constant probability as Heidi will tire during the game and may not shoot as well as at the start.

29. a)  $[P(X = 0)]^{10} = 0.735110 = 0.0461$   
b)  $P(X \leq 1) = 0.967226$   
 $P(X \leq 1)^{10} = 0.7166$   
30. a)  $P(X \geq 1) = 0.2262$   
b)  $P(X \leq 1) = 0.3019$

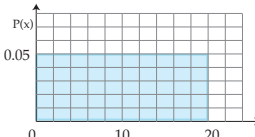
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31. a)  $P(X = 0) = 0.0183$   
b)  $P(X = 2) = 0.1465$   
c)  $P(X > 3) = 0.5665$   
d)  $P(4 \leq X \leq 6) = 0.4559$  (8)  
32. a)  $P(X = 2) = 0.0107$   
b)  $P(X > 3) = 0.9576$   
c)  $P(4 \leq X \leq 7) = 0.4106$   
d)  $P(X \geq 6) = 0.8088$   
33. a)  $P(X = 0) = 0.0334$   
b)  $P(X > 4) = 0.2558$   
c)  $P(2 \leq X \leq 5) = 0.7237$   
34. a)  $P(X = 3) = 0.2090$   
b)  $P(X = 0) = 0.3012$   
c)  $P(X \leq 5) = 0.6510$  (09)

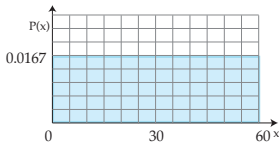
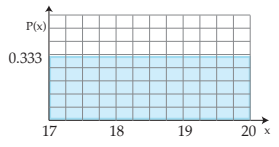
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35. a)  $\lambda = 1.609$   
b) 2 ways of 0 with 2 gives = 0.1036  
36. a)  $\lambda = 2.4$  (1 dp)  
b)  $0.2613^5 = 0.0012$   
37. a) Poisson  $\lambda = 2.4$  (rejt. / shift)  
 $P(X \leq 1) = 0.3084$   
 $P(X \leq 1)^2 = 0.0951$   
b) Constant rate of rejections.  
Rejections occur randomly.  
Rejections are independent.  
No rejections occur simultaneously.  
c) It may not be reasonable to assume the rate of car rejections is constant as there is likely to be differences between early and late shifts, also the start and end of a shift.  
The Poisson distribution has a mean of 2.4 rejections and the observation mean is similar  
 $\text{mean} = \sum x.p = 2.44$   
but the spread appears different. The Poisson S.D. =  $\sqrt{2.4} = 1.55$   
while the observations appears more spread out with a  
 $\text{S.D.} = \sqrt{\sum x^2 p - \mu^2} = 1.8$ .  
The Poisson distribution has a peak close to its mean and then steadily declines from this peak.  
The observation is bimodal with peaks at 1 and 4.  
The Poisson would predict the probability of four rejections as 0.1254 while the observation has a probability of 0.18.  
Therefore the conclusion is the observations are poorly modelled by a Poisson distribution.

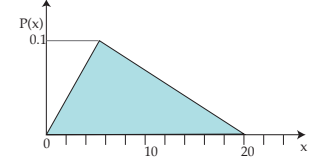
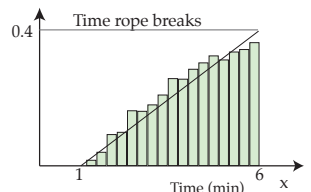
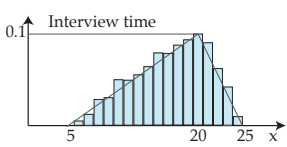
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38. a)   
b)  $P(0 < X < 2) = 0.1$   
c)  $P(5 < X < 18) = 0.65$   
d)  $P(6.5 < X < 7.5) = 0.05$

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39. a)   
b)  $P(X < 5) = 0.0833$   
c)  $P(X > 55) = 0.0833$   
d)  $P(X < 45) = 0.75$   
40. a)  $P(\text{av}) = 0.125$   
b)  $P(11 < X < 12) = 0.125$   
c)  $P(X = 11) = 0$   
Continuous distribution  
d)  $P(X > 11) = 0.875$   
41. a)   
b)  $P(1800 < X < 1815) = 0.0833$  (4 dp)  
c) Yes. Expect 6.67 of 20 but it is random so will vary.

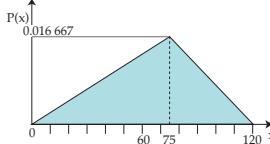
Page 144 (Rounded to 4 dp)

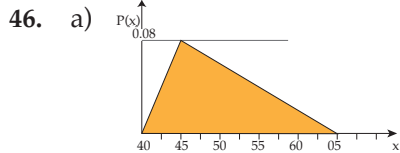
42. a)   
b)  $P(X < 5) = 0.5 \times 5 \times 0.1 = 0.25$   
c)  $f(2) = 0.04$   
 $P(0 < X < 2) = 0.5 \times 2 \times 0.04 = 0.04$   
d)  $A = 0.5 \times 13 \times (0.1 + 0.01333) = 0.7367$   
 $P(5 < X < 18) = 0.7367$   
43. a)   
b)  $A = 0.5 \times 1 \times (0.08 + 0.16) = 0.12$   
 $P(2 < X < 3) = 0.12$   
c)  $A = 0.5 \times 0.25 \times (0.4 + 0.38) = 0.0975$   
 $P(5.75 < X < 6) = 0.0975$   
44. a)   
b)  $P(X > 20) = 0.5 \times 5 \times 0.1 = 0.25$   
c)  $P(X < 10) = 0.5 \times 5 \times 0.03333 = 0.0833$

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44. d)  $P(10 < X < 20) = 0.6667$   
 $P(20 < X < 22) = 0.16$   
 $P(10 < X < 22) = 0.8267$

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45. a) 
- b) Area triangle = 0.025  
 $P(0 < X < 15) = 0.025$
- c) Area of trap. = 0.225  
 Area of triangle = 0.375  
 $P(60 < X < 120) = 0.6$
- d) Area of trap. 1 = 0.225  
 Area of trap. 2 = 0.2083  
 $P(60 < X < 90) = 0.4333$
- e) Probability of arriving after 6:15 is 0.375. Should be 0.5 for median.
- f) Find triangle with area 0.5  
 $0.5 \times T \times 0.000222T = 0.5$   
 $T = 67$  so 6:07 pm



- b)  $P(X > 45) = 0.5 \times 20 \times 0.08 = 0.8$
- c)  $P(X < 60) = 1 - P(X > 60) = 0.95$
- d) Probability of arriving after 9:45 is 0.8. Should be 0.5 for median.
- e) Let T = time to 10:05  
 Find triangle with area 0.5  
 $0.5 \times T \times 0.004T = 0.5$   
 $T = 15.8$  minutes  
 Median = 9:49.2

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47. a)  $P(0 < Z < 1.452) = 0.4268$  (7)  
 b)  $P(Z > 1.452) = 0.0732$  (3)  
 c)  $P(Z < 1.452) = 0.9268$  (7)  
 d)  $P(-1.452 < Z < 1.452) = 0.8536$  (5)

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48. a)  $P(Z < -0.973) = 0.1652$  (3)  
 b)  $P(-0.895 < Z < 1.059) = 0.3147 + 0.3552 = 0.6699$   
 c)  $P(0.652 < Z < 2.074) = 0.4810 - 0.2428 = 0.2382$   
 d)  $P(-1.953 < Z < -1.049) = 0.1217$

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49. a)  $P(X > 108) = 0.0548$   
 b)  $P(X < 95) = 0.1587$   
 c)  $P(98 < X < 107) = 0.5746$   
 d) 159
50. a)  $P(X > 4250) = 0.0345$   
 b)  $P(2600 < X < 4000) = 0.7950$  (1)  
 c) Number outside range = 205

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51. a)  $P(X < 24) = 0.0913$  (2)  
 b)  $P(X > 20) = 0.9962$   
 c)  $P(25 < X < 30) = 0.5890$  (89)  
 d) 61 days
52. a)  $P(X > 30) = 0.0159$   
 b)  $P(18.5 < X < 25) = 0.6423$  (1)  
 c)  $P(X < 17.5 \text{ or } X > 30) = 0.0790$  (1)  
 d)  $0.07898 \times 650 = 51$  or 52 boys

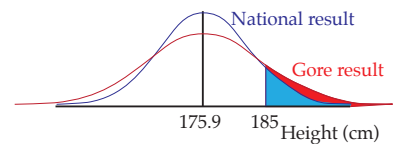
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53. a)  $P(X > 5) = 0.1587$   
 $[P(X > 5)]^3 = 0.0040$   
 b)  $P(X < 3) = 0.0668$   
 $[P(X < 3)]^3 = 0.0003$
54. a)  $P(X < 3) = 0.14199$   
 $P(Y < 3) = 0.30854$   
 Both = 0.0438  
 b)  $P(X < 2) = 0.03707$   
 $P(Y < 2) = 0.04006$   
 Either = 0.0756  
 c)  $P(X > 5) = 0.3605$   
 $P(Y < 2) = 0.0400$   
 $P(X > 5) \text{ and } P(Y < 2) = 0.0144$

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55. a)  $P(X < 0.1667) = 1.6 \times 10^{-6}$  (any answer that rounds to 4 dp).  
 Only one call so it does not call into question the parameters.  
 b)  $P(\text{three} > 15) = (0.2602)^3 = 0.0176$   
 c) Time to answer each call is independent of any other call and the parameters are constant throughout the day. This is unlikely as they will change at times of high demand.

56. a)  $P(X > 185) = 0.2222$   
 b)  $P(X < 165) = 0.1798$   
 $P(\text{three} < 165) = 0.0058$   
 c) Not reasonable as each of the friends may feel more comfortable with someone close to their own height therefore not random.  
 d) Standard deviation of 17 yo male heights in Gore must be larger (16.1) as it has more results further from the mean.



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57. 73.3 m  
 58. Distinction = 63 or better  
 Merit = 55 to 62  
 59. Lower = 117  
 Upper = 131

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60. a) Mean = 17.4 mm  
 b) Reject > 20.5 mm  
 61. a) Std. Dev. = 18.5 kg  
 b)  $P(X > 70.0) = 0.1587$

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62. a)  $P(X > 65.5) = 0.0846$   
 b)  $P(X < 57.5) = 0.2660$   
 c)  $P(57.5 < X < 61.5) = 0.3802$   
 d)  $0.0846 \times 500 = 42.3 = 42$  or 43 sacks  
 63. a)  $P(X < 1.45) = 0.1908$   
 b)  $P(X < 1.45)^2 = 0.0364$   
 c)  $P(X < 1.55)^4 = 0.1743$

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64. a)  $P(X > 5.025) = 0.3951$   
 b)  $P(X > 4.975) = 0.4250$   
 c)  $P(4.975 < X < 5.025) = 0.0299$   
 d) Lower quartile = 4.4135 kg  
 So salmon recorded as 4.40 kg and lighter.  
 65. a)  $P(X > 30.5) = 0.0401$   
 b)  $P(X > 28.5) = 0.0861$   
 2.14 times or just over twice as likely.

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- 66. a)  $P(X < 37.5) = 0.2647$
- b)  $P(X > 42.5) = 0.4296$
- c) Top 15% = 47.8 g  
so recorded as 50 g
- d)  $P(42.5 < X < 47.5) = 0.2670$   
Number 32 mice expected.
- 67. a)  $P(54.5 < X < 55.5) = 0.0498$
- b)  $P(X < 37.5) = 0.0143$  (4)
- c)  $P(X > 64.5) = 0.1175$  (5)  
Only 11.75% achieving 65 lengths.

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- 68. a)  $P(X = 1) = 0.2707$
- b)  $P(X = 1 \text{ and } X = 1) = 0.0733$
- c)  $P(X = 0 \text{ and } X = 2) = 0.1353 \times 0.2707 = 0.0366$
- 69. a)  $P(X = 10) = 0.1969$
- b)  $P(X = 8) = 0.2759$   
 $P(X = 8 \text{ and } X = 10) = 0.2759 \times 0.1969 = 0.0543$
- 70.  $P(\text{Late once}) = P(\text{Late}) \times P(\neq \text{late}) + P(\neq \text{late}) \times P(\text{Late}) = 0.04446$
- 71.  $P(X \geq 3) = 0.7619$   
 $P(X \geq 3 \text{ and } X \geq 3) = 0.7619^2 = 0.5805$

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- 72. a)  $P(X < 6)^2 = 0.0112$
- b)  $n = 5, \pi = 0.1056$   
 $P(X = 2) = 0.0798$
- 73. a)  $P(X = 0) = 0.0907$
- b)  $n = 8, \pi = 0.3012$   
 $P(X = 5) = 0.0474$
- c)  $n = 8, \pi = 0.3012$   
 $P(X \geq 5) = 0.0589$

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- 74.  $P(X \geq 1) = 1 - P(X = 0) = 0.2592$   
 $P(\text{three successive days}) = 0.2592^3 = 0.0174$
- 75.  $n = 5, \pi = 0.1649$   
 $P(X \geq 3) = 0.0345$

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- 76. a) Binomial.  
Failure<sup>5</sup> = 0.30  
Failure = 0.7860  
Success = 0.2140
- b)  $P(X = 2) = 0.2224$   
So consistent with 22 results of two 'sixes'.  
Biased as expect success to be 0.1667 not 0.2140.
- c) Mean = 1.07 sixes / throw
- d)  $P(X = 5) = 0.000448$   
Expect to throw 2232 times.
- 77. a) Unlikely to be a symmetrical distribution around the mean. Some amounts will be very common.
- b)  $P(\text{Money} > 30) = 0.22$   
 $\sigma = \$7.12$
- c) Binomial,  $\pi = 0.22, n = 4$   
 $P(X \geq 2) = 0.2122$
- d) The income of the four friends is unlikely to be independent of each other.
- 78. a) Poisson as rate proportional to interval (area), independent, distribution must be random and no simultaneous results.

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- 78. b)  $\lambda = 1.77$  worms / m<sup>2</sup>  
Pop. = 160 000 worms.
- c) Reality. Worms will depend upon the quality of soil so distribution unlikely to be random. Experiment. Potassium permanganate may not force up all worms. Selection of the 100 square plots may not be random. Other answers possible.

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- 79. a) Binomial. Independent, Two outcomes, fixed probability and probability must be constant. After 2500 spins the reality is likely to be reflected in the results.  $\pi = 0.1008$ .
- b)  $P(X = 2) = 0.0274$
- c) Reel 1  $p = 0.096$  so expect cherries = 7  
Reel 2  $p = 0.108$  so expect cherries = 8  
Reel 3  $p = 0.0984$  so expect cherries = 7
- 80. a) Poisson as rate proportional to interval, may be independent, distribution random and no simultaneous results.
- b) Mean = 1.7 texts / min. Variance = 1.8. Similar so Poisson confirmed.
- c)  $\lambda_{5\text{-minute}} = 8$  or 9 texts
- d) Binomial trials = 5,  $\pi = 0.2$   
 $P(X = 2) = 0.2048$

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Practice External Assessment – Probability Distributions

Q	Evidence	Achievement	Merit	Excellence
1 a)	$P(\text{Light} > 4) = 0.0459$ $P(\text{Regular} < 4) = 0.0408$ Approximately the same so no misquoting.	2 probabilities plus conclusion.		
b) i)	$P(\text{Light} > 3) = 0.3004$ $P(\text{Regular} > 10) = 0.1743$ Probability both = 0.0524	Correct approach with one error.	All probabilities correct.	
ii)	Binomial $P(X = 3) = 0.0657$ , So 6.5% you would get 3 out of 20 by <b>chance alone</b> so on this <b>one sample</b> there is not enough evidence to suspect the distribution.		Binomial probability correct.	Correct conclusion.
c)	1 kg is 0.91536 standard deviation above the mean. One standard deviation = 0.180 kg.	Partial calculation.	Correct answer.	

Q	Evidence	Achievement	Merit	Excellence
2 a) i)	$P(X > 2) = 0.2390$	Correct solution.		
ii)	The Binomial distribution requires the results to be independent. If the delivery van was in an accident or something occurred the independence of damage to each item would likely not hold.	Correctly identifies independence with explanation.		
iii)	Binomial probability. Insurance income = \$19.50 / van load. Only pay out if $\geq 3$ items damaged. Payout = $0.1546 \times \$50 + 0.0611 \times \$100 + 0.0181 \times \$150 + 0.0042 \times \$200 + 0.0001 \times \$250 = \$17.42$ Profit or return = $\$19.50 - \$17.42 = \$2.08$ per van load.		Correct return.	
b) i)	Mean = 1.7 and Variance = 1.8 Poisson as random, no simultaneous solution, mean approximately equals variance and results are likely to be independent.		$\lambda$ (mean) correct.	Correct justification.
ii)	$\lambda = 5.1$ . $P(X \geq 4) = 0.7488$	$\lambda$ (mean) correct.	Correct solution.	
3 a)	Triangular distribution. Mode 8, Height = 0.1667 $P(X > 7) = 1 - P(X < 7)$ $P(X > 7) = 0.8125$	Correct solution.		
b)	i) First office $P(X > 12) = 0.167$ so second office slower. ii) Mode = 9.33 minutes.	i) Correct.	i) Correct solution and conclusion.	i) Correct and conclusion plus mode.
c)	Assuming a binomial distribution with $\pi = 0.5$ and $n = 18$ probability of getting a result from 0 to 6 answers is 0.119. Therefore if this occurs 12% of the time by chance alone we cannot conclude it was not chance. Also only one trial.		Some evidence but small errors.	Correct conclusion with evidence.
d) i)	$\lambda = 4.5 / 15$ min., $P(X > 4) = 0.4679$	Correct solution.		
ii)	$P(X = 0) = 0.16$ implies $\lambda = 1.832 / 5$ min. $\lambda = 5.5 / 15$ min., $P(X > 4) = 0.6425$	A correct $\lambda$ .	Correct solution.	

### Practice Assessment – Probability Distributions

In the external examinations NZQA uses a different approach to marking based on understanding (u), relational thinking (r) and abstract thinking (t). They then allocate marks to these concepts and add them up to decide upon the overall grade. This approach is not as easy for students to self mark as the NuLake approach, but the results should be broadly similar.

**Sufficiency.** For each question award yourself a score out of 8 using this table. Add the three scores for a score out of 24 and compare to the cut scores. All answers must include evidence / justification where appropriate.

Quest.	N0	N1	N2	A3	A4	M5	M6	E7	E8
ONE	Nil correct	Part correct	1 A	2A or equiv.	3A or equiv.	1M + 1M minor error	2M	1E Minor error	1E all correct
TWO	Nil correct	Part correct	1 A	2A or equiv.	3A or equiv.	1M + 1M minor error	2M	1E Minor error	1E all correct
THREE	Nil correct	Part correct	1 A	2A or equiv.	3A or equiv.	1M + 1M minor error	2M	1E	2E all Minor error
Cut Scores									
Not Achieved		Achievement		Achievement with Merit		Achievement with Excellence			
0 – 6		7 – 13		14 – 20		21 – 24			