Answers

Page 4

Equivalent answers are acceptable.

- 1. C = No. children C - 5 = 68
- 2. B = No. boys & G = No. girls B = 0.7G
- 3. Q = Quantity of petrol2.15Q = 300
- 4. A = Net cost of alteration 0.15A = 130
- 5. R = Rate of pay8.5R = M
- 6. B = No. business seats 4B + B = 420
- 7. F = Fuel used $F = \frac{16}{100}D$
- 8. S = average speed $S = \frac{2D}{5.5}$

9. C = cost price and S = selling priceS = 1.5C

10. S = No. sacks $S = \frac{1000T}{20}$

Page 6

Equivalent answers are acceptable.

- 11. S = side length $S^2 = 20$
- **12.** H = Height2(H + 3H) = 144
- **13.** B = base angle2B + 112 = 180
- 14. L = side length4L = P
- **15.** A =smaller angle A + 3A = 180
- **16.** A = Side area A = $(\sqrt[3]{V})^2 = V^{2/3}$
- 17. N = number sides $N \times k = 360$
- **18.** H = Height of side $H^2 + (5 + H)^2 = D^2$

Page 8

Equivalent answers are acceptable.		51.	T
19.	Let A and B be two numbers		1
	A + B = 32		2
	AB = 175		S

Page 8 cont...

- **20.** G = No. girls, B = No. boysG + B = 180B = G - 30
- 21. Y = No. Yes, N = No. No $Y + N = 0.8 \times 160$ (Y + N = 128)Y = 2N
- 22. A = Angle 1, B = Angle 2A = 31 + BA + B + 89 = 180
- 23. H = Cost hamburger and T = Cost of thick shake 2H + T = 152H = 9 + T
- 24. A = Cost of adult and T = Cost of child 2A + 3C = 86A + 2C = 50
- 25. C = No. correct and W = No. wrong 5C - 2W = 58C + W = 20
- **26.** A = Age Anna and T = Age of mum M - 22 = A0.5(M + 6) = (A + 6)

Page 9

27. Let A = Anthea's age and B = Bart's age A - 18 = B + 18 so A - B = 36 A + 25 = 3(B - 25) so A - 3B = -100
28. Let X = money invested at 5% and Y = money invested at 5%

- and Y = money invested at 3%X + Y = 50 000 0.05X + 0.07Y = 2800
- 29. Let S = small jug and L = large jug
 2L + 2S = 19
 L S = 2.5
- 30. Let T = true / false questionsand M = multichoice questionsT + M = 302T + 6M = 132

31. Let S = son's age and F = father's age2S + F = 97S + F = 73 Page 9 cont...

32. Let X = the ones digit and Y the tens digit X = 3Yso X - 3Y = 0 10X + Y = 10Y + X + 54so 9X - 9Y = 54

Page 13





34. x = 4 and y = 6



35. A = 6 and B = -2



Page 13 cont...





Page 14







Lines are parallel and will not 39. intersect so there is no solution



Page 14 cont...

40. Lines are congruent (on top of each other or the same line) so there are infinite solutions.



Page 15 **41.** 2y - x = 3y = 3x - 6Pt of int. = (3, 3)42. y = x + 33y + x = -1Pt of int.= (-2.5, 0.5)43. x + y = 2x = 2yPt of int.= $\left(\frac{4}{3}, \frac{2}{3}\right)$ [(1.333, 0.667)] **44.** 3x - 2y = 104x + y = 6Pt of int. = (2, -2)Page 16 **45.** A + B = 8A - B = 2Pt of int. A = 5, B = 3**46.** 2P + Q = 4P - Q = -1Pt of int. P = 1, Q = 247. 2M + N = 4M + 3N = 7Pt of int. M = 1, N = 2**48**. 2T - S = 1T + 2S = -7Pt of int. S = -3, T = -1Page 18 **49.** x = 6, y = 2

Page 19

- W = -3, Z = 456.
- a = 1, b = -0.557.
- c = 1, d = -358.
- 59. You get an algebraic statement that is always true such as 6 = 6 which means it is true for all e and f. The two equations represent the same line.
- 60. You get an algebraic statement that is never true such as -6 = 5. The two equations never intersect, they are parallel lines.

Page 22

61.
$$A - 4B = 1$$

 $^{-}A + 2B = ^{-}2$
 $A = 3, B = \frac{1}{2}(0.5)$
62. $3C + D = 2$
 $^{-}6C + D = ^{-}10$
 $C = \frac{4}{3}(1.333), D = ^{-}2$
63. $2E - F = 1$
 $^{-}4E + 5F = ^{-}5$
 $E = 0, F = ^{-}1$

64.
$$2G + 2H = 1$$

 $4G - 8H = 11$
 $G = \frac{5}{4}(1.25), H = \frac{-3}{4}(-0.75)$

65.
$$J - 4K = {}^{-}5$$

 ${}^{-}J + K = 5$
 $J = {}^{-}5, K = 0$

66.
$$5L + 2M = 0$$

 $^{-1}0L + 6M = ^{-5}$
 $L = \frac{1}{5}(0.2), M = \frac{^{-1}}{2}(^{-0.5})$

67.
$$0.4N + P = 0.8$$

 $N + 0.5P = ^{-2}$
 $N = ^{-3}, P = 2$
68. $5Q + R = 9$
 $^{-1}0Q + R = 6$
 $R = \frac{1}{5}(0.2), Q = 8$

69.
$$2T + 3S = 2$$

 $6T - 6S = 1$
 $T = \frac{1}{2}(0.5), S = \frac{1}{3} (0.333)$
70. $5X + Y = 0$
 $^{-6}X + Y = 0$
 $X = 0, Y = 0$

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50. G = 4, H = -5

51. J = -3, K = 5

53.

52. M = 0.5, N = -1.5

P = -5, Q = -1254. R = -3, S = -2

IAS 2.14 - Systems of Equations

Page 24

- 71. Let B = Budget and S = Super B + S = 72 6B + 10S = 556 B = 41, S = 31 so Number of drivers that select a budget wash is 41.
- 72. Let C = chip cost and P = popcorn cost. 3C + 7P = 186C + 2P = 18C = 2.5, P = 1.5 so Cost chips = \$2.50 and cost popcorn = \$1.50.
- 73. Let J = Jack's age and F = father's age. F = J + 32 J + F = 60 J = 14, F = 46 so Jack was born 14 years ago.
- 74. Let S = smaller angle and L = larger angle S + L = 180 L = 2S S = 60, L = 120 so larger angle is 120°.
- 75. Let D = Diana's votes and B = Brett's votes D + B = 125 D = B + 17 D = 71, B = 54 so Brett got 54 votes.
- 76. Let D = Donna's fare and M = mother's fare D + M = 430 D = M - 40 D = 195, B = 235 so Donna's fare cost \$195.

Page 25

- 77. Let F = first property and S = second property F + S = 640 000 $0.8F + 1.2S = 610\ 000$ F = 395 000, S = 245 000 so first property cost \$395 000.
- 78. Let F = flag fall and K = cost per km F + 25K = 58 F + 18K = 42.6 F = 3, K = 2.2 so flag fall is \$3.

Page 25 cont...

- 79. Let K = time on the bike and B = time on the bus K + B = 0.5 19K + 54B = 20 K = 0.2, B = 0.3 so time on the bus is 18 minutes (0.3 hours).
- 80. Let P = per person cost and C = cost to hire minibus 12P = C12(P + 3) + 42 = 2CP = 6.5, C = 78 so cost of the van is \$78.
- 81. Let n = normal rate and v = overtime rate 8n + 4v = 244 5n + 7v = 265 n = 18, v = 25 so normal rate \$18 and overtime rate \$25.
- 82. Let W = time to run width and L = time to run length 6L + 6W = 42 10L = 36 W = 3.4, L = 3.6 so width is 680 metres.

Page 27

83. x = 0, y = 0 and x = 5, y = -5.



84. x = 0, y = -4 and x = 4, y = 4.



Page 27 cont...





86. A = 6, B = 1 and A = -3, B = -2.



Page 28



88. F = -4, E = 1 and F = 6, E = -4.





01. $x^2 + (3x - 30)^2 = 100$ x = 8, y = -6x = 10, y = 0 $x^2 + (3x - 5)^2 = 25$ 02. x = 0, y = -5x = 3, y = 4 $^{-}2 + 2x = x^2 - 2x + 2$ 03. $x^2 - 4x + 4 = 0$ x = 2, y = 2 $2x + 5 = x^2 - 6x + 5$ 04. $x^2 - 8x = 0$ x = 0, x = 8x = 0, y = 5x = 8, y = 2105. 9 $4x^2 - 9x - 9 = 0$ x = 3, x = -0.75x = 3, y = 0.3333x = ⁻0.75, y = ⁻1.3333 $\frac{2x-8}{-}=x^2-2x+1$ 06. 5 $5x^2 - 12x + 13 = 0$ No solution, line and parabola do not intersect. age 35 07. (2)y = 2xSubstitute into (1). 2x + 3 = 2(x - 1)2x + 3 = 2x - 23 = -2 As this cannot be true there are no solutions. 3y + 9 = x + 108. (1)but 3y = x + 2(2)Substitute into (1) x + 2 + 9 = x + 111 = 1As this cannot be true there are no solutions. Page 36 09. y = 10 - 4x(1)Substitute into (2) 10 - 4x + 4x + 3 = 1313 = 13As this is always true there are infinite solutions. 110. 3y - 6 = x + 1(1)

x = 3y - 7 (1) Substitute into (2) 3y - (3y - 7) - 7 = 00 = 0As this is always true there are infinite solutions.

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Page 36 cont...

111.
$$y + 3 = \frac{-3}{4}(x - J)$$
 (1)
 $4y + 3x = 0$ (2)
 $y = \frac{-3}{4}(x - J) - 3$ (1)
 $4\left(\frac{-3}{4}(x - J) - 3\right) + 3x = 0$ (2)
 $3J = 12.$

If $J \neq 4$ this is an untrue statement and there will be no solutions.

112.
$$3y = -2x + k$$
 (1)
 $8 - 2 \times 3y - 4x = 0$ (2)
 $8 - 2 \times (-2x + k) - 4x = 0$ (2)
 $2k = 8.$
If $k = 4$ this is a true statement

If k = 4 this is a true statement and there will be infinite solutions and if $k \neq 4$ there are no solutions so there is no unique solution.

Page 37

113.
$$3x - 2y = 5$$
 (1)
 $6x + ky = 4$ (2)
 $3x = 5 + 2y$ (1)
 $2(5 + 2y) + ky = 4$ (2)
 $10 + 4y + ky = 4$
 $(k + 4)y = ^{-6}$
If $k = ^{-4}$, then $0 = ^{-6}$ and the
two lines would be parallel
and there would be no
solution.
114. $2x - y = 2$ (1)
 $5x + ky = 5$ (2)
 $2x = 2 + y$ (1)
 $2.5(2 + y) + ky = 5$ (2)
 $5 + 2.5y + ky = 5$
 $(k + 2.5)y = 0$
If $k = ^{-2}.5$, then $0 = 0$ and the
two lines are the same and
there would be an infinite
number of solutions.
115. $3x + ky = 3$ (1)
 $3x - 4y = 5$ (2)
 $3x = 3 - ky$ (1)
 $(3 - ky) - 4y = 5$ (2)
 $(^{-1}k - 4)y = 2$
If $k = ^{-4}$, then $0 = 2$ and the two

lines would be parallel and there would be no solution. Page 37 cont...

116.
$$^{-}4x + ky = 2$$

 $y - 2x - 1 = 0$
 $4x = ky - 2$
 $y - 0.5(ky - 2) - 1$

- 0.5(Ky v - 0.5kv + 1 - 1 = 0

$$(1 - 0.5k)y = 0$$

If k = 2, then 0 = 0 and the two lines are the same and there would be an infinite number of solutions.

Page 40

y = -3x - 10 (1) **Page 41** 117. 12 $x^{2} + (-3x - 10)^{2} = 10$ (2) $10x^2 + 60x + 90 = 0$ $x^2 + 6x + 9 = 0$ x = -3x = -3, y = -1Since there is only one solution the line must just touch the circle and therefore it must be a

tangent to the circle.
118.
$$y = 2x + 1$$
 (1)
 $xy = -8$ (2)
 $x(2x + 1) = -8$
 $2x^2 + x + 8 = 0$
 $\Delta = b^2 - 4ac$
 $\Delta = -63$
The discriminant Δ is negative
we cannot take its square root
to get the solution.
Since there is no solution to
this quadratic there is no point
of intersection.
119. $y = 2x + k$ (1)

$$x^2 + y^2 = 20$$
 (2)

Substitute (1) into (2) $20 = x^2 + (2x + k)^2$ $20 = x^2 + 4x^2 + 4kx + k^2$ $20 = 5x^2 + 4kx + k^2$ $\Delta = 0$ for tangent $0 = (4k)^2 - 4 \times 5 (k^2 - 20)$ $0 = -4k^2 + 400$ $k = \pm 10$ As the discriminant must be zero.

Page 40 cont...

(1)

(2)

(1)(2)

120.
$$y = 3x - 11$$
 (1)
 $10 = (x - 1)^2 + (y - 2)^2$ (2)
Substitute (1) into (2)
 $10 = 10x^2 - 80x + 170$
 $0 = x^2 - 8x + 16$
 $0 = (x - 4)^2$
 $x = 4$

Since there is only one solution the line must just touch the circle and therefore it must be a tangent to the circle.

1.
$$x = 2y - 8$$
 (1)
 $x^{2} + y^{2} = 9$ (2)
Substitute (1) into (2)
 $(2y - 8)^{2} + y^{2} = 9$
 $5y^{2} - 32y + 55 = 0$
 $\Delta = b^{2} - 4ac$

$$\Delta = -76$$

The discriminant Δ is negative therefore there are no solutions.

$$x = 10 - 2y \quad (1)$$
$$y = \frac{8}{x - 2} \quad (2)$$
Substitute (1) into (2)
$$y(10 - 2y - 2) = 8$$
$$^{-}2y^{2} + 8y - 8 = 0$$
$$(y - 2)^{2} = 0$$

Since there is only one repeated solution the line must be a tangent to the curve.

123.
$$y = \frac{10}{x-1}$$
 (1)

$$y = \frac{12 - x}{4} \tag{2}$$

Substitute (1) into (2)

$$\frac{10}{x-1} = \frac{12-x}{4}$$
$$40 = (12-x)(x-1)$$
$$x^2 - 13x + 52 = 0$$
$$\Delta = b^2 - 4ac$$
$$\Delta = -39$$

IAS 2.14 – Systems of Equations

Page 41 Q123 cont...

The discriminant Δ is negative, **127**. we cannot take its square root to get the solution. Since there is no solution to this quadratic there is no point of intersection.

124.
$$y = 3x + k$$
 (1)

 $x^2 + y^2 = 40$ (2)Substitute (1) into (2) $40 = (3x + k)^2 + x^2$ $0 = 10x^2 + 6kx + k^2 - 40$ Minimum value is when $\Delta = 0$

 $(6k)^2 - 4 \times 10(k^2 - 40) = 0$ $k = \pm 20.$ Largest value of k = 20. y = 3x + 20

Page 42

125. y = x + 1(1) $x^2 + (x + 1)^2 = 25$ (2) $2x^2 - 2x - 24 = 0$ $2(x^2 - x - 12) = 0$ x = -3 and 4

Solutions (4, ⁻3) and (⁻3, 4).

The line cuts the circle in two places hence there are two solutions. This can also be confirmed by calculating the discriminant which is greater than zero.

 $x^2 + y^2 = 13$ (2)Substitute (1) into (2) $13 = (k + 2y)^2 + y^2$ $13 = k^2 + 4ky + 4y^2 + y^2$ $0 = 5y^2 + 4ky + (k^2 - 13)$ $\Delta = 0$ for tangent $0 = (4k)^2 - 4 \times 5 (k^2 - 13)$

x = k + 2y

$$0 = -4k^2 + 260$$

$$k = \pm \sqrt{65}$$

Page 42 cont... y = 6x - 5(1) $y = 5x^2 - 14x + 15$ (2)Substitute (1) into (2) $6x - 5 = 5x^2 - 14x + 15$ $0 = 5x^2 - 20x + 20$ $0 = 5(x^2 - 4x + 4)$ $0 = (x - 2)^2$ So point of contact x = 2, y = 7As the discriminant of the quadratic $5x^2 - 20x + 20$ is 0 there can be only one point of contact hence the line y = 6x is tangent to the quadratic $y = 5x^2 - 14x + 15.$ y = 2x + k(1)128. $x^2 + y^2 = 9$ (2)Substitute (1) into (2) $9 = (2x + k)^2 + x^2$ $0 = 5x^2 + 4kx + k^2 - 9$

> Minimum value is when $\Delta = 0$ $(4k)^2 - 4 \times 5(k^2 - 9) = 0$ $k = \pm \sqrt{45}$. Smallest value of $k = \sqrt{45}$.

 $v = 2x - \sqrt{45}$

Page 43

129. T = 3.29 (hours) (3.3 off a graph) **130.** Teacher = 33, student = 8. Width = 63 m, length = 84 m. 131.

132. Width = 31 m, length = 68 m.

Page 44

(1)

- 133. Height = 35 cm, width = 20 cm
- **134.** When x = 2 and x = 10 they are both at same height.

135. $t^2 - 10t + 11 + k$ Discriminant < 0100 - 44 - 4k < 0k > 14 Pages 45 - 46

136. a) Intersection up the slope is
at position (20.7, 32.1). X
intercept of slope is (10, 0).
Distance up slope
$$D = \sqrt{(20.7 - 10)^2 + 32.1^2}$$
$$D = 33.8 \text{ m}$$
b) Intersections between
$$y = x + 18 \text{ and}$$
$$y = \frac{x(30 - x)}{6}$$
(6, 24) and (18, 36).
c) One shot laser must hit
bank below projectile
$$m = \frac{32.1 - 18}{20.7 - 0}$$
$$m = 0.681$$
d) To never hit the bank it
must be parallel to the bank
so m = 3.
e) Gradient to be a tangent
$$m = 1.536$$
So range for m is
0.681 < m < 1.536
Pages 47 - 48

137. a) Form the equation y = -0.2x + 250Intersection

- (100, 230)b) Intersections circle and the new line (50, 130) and (90, 210)
 - giving the distance over the swamp at 89.4 m.
- c) Find the equation of the intersection between the circle and y = -0.2x + 250And show the discriminant is negative (no intersection).

For **Achievement** a student must have correctly selected and applied systems of equations in solving problems.

Also they must have demonstrated knowledge of concepts and terms and communicated using appropriate representations.

For example:

The student has formed a correct pair of linear equations

3Y + 2X = 180 and

Y = 2X + 120

to represent the flight paths.

The student has found a solution to their system of linear equation of X = -22.5 and Y = 75

The student has interpreted their solution by calculating the distance to the intersection point at 37.5 km. Units required.

For **Achievement with Merit** a student must have demonstrated relational thinking in solving problems.

The student must have connected concepts or representations and selected and carried out a logical sequence of steps.

The student must have related their findings to the context or communicated their thinking using appropriate mathematical statements.

For example:

The student has formed a model for the situation involving the plane from Napier to NP. They have used appropriate methods to find the points of intersection of (-90, 120) and (145.38, -36.92) and hence determined the distance inside the circle as 282.9 km. They then have calculated the time to fly this distance as 80.8 minutes (1.35 hours).

For **Achievement with Excellence** a student must have applied systems of equations, using extended abstract thinking, in solving problems.

Also the student must have used correct mathematical statements or communicated mathematical insight.

The solution must involve a chain of logical reasoning and be explained using correct mathematical statements.

The student has formed a model for the situation involving the plane from PN to Tauranga and proven using the discriminant that the resulting equation has no solution.

Pages 52 – 54 Practice Internal Assessment Task 2 – Systems of Equations 2.14

For **Achievement** a student must have correctly selected and applied systems of equations in solving problems.

Also they must have demonstrated knowledge of concepts and terms and communicated using appropriate representations.

For example:

The student has formed a correct pair of linear equations

A + T = 2000 and

 $25A + 15T = 35\ 000$

The student has found a solution to their system of linear equations of A = 500 and T = 1500

The student has interpreted their solution by calculating the ratio of teenagers to adult as 3.

For **Achievement with Merit** a student must have demonstrated relational thinking in solving problems.

The student must have connected concepts or representations and selected and carried out a logical sequence of steps.

The student must have related their findings to the context or communicated their thinking using appropriate mathematical statements.

For example:

The student has formed a model for the situation involving the total number A + T = 2000 and the promoter's equation. They have found the points of intersection of

A = 1815, T = 185 and A = 155, T = 1845.

The first solution raises plenty of money \$48 150 but has few teenagers while the second has many teenagers but only raises \$31 550.

For **Achievement with Excellence** a student must have applied systems of equations, using extended abstract thinking, in solving problems.

Also the student must have used correct mathematical statements or communicated mathematical insight.

The solution must involve a chain of logical reasoning and be explained using correct mathematical statements.

The student has formed a model for the situation involving the promoter's equation and the total raised equals \$35 000 (25A + 15T = 35 000) and solved it. The best solution for this pair of equations has only 322 teenagers and 1206 adults as the other solution exceeds the capacity of the hall (2000 total). The best overall solution is the 155 adults and 1845 teenagers as it entertains the teenagers, fills the hall and is only \$3450 short of the target.