Answers

Page 4

1. 1.070 (3 dp)
2. 0.785 (3 dp)
3. 3.547 (3 dp)
4. 6.283 (3 dp)
5. 0.838 (3 dp)
6. 4.294 (3 dp)
7. 10.036 (3 dp)
8. 18.047 (3 dp)
9. $\frac{\pi}{2}$
10. π
11. $\frac{\pi}{6}$
12. $\frac{\pi}{12}$

Page 5

13. $\frac{\pi}{4}$
14. $\frac{7\pi}{6}$
15. $\frac{\pi}{3}$
16. $\frac{5\pi}{12}$
17. $\frac{3\pi}{10}$
18. $\frac{3\pi}{4}$
19. $\frac{26\pi}{9}$
20. $\frac{14\pi}{3}$
21. 114.6° (1 dp)
22. 31.0° (1 dp)
23. 120.0° (1 dp)
24. -71.0° (1 dp)
25. 235.0° (1 dp)
26. 345.0° (1 dp)
27. 470.0° (1 dp)
28. 896.7° (1 dp)
29. 180°
30. 45°
31. 90°
32. 60°
33. 30°
34. 225°

Page 5 cont...

35. 240°
36. 270°
37. 540°
38. 210°
39. 80°
40. 162°

Page 6

41. The speed of the car in m/s

Speed $= 90\,000 / 3600$
$= 25$ m/s

Circumference of wheel

$C = 2\pi \times 0.37$
$= 2.3248$ m

Revolutions per second

Revs $= 25 / 2.3248$
$= 10.754$ revs / s

Time to Wellington

Time $= 3.75 \times 3600$
$= 13\,500$ s

Revolutions to Wellington

Revs $= 13\,500 \times 10.754$
$= 145\,200$ (4 sf)

The weak spot will touch the road about 145 200 times (4 sf).

42. We need the angle in radians to calculate the arc length. Angle cut out

Angle out $= 0.890$
Angle A $= 2\pi - 0.890$
$= 5.393$ radians

Arc length of green

Arc $= 5.393 \times 0.652$
$= 3.5163$ m (5 sf)

Perimeter + wedge

Dist. $= 3.5163 + 2 \times 0.652$
$= 4.8203$ m (5 sf)

Area of gold

Area $= 4.8203 \times 0.845$
$= 4.0731$ m^2 (5 sf)

Cost of foil

Cost = \$305.49 (2 dp)

Page 8

43. 5.97 m (3 sf)
44. 1.235 radians (3 dp)
70.8° (1 dp)

Page 9

45. a = 6.3 m (2 sf)
46. 1.47 radians or 84.0° (3 sf)

Page 9 cont...

47. Arc = 5.52 m (3 sf)
Sector = 6.84 m^2 (3 sf)
48. Arc = 153 cm (3 sf)
Sector = 1940 cm^2 (3 sf)
49. Perimeter = 545 m (3 sf)
50. r = 9.80 m (2 sf)
51. 72 000 mm^2 (3 sf)
52. Arc = 3.29 m (3 sf)
Area = 2.39 m^2 (3 sf)

Page 10

53. a) 2.71 radians
b) 0.61 m^2 (2 sf)
54. Angle = 4.5 radians
or = 256.9°
Area = 6800 mm^2 (2 sf)
55. a) 1300 m^2 (2 sf)
b) 152 m
c) 105 m

Page 11

56. Let r = radius hat,
R = radius card.

$\theta R = 2\pi r$
$32\theta = 2\pi 11$
$\theta = \frac{11}{16}\pi$ (2.16)

cut out $= \frac{21}{16}\pi$ (4.12)
Area $= 1100$ cm^2

57. Angle of one sector.

Angle $= 0.314\,16$ rad.
Area in $= 0.5\pi(143^2 - 132^2)$
$= 475$ mm^2 (0 dp)
Area out $= 0.5\pi(223^2 - 212^2)$
$= 752$ mm^2 (0 dp)

Total winning area

$= 2(475 + 752)$
$= 2454$ mm^2

Ratio win to total

$= 305\,800 : 2454$
$= 125 : 1$

Therefore given odds of \$100 : \$2 or 50 : 1 are not good particularly as you may miss the entire board.

Page 14

58. Area = 32 m^2 (2 sf)
59. Area = 100 cm^2 (2 sf)
60. Area = 75.9 m^2 (1 dp)
61. Area = 427 cm^2 (3 sf)
62. Area = 26.8 m^2 (3 sf)

Page 14 cont...

63. Area = 38 000 m² (3 sf)

64. Area = 38 000 m² (3 sf)

65. Area = 2.1 m² (2 sf)

66. Length = 27.5 km (3 sf)

Page 15

67. a) sector = 57.1 m² (3 sf)
b) trian. = 42.8 m² (3 sf)
c) seg. = 14.3 m² (3 sf)

68. radius = 17.5 cm (3 sf)

69. area whole = 235.06 cm²
segment = 21.35 cm²
area part = 214 cm² (3 sf)

70. AOB = 0.667 radians
AOB = 38.2°

71. Angle = 1.4765 radians
Area = 89.3 m² (1 dp)
Pavers = 4912
including 10% extra.
Cost = $46 660.00
Answers will vary slightly depending on rounding and what angle is used to find the area.

72. Sector radius 64 mm and chord - 32 mm. Let angle = A

$$\sin 0.5A = \frac{16}{64}$$

$A = 0.5054$ rad. (4 sf)
Area 1 $= 0.5 \times 64^2 \times 0.5054$
$= 1035$ mm² (4 sf)
All area = 4140 mm² (3 sf)

Page 16

73. Area triangle
Area $= 0.5 \times 8.45 \times 7.45 \sin G$
$\sin G = 0.1979$
$G = 11.4°$ (0.1990 rad.)
or $G = 180 - 11.4°$
$= 168.6°$ (2.9426 rad.)

74. Angle = 1.1519 rad.
Two identical segments
Area $= 0.5 \times r^2 (A - \sin A)$
$= 1.259$
Tot. area = 2.52 cm² (3 sf)

75. Missing segment
Angle = 1.3606
Area $= 0.5 \times r^2 (A - \sin A)$
$= 735.4$ mm²
Circle = 12 076 mm²
Shape = Circle – 2 seg.
$= 10\,600$ mm² (3 sf)

Page 16 cont...

76. Circle $= \pi r^2$
Octagon
Angle $= 45°$ (0.7854 rad.)
1 area $= 0.5\ r^2 \sin 0.7854$
8 areas $= 4\ r^2 \sin 0.7854$

$$\text{Ratio} = \frac{4r^2 \sin 0.7854}{\pi r^2}$$

$= 0.9003$
Percent = 90.0%

Page 18

77. x = 522 mm

78. G to A = 173 km (3 sf)

Page 19

79. 6.8 cm (1 dp)

80. 14.8 cm (1 dp)

81. 10.2 cm (1 dp)

82. 3.6 km (1 dp)

83. x = 10.8 km (1 dp)
y = 11.5 km (1 dp)
Total = 26.8 km (1 dp)

84. a) 6.6 m (1 dp)
b) 6.0 m (1 dp)
c) 30 sheet widths
d) 189 metres. Cost $3200

Page 20

85. 33.6° (1 dp)

86. 43.9° (1 dp)

87. 57.3° (1 dp)

88. A = 59.2° (1 dp)

89. M = 16.0° (1 dp)
N = 12.8° (1 dp)

90. D = 65.5 ° (1 dp)
x = 3.6 km (1 dp)

91. X = 38.8° (1 dp)
Y = 88.1° (1 dp)
y = 4.6 m (1 dp)

Page 21

92. p = 19.7 m (3 sf)
q = 13.3 m (3 sf)

93. K = 58.7° (1 dp)
or K = 121.3° (1 dp)

94. G = 69.9° (1 dp)
H = 41.7° (1 dp)

95. A = 61.1° (1 dp)
y = 3.66 m

96. X = 34.6°, Y = 37.4°,
y = 3.64 m (2 dp)

Page 23

97. 10.4 cm (1 dp)

98. 6.7 cm (1 dp)

99. 11.4 m (1 dp)

Page 24

100. Guy rope is 26.4 m (1 dp)
Mast is 21.2 m.

101. 121 km (3 sf)

102. a) 7.7 km (1 dp)
b) 157°

103. Blue triangle
Sides = 128 mm (0 dp)
and 305 mm (0 dp)
Green triangle outside
side = 464 mm (0 dp)
Orange triangle
Angle = 123°
Side 409 mm (0 dp)
Perim. = 413 + 464 + 409
= 1286 mm (0 dp)

Page 25

104. 76.2° (1 dp)

105. 38.9° (1 dp)

106. 119.8° (1 dp)

107. X = 48.9° (1 dp)
Y = 59.1° (1 dp)
Z = 72.0° (1 dp)

108. 076°

109.a) Using the cosine rule, A = 114°. Similarly B = 46° and C = 21° to the nearest degree.
b) Angle ZCX is a right angle. BCX = 69.3° which gives CX = 4.67 m and XB = 12.35 m (2 dp)

Page 26

110. Largest angle = 68.3°

111. x = 10.6 cm (3 sf)

112. y = 220 mm (3 sf)

113. X = 29.5° (1 dp)
y = 36.1° (1 dp)
Z = 114.4° (1 dp)

114. BAC = 53.1°
ABC = 36.9°
ACB = 90°

Page 26 cont...

115. a) Angle = 107.8° (1 dp)

b) Area = 17 900 km² (3 sf)

116. a) Angle = 46.4° (1 dp)

b) Area = 243 m² (3 sf)

Page 27

117. a) Calculate all known lengths (60 s = 1 min)

C1 = 4.1 x 32.3 x 60
= 7 945.8 m
C2 = 10 701.0 m
J1 = 5 054.5 m
J2 = 12 823.2 m

Clint's distance
D = 99°
Using cosine rule
SE = 14 291.6 m
SE = 14 300 m (3 sf)

Jenny's distance
SL = 16 900 m (3 sf)

b) Angle DSK is a right angle. Using cosine rule
ESD = 47.7°
LSK = 30.7°
Therefore
ESL = 11.6°
Distance apart is from triangle ESL
EL = 4090 m (3 sf)

Page 29

118. Acute angle is 7.3°
Obtuse angle is 165.4°
ASA so sine rule.

$$\frac{x}{\sin 7.3^\circ} = \frac{3150}{\sin 165.4^\circ}$$

x = 1590 mm (3 sf)

119. SSS so cosine rule

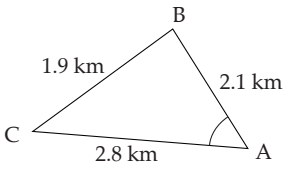
$$48^2 = 43^2 + 17^2 - 2 \times 43 \times 17 \times \cos A$$

Angle = 96.5°

120. a) SAA so sine rule
h_1 = 41.2 m

b) h_2 = 850 sin 18.1
= 264 m

121. SSS so cosine rule



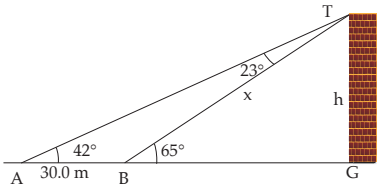
BtoAtoC = 42.7°

Page 30

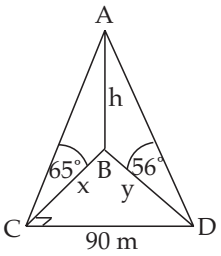
122. Angle ATB = 23°
Ext. $\angle$ tri. = 2 Opp. L's.
Using sine rule as SAA
Let BT = x

$$\frac{x}{\sin 42^\circ} = \frac{30}{\sin 23^\circ}$$

x = 51.38 m.
Using Rt. $\angle$ tri. BTG
h = 46.6 m (3 sf)



123.



Let the distance from C to the mast be x and from D to the mast be y. If the height of the radio mast is h then

$$\tan 65^\circ = \frac{h}{x} \text{ and } \tan 56^\circ = \frac{h}{y}$$

$$\text{so } x = \frac{h}{\tan 65^\circ} \text{ and } y = \frac{h}{\tan 56^\circ}$$

For the triangle CBD

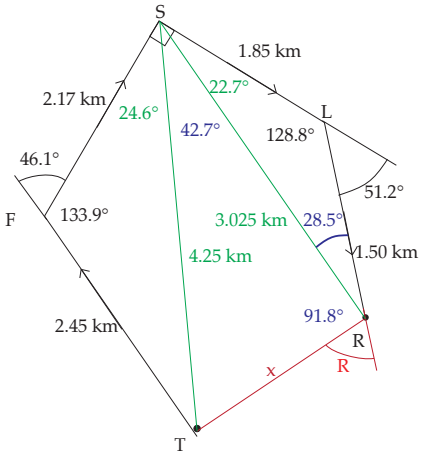
$$x^2 + 90^2 = y^2$$

$$90^2 = h^2\left(\frac{1}{\tan^2 56^\circ} - \frac{1}{\tan^2 65^\circ}\right)$$

h = 185 m (3 sf)

Page 31

124. Distance TS using cosine rule
TS = 4.25 km



Page 31 Q124 cont...

124. Distance SR using cosine rule
SR = 3.025 km
Angle FST and RSL
FST = 24.6°
RSL = 22.7°
This gives TSR
TSR = 90 – 22.73 – 24.54
= 42.7°
Using cosine rule we now find the distance home (x)
x = 2.89 km
Calculate angles LRS and SRT and found to get R the angle home.
R = 180 – 91.8 – 28.5
= 59.7°

125. Angle OHW = 178.29°
Adj. angles str. line. = 180°
Dist. = Speed x time

$$OH = 8.52\frac{53.2}{60}$$

= 7.55 km
Sine rule to get angle HWO
HWO = 1.09°
HOW = 0.62°
Angle sum Tri. = 180°
Side HW = 4.30 km
using sine rule.
Time = 17.5 min so
Speed = 14.7 km/h

Page 33

126. Distance apart is 5.8 km (2 sf)

127. a) Angle between two completed legs is 137°.
Distance from Christchurch is 3329 km.

b) 3 hours 23 minutes

c) Bearing back 187.2°

128. Area of each of 4 triangles

A_1 = 0.5 x 23.5 x 27.2 x sin 100°
= 314.7 m²

A_2 = 0.5 x 18.6 x 27.2 x sin 78°
= 247.4 m²

A_3 = 0.5 x 18.6 x 32.6 x sin 89°
= 303.1 m²

A_4 = 0.5 x 32.6 x 23.5 x sin 93°
= 382.5 m²

Total = 1250 m² (3 sf)

Page 34

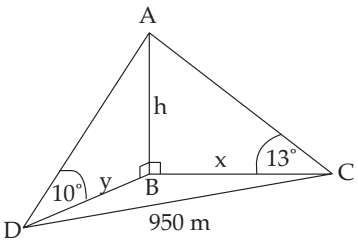
129. Angle DBC = 138°,

x = h tan 77°and y = h tan 80°

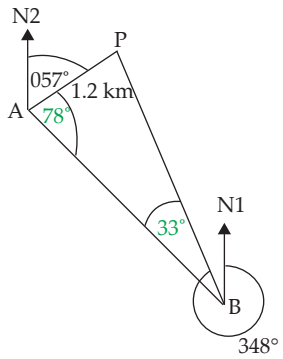
Using the cosine rule

$950^2 = h^2(\tan^2 77° + \tan^2 80° - 2 \tan 77° \tan 80° \cos 138°)$

h = 102 m (3 sf)



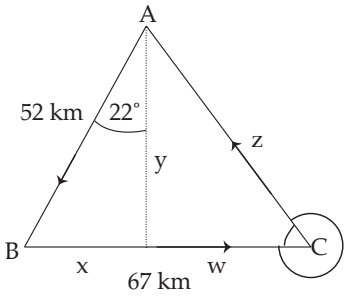
130. N2AB = 135° (SE direction)
ABN1 = 45° Co-int. // =180°
Using the sine rule



Distance 2.1 km (1 dp)

Page 35

131.



x = 19.479 km, y = 48.213 km

w = 47.520 km, z = 67.695 km

C = 45.4° (1 dp)

Bearing is 315.4°

Distance 67.7 km

132. a = Boat A, b = Boat B and c = Lighthouse.

Using the cosine rule to find the distance from a to b we get 8.793.... Dist = 8.8 km (1 dp).

To find angle B we use the cosine rule = 45.16...°

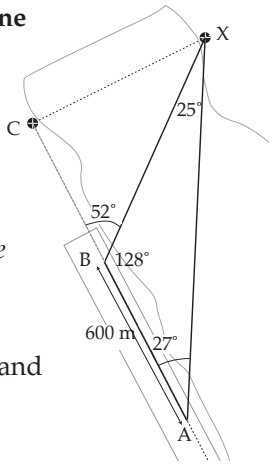
Bearing = 180 + 100 + 45.16...
= 325°

Pages 36 – 39

Practice Internal Assessment 1 – Trigonometric Relationships 2.4

Question One

Measured angles and lengths (will vary depending upon points selected in the rectangle.)



BA = 600 m and CBX = 52°

BXA = 25°

XBA = 128° Adj. *L*'s str. / 180°

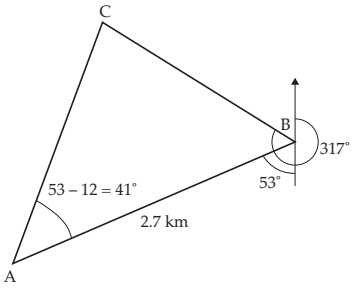
BAX = 27° *L* sum Tri. = 180°

BX = 644.5 m

Using Rt. triangle BCX

CX = 508 m ± 10 m

Assessment 1 cont...
Question Two



CAB = 53 – 12
= 41°

ABC = 317 – 180 – 53
= 84°

ACB = 55° Angle sum Tri = 180°

Using sine rule

BC = 2.16 km

AC = 3.28 km

Distance = 8.1 km

Area = 2.9 km^2

Assessment 1 cont...
Question Three

First sail

Angles PRQ = 39.7°

Area = 21.4 m^2

New sail

Increased area = 25.7 m^2

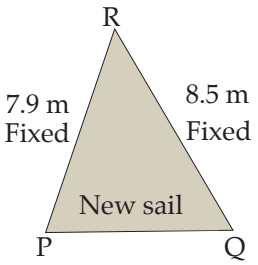
Area = 0.5 x 7.9 x 8.5 x sin PRQ

25.7 = 0.5 x 7.9 x 8.5 x sin PRQ

PRQ = 49.9°,

Using cosine rule PQ = 6.9 m

RPQ = 69.5° and PQR = 60.6°.



Evidence for Achievement	Evidence for Merit	Evidence for Excellence
The student has applied trigonometric relationships in solving problems. The student correctly selects and uses trigonometric relationships. They have demonstrated knowledge of trigonometric concepts and terms and communicated using appropriate representations. Any appropriate and correct rounding is acceptable (including truncation) and does not need to be stated. Some evidence of correct units is required.	The student has applied trigonometric relationships, demonstrating relational thinking in solving problems. The student has related their findings to the context or communicated their thinking using appropriate mathematical statements. Any appropriate and correct rounding is acceptable (including truncation) and does not need to be stated. Correct units are required.	The student has applied trigonometric relationships, demonstrating extended abstract thinking in solving problems. The student has used correct mathematical statements or communicated mathematical insight. Any appropriate and correct rounding is acceptable (including truncation) and does not need to be stated. Correct units are required.
Question One *Correct approach including triangle drawn. Some calculations correct.*	*All angles correct and at least one side correctly calculated. Answer related to context.*	*Method correctly communicated and answer completely correct within limits.*
Question Two *Diagram drawn correctly and all angles calculated.*	*All angles correct and at least one distance correctly calculated. Answer in context.*	*Distance and area correct and method correctly communicated.*
Question Three *Initial vertex angle and area correctly calculated.*	*Diagram drawn and the problem approached correctly. The new area and new vertex angle calculated.*	*Method appropriate for the problem and communicated. The full answer is completely correct.*
Sufficiency. Overall students require: **Two or more Achievement questions or better for overall Achievement.** **Two or more Merit questions or better for overall Merit.** **Two or more Excellence questions or better for overall Excellence.**		

Pages 40 – 42

Practice Internal Assessment 2 – Trigonometric Relationships 2.4

Middle course

$BD^2 = 67.5^2 + 183^2 - 2(67.5)(183)\cos 88^\circ$

$BD = 192.8$ m

Course $= 67.5 + 183 + 193$

$= 443$ m (3 sf)

Junior course

Using triangle ABD. Let angle ABD = x.

$$\frac{\sin x}{183} = \frac{\sin 88}{192.8}$$

$x = 71.5^\circ$

Using triangle CBD.

Angle BCD using cosine rule

$BCD = 101.9^\circ$

Let angle CBD = y.

$$\frac{\sin y}{144} = \frac{\sin 101.9}{192.8}$$

$x = 47.0^\circ$

Assessment 2 cont...

Angle ABC = 118.5°

Length of junior course AC + 169.5

$AC = 146.7$ m (using Cosine rule)

Course $= 316$ m (3 sf)

Area enclosed is triangles ABD plus CBD.

Area ABD $= 0.5(67.5)(183)\sin 88$

$= 6172.5$ m^2

Area CBD $= 7186.5$ m^2

Total area $= 13\,400$ m^2 (3 sf)

New position of B

Angle BAC = 37.7°

Let AB = x and BC = y then

$x + y = 173$ (500 – 183 – 144)

$y^2 = x^2 + 146.7^2 - 2(x)(146.7)\cos 37.7$

Substitute for y and solve with Solver on calculator.

$x = 73.8$ m

AB = 73.8 m and BC = 99.2 m

Line AB is extended in a straight line to 73.8 m. From there to C should be 99.2 m.

Evidence for Achievement	Evidence for Merit	Evidence for Excellence
The student has applied trigonometric relationships in solving problems. The student correctly selects and uses trigonometric relationships. They have demonstrated knowledge of trigonometric concepts and terms and communicated using appropriate representations. Any appropriate and correct rounding is acceptable (including truncation) and does not need to be stated. Some evidence of correct units is required.	The student has applied trigonometric relationships, demonstrating relational thinking in solving problems. The student has related their findings to the context or communicated their thinking using appropriate mathematical statements. Any appropriate and correct rounding is acceptable (including truncation) and does not need to be stated. Correct units are required.	The student has applied trigonometric relationships, demonstrating extended abstract thinking in solving problems. The student has used correct mathematical statements or communicated mathematical insight. Any appropriate and correct rounding is acceptable (including truncation) and does not need to be stated. Correct units are required.
Course lengths *Correct Middle school length. Approach to Junior school correct.*	*Correct Middle school length and most of calculations for Junior school correct plus answer given in context.*	*Method correctly communicated and answer completely correct within limits of accuracy.*
Area *Approach correct and at least one area correctly calculated.*	*Total area correct.*	*Total area correct with units and correct rounding.*
New position for B *Angle BAC correct and the sum of the two sides is 173 m.*	*Angle BAC correct and the sum of the two sides is 173 m. Approach to solving for sides is correct, but side length is not solved.*	*Answer correct and instructions to groundsman sufficient and in context.*
Sufficiency. Overall students require: **Two or more Achievement questions or better for overall Achievement.** **Two or more Merit questions or better for overall Merit.** **Two or more Excellence questions or better for overall Excellence.**		

Pages 43 – 47

Practice Internal Assessment 3 – Trigonometric Relationships 2.4

A Block of Land

Finding angle

Arc $= r\theta$

$80.2 = 46\theta$

$\theta = 1.743$ radians

Area of segment

$= 0.5r^2(\theta - \sin\theta)$

$= 801.7\ m^2$

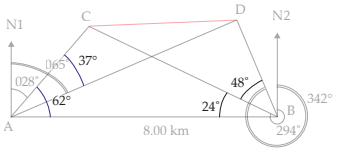
Land area = Trapezium – Segment

$= (110 + 154) \div 2 \times 55 - 801.7$

$= 6460\ m^2$ (3 sf)

Area is less than the $6900\ m^2$ advised. It is $6460\ m^2$ (3 sf)

Lighthouses and Ships



Page 44 Assessment 3 cont...

Angle CAD = 37°

Angle CAB = 62°

Angle CBA = 24°

Angle DBC = 48°

Angle DBA = 72°

Angle ACB = 180 – 62 – 24

$= 94°$

Length AC

$$\frac{AC}{\sin 24°} = \frac{8}{\sin 94°}$$

$AC = 3.262$ km

Length AD

$$\frac{AD}{\sin 72°} = \frac{8}{\sin 83°}$$

$AD = 7.666$ km

Length CD (Dist. between ships) using cosine rule on triangle CAD.

$= 5.4$ km (2 sf)

The ships are 5.4 km (2 sf) apart.

Page 46 Assessment 3 cont...

Width of a Stream

$$\tan 18° = \frac{H}{x},\ \tan 20° = \frac{H}{y}.$$

$$\text{or } x = \frac{H}{\tan 18°},\ y = \frac{H}{\tan 20°} \quad (1)$$

Using Pythagoras

$x^2 = w^2 + 8^2$ so $w^2 = x^2 - 64$

$y^2 = w^2 + 6^2$ so $w^2 = y^2 - 36$

Substituting x and y from above gives.

$$w^2 = \frac{H^2}{\tan^2 18°} - 64 \text{ and}$$

$$w^2 = \frac{H^2}{\tan^2 20°} - 36$$

Equating and solving with the graphics calculator solver gives

H = 3.815 m and w = 8.60

Width of the river is 8.6 m (1 dp)

Evidence for Achievement	Evidence for Merit	Evidence for Excellence
The student has applied trigonometric relationships in solving problems. The student correctly selects and uses trigonometric relationships. They have demonstrated knowledge of trigonometric concepts and terms and communicated using appropriate representations. Any appropriate and correct rounding is acceptable (including truncation) and does not need to be stated. Some evidence of correct units is required.	The student has applied trigonometric relationships, demonstrating relational thinking in solving problems. The student has related their findings to the context or communicated their thinking using appropriate mathematical statements. Any appropriate and correct rounding is acceptable (including truncation) and does not need to be stated. Correct units are required.	The student has applied trigonometric relationships, demonstrating extended abstract thinking in solving problems. The student has used correct mathematical statements or communicated mathematical insight. Any appropriate and correct rounding is acceptable (including truncation) and does not need to be stated. Correct units are required.
A Block of Land *Correct radian measure and correct approach.*	*Correct calculations (ignore minor error) and the answer given in context.*	*Method correctly communicated and answer completely correct within limits of accuracy. Comparison with $6900\ m^2$ made.*
Lighthouses and Ships *Angles all correct and at least one length correct.*	*Correct calculations (ignore minor error).*	*Answer correct with good communication and context.*
Width of Stream *Tangent expressions and Pythagoras correctly expressed but not solved.*	*Approach correct and x and y eliminated.*	*Answer for width correct with good communication and context.*
Sufficiency. Overall students require: **Two or more Achievement questions or better for overall Achievement.** **Two or more Merit questions or better for overall Merit.** **Two or more Excellence questions or better for overall Excellence.**		