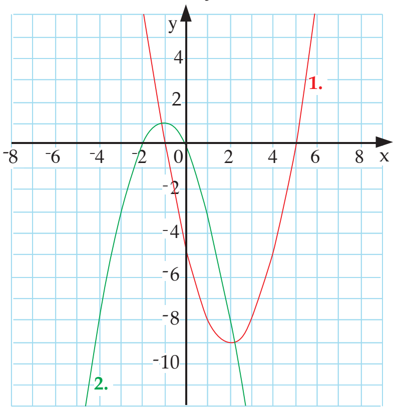
Answers

Page 6

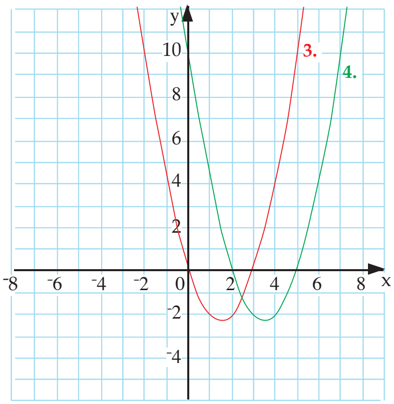
1. $x = {}^{-}1, 5$ and $y = {}^{-}5$

2. $x = {}^{-}2, 0$ and $y = 0$



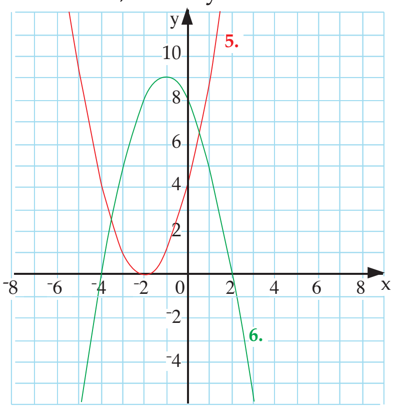
3. $x = 0, 3$ and $y = 0$

4. $x = 2, 5$ and $y = 10$



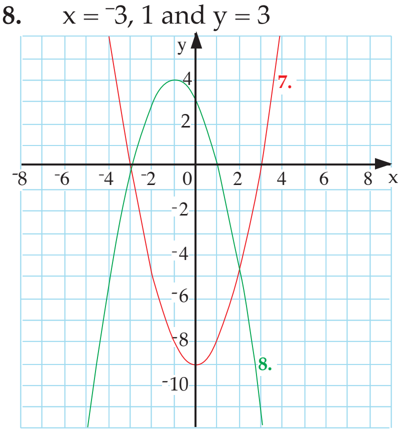
5. $x = {}^{-}2$ and $y = 4$

6. $x = {}^{-}4, 2$ and $y = 8$



7. $x = {}^{-}3, 3$ and $y = {}^{-}9$

8. $x = {}^{-}3, 1$ and $y = 3$



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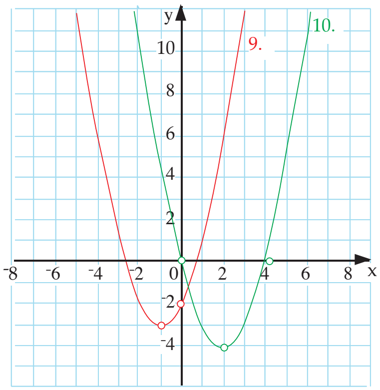
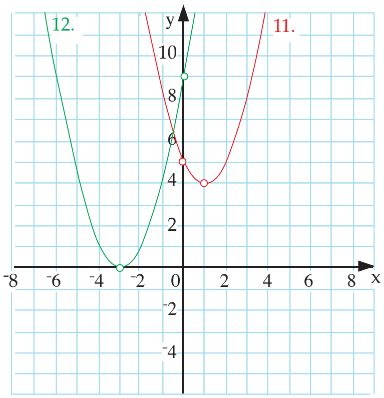
9. T.P. = $({}^{-}1, {}^{-}3)$ and $y = {}^{-}2$

10. T.P. = $(2, {}^{-}4)$ and $y = 0$



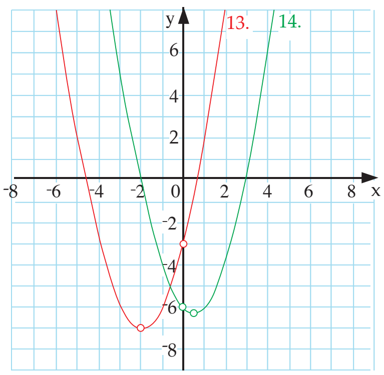
11. T.P. = $(1, 4)$ and $y = 5$

12. T.P. = $({}^{-}3, 0)$ and $y = 9$



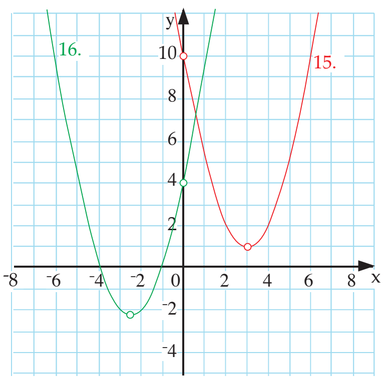
13. T.P. = $({}^{-}2, {}^{-}7)$ and $y = {}^{-}3$

14. T.P. = $(0.5, {}^{-}6.25)$ and $y = {}^{-}6$



15. T.P. = $(3, 1)$ and $y = 10$

16. T.P. = $({}^{-}2.5, {}^{-}2.25)$ and $y = 4$



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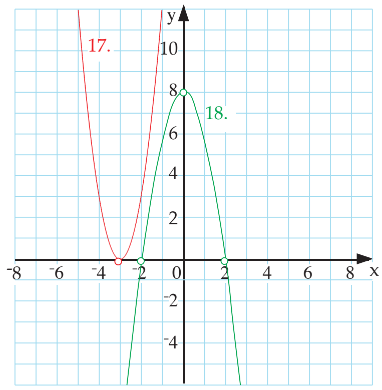
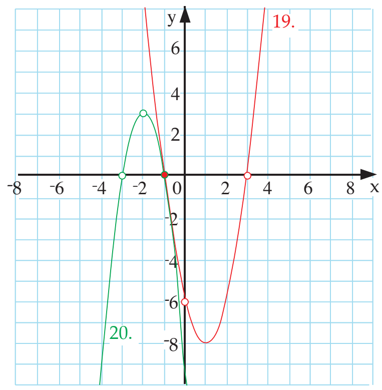
17. Turning point = $({}^{-}3, 0)$
y intercept – off scale $(0, 27)$
x intercept = $({}^{-}3, 0)$

18. Turning point = $(0, 8)$
y intercept = $(0, 8)$
x intercept = $({}^{-}2, 0), (2, 0)$



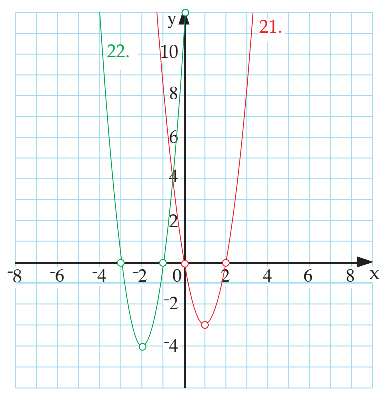
19. Turning point = $(1, {}^{-}8)$
y intercept = $(0, {}^{-}6)$
x intercept = $({}^{-}1, 0), (3, 0)$

20. Turning point = $({}^{-}2, 3)$
y intercept = $(0, {}^{-}9)$
x intercept = $({}^{-}3, 0), ({}^{-}1, 0)$



21. Turning point = $(1, {}^{-}3)$
y intercept = $(0, 0)$
x intercept = $(0, 0), (2, 0)$

22. Turning point = $({}^{-}2, {}^{-}4)$
y intercept – off scale $(0, 12)$
x intercept = $({}^{-}1, 0), ({}^{-}3, 0)$



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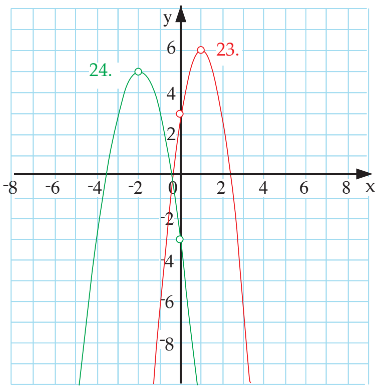
23. Turning point = (1, 6)
y intercept = (0, 3)
x intercept – not integers.

24. Turning point = ($^-2$, 5)
y intercept = (0, $^-3$)
x intercept – not integers.



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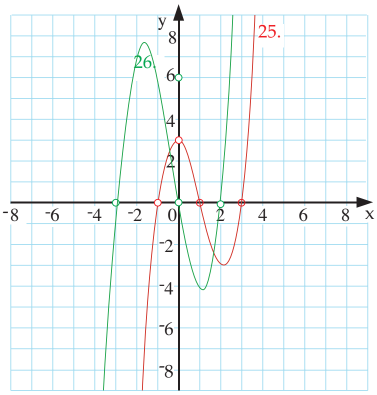
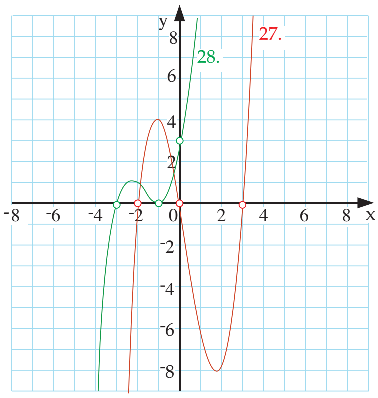
25. y intercept = (0, 3)
x intercept = ($^-1$, 0), (1, 0), (3, 0)

26. y intercept = (0, 0)
x intercept = ($^-3$, 0), (0, 0), (2, 0)



27. y intercept = (0, 0)
x intercept = ($^-2$, 0), (0, 0), (3, 0)

28. y intercept = (0, 3)
x intercept = ($^-3$, 0), ($^-1$, 0), ($^-1$, 0)



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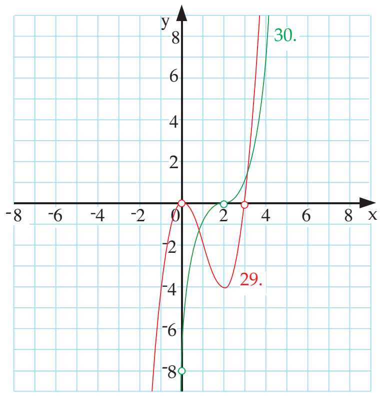
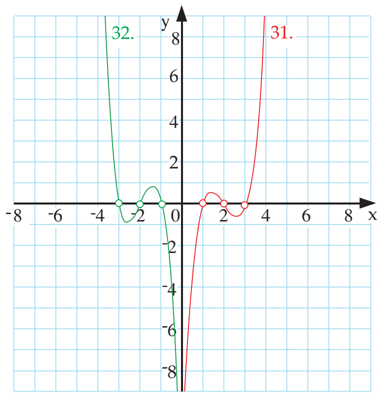
29. y intercept = (0, 0)
x intercept = (0, 0), (3, 0)

30. y intercept = (0, $^-8$)
x intercept = (2, 0)



31. y intercept = (0, $^-12$)
x intercept = (1, 0), (2, 0), (3, 0)

32. y intercept = (0, $^-18$)
x intercept = ($^-3$, 0), ($^-2$, 0), ($^-1$, 0)



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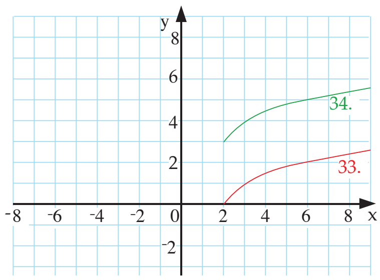
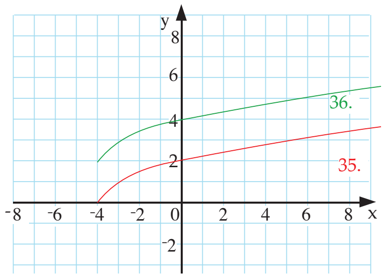
33. Translation over 2 up 0.

34. Translation over 2 up 3.



35. Translation over $^-4$ up 0.

36. Translation over $^-4$ up 2.



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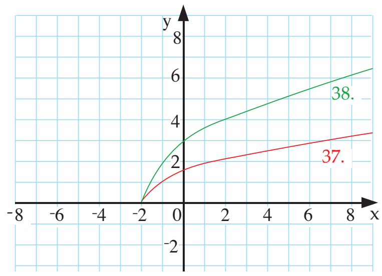
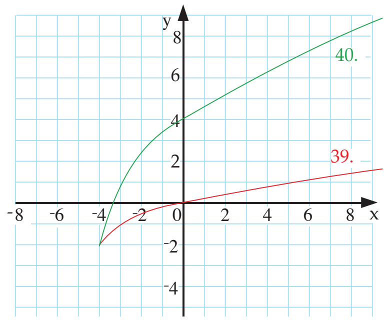
37. Translation over $^-2$ up 0.

38. Translation over $^-2$ up 0.
Stretch from x axes (y = 0) scale factor 2.



39. Translation over $^-4$ down 2.

40. Translation over $^-4$ down 2.
Stretch from y = $^-2$ scale factor 3.



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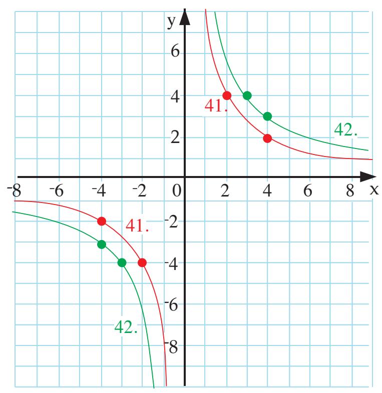
41.
The centre of the hyperbola is (0, 0), the vertical asymptote is x = 0 and the horizontal asymptote is y = 0.

42.
The centre of the hyperbola is (0, 0), the vertical asymptote is x = 0 and the horizontal asymptote is y = 0.



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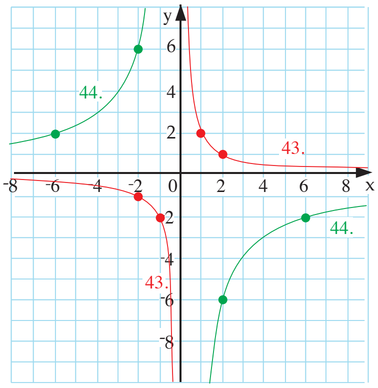
43. The centre of the positive hyperbola is (0, 0), the vertical asymptote is x = 0 and the horizontal asymptote is y = 0.

44. The centre of the negative hyperbola is (0, 0), the vertical asymptote is x = 0 and the horizontal asymptote is y = 0.



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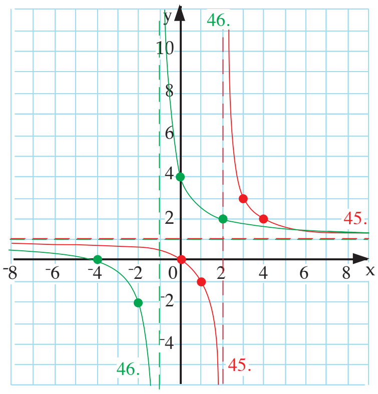
45. The hyperbola has half-turn symmetry about the centre (2, 1), the vertical asymptote is x = 2 and the horizontal asymptote is y = 1.

46. The hyperbola has half-turn symmetry about the centre ($^{-}1$, 1), the vertical asymptote is x = $^{-}1$ and the horizontal asymptote is y = 1.



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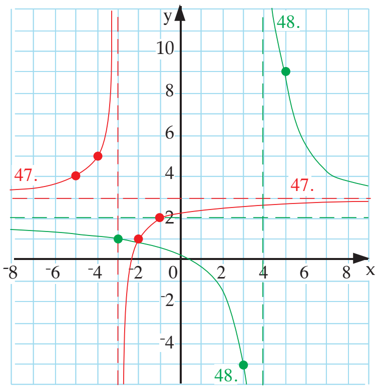
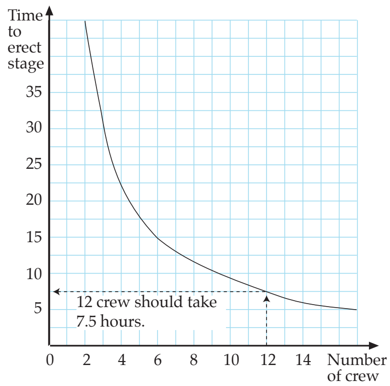
47. The hyperbola has half-turn symmetry about the point ($^{-}3$, 3), the vertical asymptote is x = $^{-}3$ and the horizontal asymptote is y = 3.

48. The hyperbola has half-turn symmetry about the point (4, 2), the vertical asymptote is x = 4 and the horizontal asymptote is y = 2.



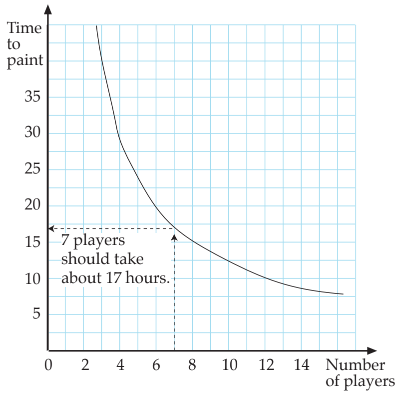
49. Equation is $\text{Time} = \dfrac{90}{\text{crew}}$

Time for 12 crew is 7.5 hours.



50. Eqn. is $\text{Time} = \dfrac{120}{\text{No. players}}$

Time to paint with 7 players is 17.1 hours (1 dp).



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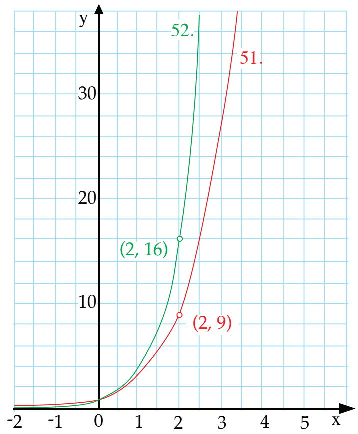
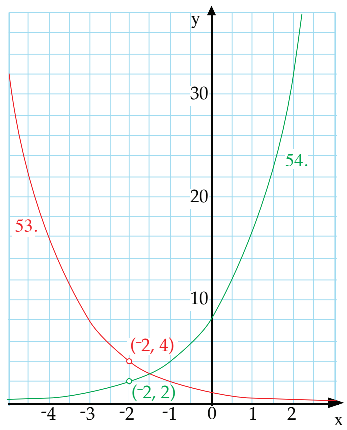
51. (2, 9)

52. (2, 16)



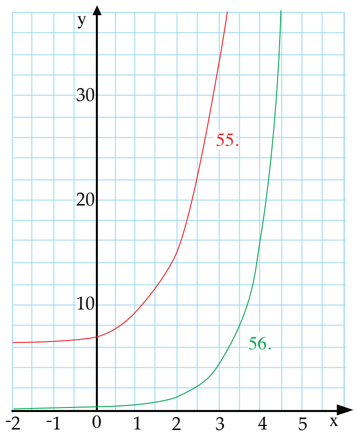
53. ($^{-}2$, 4)

54. ($^{-}2$, 2)



55. Translation of $y = 3^x$ across 0 up 6.

56. Translation of $y = 4^x$ across 2 up 0.



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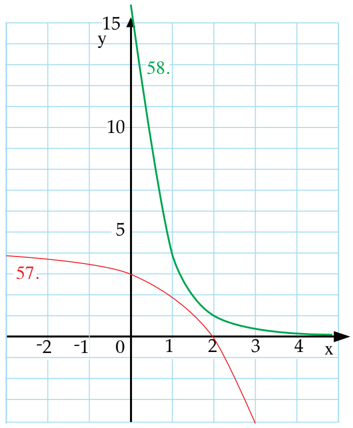
57. Reflection of $y = 2^x$ in the line $y = 2$.

58. Reflection of $y = 4^x$ in the line $x = 1$.



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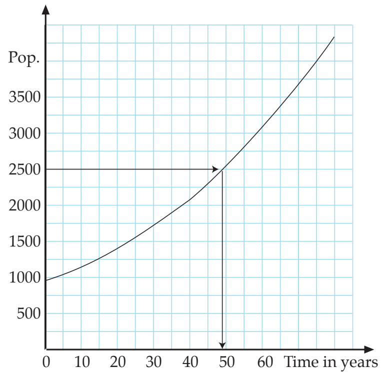
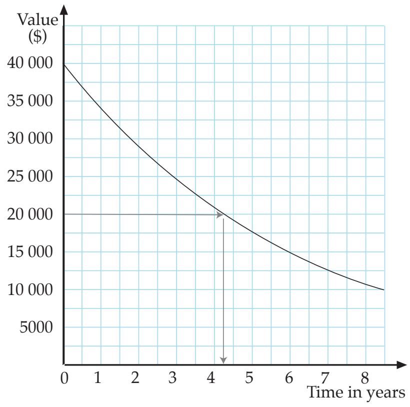
59. At about 49 years the population will reach 2500.



60. Approximately 4.3 years.



61. A = 350 (000) and R = 1.1

$V = 350 \times 1.1^t$

at t = 16, V = 1 608 (000)

Inflation of property prices will not stay steady for a long period of time. Depending upon the economic situation they could even become negative.

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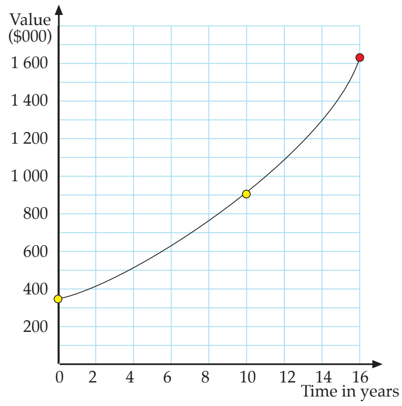

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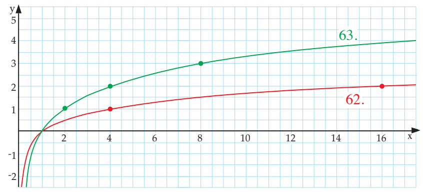
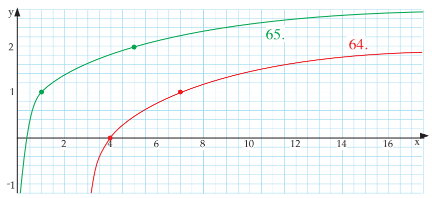
62. and 63.



64. Crosses the x axes at 4 and passes through the point (7, 1). Other features possible.

65. Crosses the x axes at 0.2 and passes through the point (1, 1). Other features possible.



66. $y = \log_7 (x)$

67. $y = \log_3 (x - 4)$

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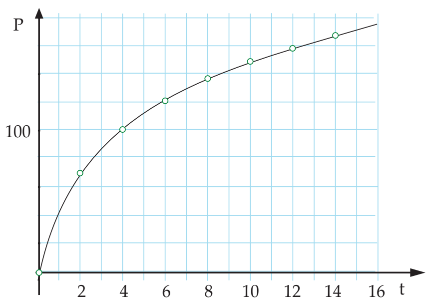
68. a)



b) $P = 100 \log_5 (t + 1)$

c) No horizontal asymptote.

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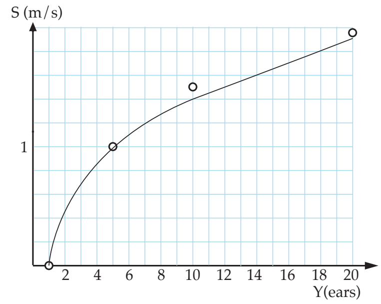
69. a)



b) $S = \log_5 Y$. B = 5 as (5, 1)

c) They both pass through (1, 0) and (5, 1) but at Y = 10 and Y = 20 the data points are a little higher.

d) The fit to the model is more likely to be chance as running records increase suddenly and it is unlikely to follow a pattern. A new record in Year 21 could be substantially faster.

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70. 1.070 (3 dp)

71. 0.785 (3 dp)

72. 3.547 (3 dp)

73. 6.283 (3 dp)

74. $\frac{\pi}{2}$

75. π

76. $\frac{\pi}{6}$

77. $\frac{\pi}{12}$

78. $\frac{\pi}{4}$

79. $\frac{7\pi}{6}$

80. $\frac{\pi}{3}$

81. $\frac{5\pi}{12}$

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82. 114.6° (1 dp)

83. 31.0° (1 dp)

84. 120.0° (1 dp)

85. ⁻71.0° (1 dp)

86. 180°

87. 45°

88. 90°

89. 60°

90. 30°

91. 225°

92. 240°

93. 270°

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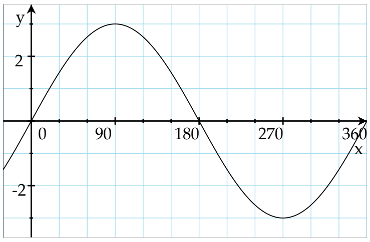
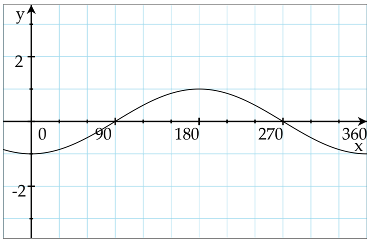
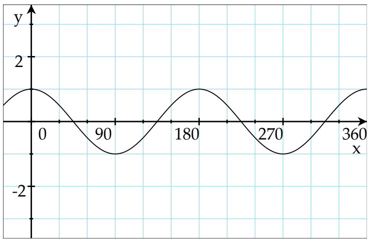
94.



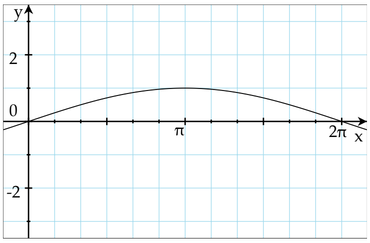
95.



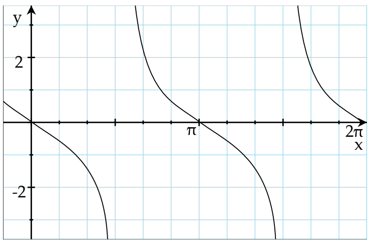
96.



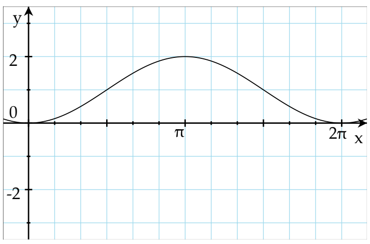
97.



98.



99.



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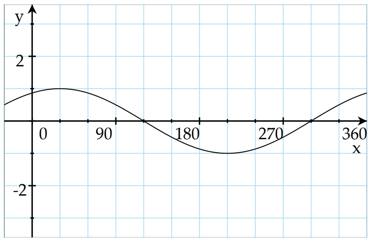
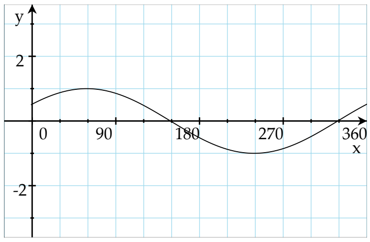
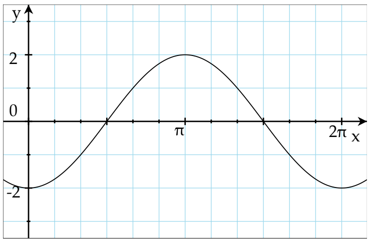
100. Translation of y = sin x across -60° up 0.



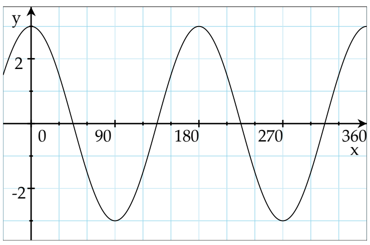
101. Translation of y = cos x across 60° up 0.



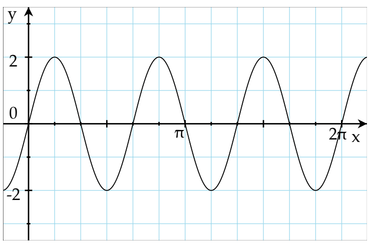
102. Translation of y = cos x across -π up 0, and a vertical enlargement scale factor 2.



103. For y = cos x, a vertical enlargement scale factor 3 and a horizontal enlargement scale factor $\frac{1}{2}$.



104. For y = sin x, a vertical enlargement scale factor 2 and a horizontal enlargement scale factor $\frac{1}{3}$.



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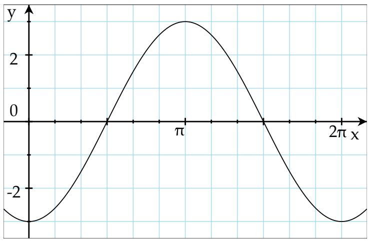
105. Translation of y = sin x across $\frac{\pi}{2}$ up 0, and a vertical enlargement scale factor 3.



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106. x = 0.985, 2.156 (3 dp)

107. x = 2.356, 5.498 (3 dp)

108. x = 3.665, 5.760 (3 dp)

109. x = 0.524, 2.618, 3.665, 5.760 (3 dp)

110. x = -1.920, -1.222, 0.175, 0.873, 2.270, 2.967 (3 dp)

111. x = -3.142, 0, 3.142 (3 dp)
It is important that the graph includes the end points.

112. x = 225°, 315°

113. x = 0°, 120°, 240°, 360°

114. x = 90°, 270°

115. x = 10°, 50°, 70°, 110°, 130°, 170°, 190°, 230°, 250°, 290°, 310°, 350°

116. x = -93.4°, 86.6° (1 dp)

117. x = -136.4°, -73.6°, 43.6°, 106.4° (1 dp)

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118. a) $1500
b) Week 5
c) t = 3 and 7
d) Periodic function with a period of 20. The graph has an amplitude of 20 either side of the Profit = 15 line.

119. a) 95 cents
b) 98.7 cents
c) d = 25 (0 dp)
d) Because after d = 30 the share price values rise far too quickly, i.e. when d = 30, V = 151.4 and when d = 31, V = 287.3.

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120. a) Amplitude = 4
Period = 52 weeks
b) Sales(10) = 11.4
i.e. 10 or 11 sales

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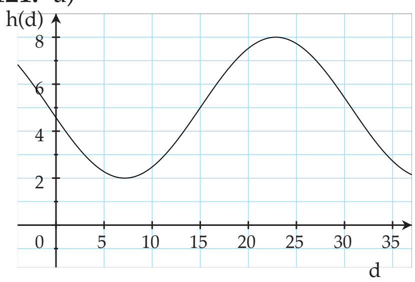
120. c) Sales = 9 in
w = 15 and 37 (0 dp)
Under 9 sales for 22 weeks. (0 dp)

d) Although sales in winter are likely to be less, events such as holidays are likely to affect the sales of real estate which is not reflected in this model.

121. a)



b) Maximum height = 8 m
Occurs d = 22.85 metres

c) height = 7 m at
d = 18.65 m to 27.06 m
18.65 m ≤ d ≤ 27.06 m

d) It would be too regular with the same low and high points being repeated.

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122. a) $y = (x - 3)(x - 5)$
b) $y = 1.5(x - 2)(x + 2)$
c) $y = {}^-2(x + 3)(x + 7)$

123. a) $y = (x - 5)^2 + 1$
b) $y = {}^-0.5(x - 4)^2$
c) $y = {}^-1.5(x + 5)^2 + 6$

124. a) $y = (x + 7)(x + 5)(x + 4)$
b) $y = 2(x + 3)(x + 1)^2$
c) $y = {}^-(x - 4)(x - 6)(x - 7)$

125. a) $y = \frac{^-8}{(x+2)}$

b) $y = \frac{8}{x-1}$

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126. a) $y = \frac{1}{x} + 3$

b) $y = \frac{^-6}{(x-1)} + 3$

127. a) $y = 3^x$
b) $y = 2^x$

128. a) $y = 4^x + 5$
b) $y = 1.414^x - 1$

129. a) $y = \log_2 (x + 1)$
b) $y = \log_4 (x - 1.5)$

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130. a) (5, $^-5$)
b) ($^-1$, 7)
c) $x < {}^-1$ or $x > 5$
d) y = 6
e) Half turn rotational symmetry about (2, 1).

131. a) Y approaches 0 (y = 0).
b) y = 0
c) x = 0
d) Domain is x > 0 and the range is all real numbers.

132. a) $x = {}^-2$
b) y = 1
c) (4, 0) and (0, $^-2$)
d) ($^-2$, 1)
e) All values except $x = {}^-2$ for which the graph is not defined.

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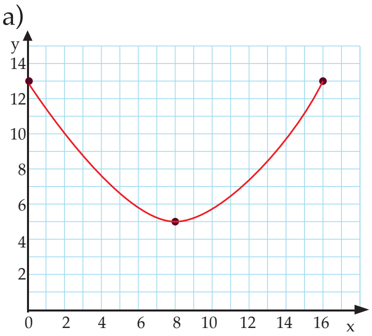
133. a)

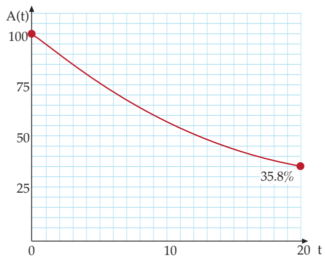


b) The best equation of the cable is a parabola
$h(x) = 0.125(x - 8)^2 + 5$

c) Minimum turning point at (8, 5) with domain of all numbers and range y ≥ 5.

d) From x = 2 to 14 m.
Height is 9.5 m (parabola).

134. a)



b) $A(t) = 100 \times 0.950^t$
c) Asymptote at A(t) = 0.
d) Tends to 0
e) Half-life = 13.5 years.

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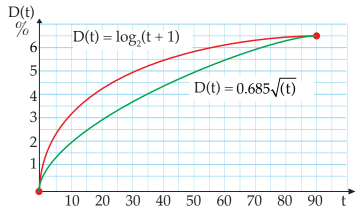
135. a) $\log_2(90 + 1) = 6.5\%$ (2 sf)
b) k = 0.685 (3 sf)
$D(t) = 0.685\sqrt{(t)}$

c)

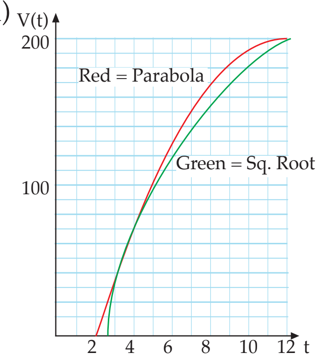


d) The logarithmic function increases fastest at the start and therefore would be the most appropriate. At t = 10 it predicts 3.46% while the square root function predicts 2.17% damage.

136. a) $V(t) = 200 - 2(t - 12)^2$
b) a = 2.663 and k = 65.45
$V(t) = 65.45\sqrt{(t - 2.663)}$

c) Parabola start time is 2 minutes.
Sq. root function start time is 2.663 minutes.

d)



e) The square root function will continue to increase while the parabola will start to decrease at t = 12. The main difference is the starting value and between 6 and 10 minutes. More information would be required to make a decision.

Practice External Assessment Tasks Graphical Models

The practice assessment should be teacher marked using these resources and will be graded using these criteria.

Concepts	Achieved
Not Achieved	The student has correctly selected and used the properties of one function but has not correctly found the modelling equation.
Low Achieved	The student has correctly selected and used one function, its features, equation and properties. The function has been correctly graphed. The student has demonstrated knowledge of the properties of the function and graph and communicated using appropriate representations.
High Achieved	The student has correctly selected and used **two or more** different functions, their features, properties and equations. These functions have been correctly graphed. The student has also demonstrated knowledge of the properties of functions and graphs and communicated using appropriate representations.
Merit	The student has formed and used multiple models in determining appropriate functions in all parts of the problem. These models have been correctly graphed. The findings are related to the context and thinking has been communicated using appropriate mathematical statements.
Excellence	The student has identified relevant concepts in context and formed a generalisation in finding an appropriate model for a generalised problem. Correct mathematical statements have been used in the response.

Pages 64 – 67
Practice External Assessment Task 1 – Graphical Models 2.2
Part One

a) Using an exponential function.
The function has to be moved down 1 as $k^0 = 1$

$$H(x) = k^t - 1$$

through (1, 1.8) so k = 2.8

$$H(x) = 2.8^t - 1$$

Not flat at A nor steep at B. It has an asymptote at $H(x) = {}^-1$.

b) Using a cubic model

$$H(x) = kx^3$$

through (1, 1.8) so k = 1.8

$$H(x) = 1.8 \times x^3$$

Flat at A and closer to our curve but not as steep at B.

c) Using a hyperbola with vertical asymptote x = 1.1 moved down so it passes through (0, 0).

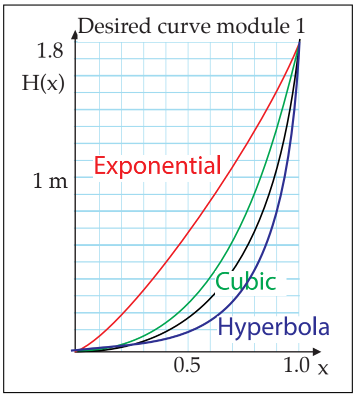
$$h(x) = \frac{{}^-k}{(x-1.1)} - \frac{k}{1.1}$$

through (1, 1.8) so k = 0.198

$$H(x) = \frac{{}^-0.198}{(x-1.1)} - 0.18$$

The hyperbola is both flat at A (has an asymptote at $H(x) = {}^-0.18$ and steep at B. It has an asymptote at x = 1.1 m.

The best model is the hyperbola as it fits the conditions at A and B and is close to the curve. The cubic model could be an alternative.



Part Two

a) Using a parabola to represent modules 2 and 3. The parabola has its base at (3, 0) and passes through (5, 1.8)

$$H(x) = k(x-3)^2 + b$$

through (3, 0) so b = 0

$$H(x) = 0.45(x-3)^2$$

So modelling equations for each module:

$$H(x) = 0.45(x-3)^2 \quad 1 \le x \le 3$$
$$H(x) = 0.45(x-3)^2 \quad 3 \le x \le 5$$

Flat at the turning point C and not too steep at D (or B).

b) Using an exponential model for module 3 and reversing it for module 2.

$$H(x) = k^{(x-3)} + b$$

through (3, 0) so $b = {}^-1$ and through (5, 1.8) so k = 1.6733

$$H(x) = 1.6733^{(x-3)} - 1$$

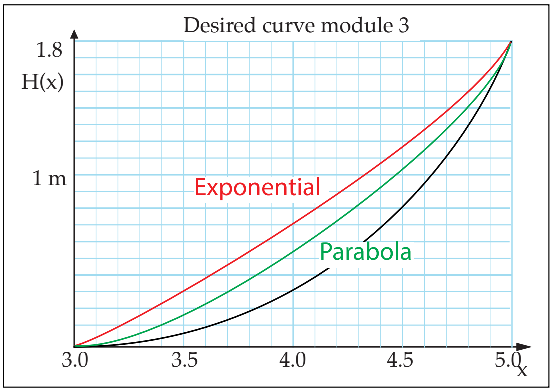
So modelling equations for each module:

$$H(x) = 1.6733^{(3-x)} - 1 \quad 1 \le x \le 3$$
$$H(x) = 1.6733^{(x-3)} - 1 \quad 3 \le x \le 5$$

Page 64 Part Two cont...

Not flat at C and not as steep at D.

Best model is the parabola as it meets the conditions at C , B and D and is closer to the manufacturer's sketch.



Part Three

The generalised models Part One are:

a) Exponential function. The function has to be moved down 1 as $k^0 = 1$

$$H(x) = k^t - 1$$

through (1, h) so $k - 1 = h$

$$H(x) = (h + 1)^t - 1$$

b) Cubic model

$$H(x) = kx^3$$

through (1, h) so $k = h$

$$H(x) = h \times x^3$$

c) Hyperbola with vertical asymptote $x = 1.1$ moved down so it passes through (0, 0)

$$h(x) = \frac{^{-}k}{(x-1.1)} - \frac{k}{1.1}$$

through (1, h) so $k = \frac{1.1h}{10}$

$$H(x) = \frac{^{-}h}{(x-1.1)} - \frac{h}{10}$$

The hyperbola is both flat at $x = 0$ (has an asymptote at $H(x) = {}^{-}0.18$ and is steep at $x = 1$ m. It has an asymptote at $x = 1.1$ m.

The best model is the hyperbola as it fits the conditions at A and B.

The generalised models in Part Two are:

a) The parabola has its base at (3, 0) and passes through (5, h)

$$H(x) = k(x - 3)^2 + b$$

through (3, 0) so $b = 0$ and $k = 0.25h$

$$H(x) = 0.25h(x - 3)^2 \qquad 1 \le x \le 3$$

$$H(x) = 0.25h(x - 3)^2 \qquad 3 \le x \le 5$$

Page 64 Part Three cont...

b) Exponential model for module 3 (reversed for module 2).

$$H(x) = k^{(x-3)} + b$$

through (3, 0) so $b = {}^{-}1$ and through (5, h) so

$$k = \sqrt{h+1}$$

$$H(x) = \left(\sqrt{h+1}\right)^{(x-3)} - 1$$

So modelling equations for each module:

$$H(x) = \left(\sqrt{h+1}\right)^{(3-x)} - 1 \quad 1 \le x \le 3$$

$$H(x) = \left(\sqrt{h+1}\right)^{(x-3)} - 1 \quad 3 \le x \le 5$$

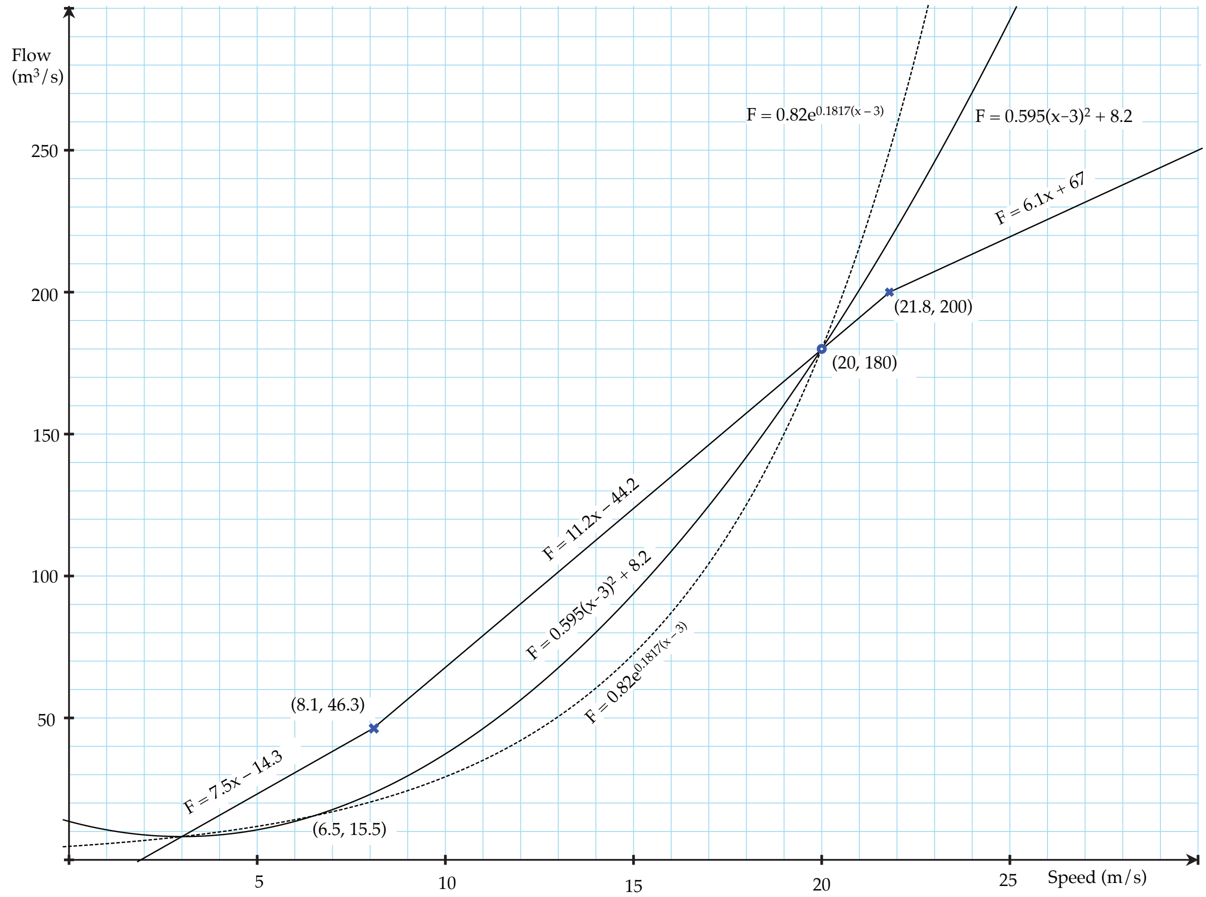
Not flat at C and but not as steep at D.

Best model is the parabola as it meets the conditions at C , B and D and is closer to the manufacturer's sketch.

Pages 68 – 71 Practice External Assessment Task 2 – Graphical Models 2.2

Resources that enable you to assess the answer.

Graphs of three Speed versus Flow models



Models

Piecewise function

$$F = \begin{cases} 7.5x - 14.3 & 2 < x < 8.1 \\ 11.2x - 44.2 & 8.1 \le x < 21.8 \\ 6.1x + 67 & x \ge 21.8 \end{cases}$$

$F_{Quad.} = 0.5945(x - 3)^2 + 8.2$

$F_{Exp.} = 8.2\, e^{0.1817(x - 3)}$

Trig where Jan = 0 and $0 \le M \le 11$ and M is in radians. The model is $F = 30 - 6\cos(0.5235M)$ Mm^3/mth.

Intersections

Solves

$7.5x - 14.3 = 11.2x - 44.2$

Gets (8.081, 46.31) and

$11.2x - 44.2 = 6.1x + 67$

Gets (21.8, 200)

Non-linear

(6.5, 15.5) Third point

(20, 180) all three models.

Solutions For flood

Piecewise $F = 235.8$ m^3/s

Quad. $F = 295.9$ m^3/s

Expont. $F = 446.5$ m^3/s

Trig. November (M = 10)

Flow = 27 million m^3/mth.

Model descriptions could include some of the following but other points are possible.

Piecewise.

- Speed less than 2 m/s results in a negative flow.
- After 8.1 m/s every 1 m/s increase in speed results in 11.2 m^3/s of water.
- After the bank overflows every 1 m/s increase in speed results in 6.1 m^3/s of water flow.
- Linear seems a good model but you would expect that double the speed you would get double the volume and this does not fit this data.

Quadratic

- Initially the model does not predict much change of flow as the speed increases from 3 m/s.
- For speeds less than 3 m/s the flow would increase.
- In the flood, speed is up 25% on last data point but flow predicted is up 64%.

Exponential

- Even slower than quadratic to start showing an increased flow as the current increases.
- Prediction for the flood is not reasonable.

Judgements

As a river increases in speed the cross sectional area would be expected to increase so it would be expected that doubling the speed would result in greater than a double in flow.

Of the models here the two non-linear models increase too slowly at the start and too fast after 20 m/s. The best answer is probably Flow = 11.2x – 44.2 [Linear with no overflow of stopbanks] as an increase of 25% in speed from (20, 180) it predicts a flow of 236 m^3/s an increase of 31%.

The prediction for November makes an assumption about the weather being consistent which is not supported by reality and global weather change.