Answers

Please Note: In this Achievement Standard there is often more than one correct answer, especially when dealing with paths or circuits.

Page 4

1. Number of edges: 8
Number of vertices (nodes): 5
Degree of vertex A: 3
Degree of vertex B: 3
Degree of vertex C: 3
Degree of vertex D: 3
Degree of vertex E: 4

2. Number of edges: 10
Number of vertices (nodes): 8
Degree of vertex A: 2
Degree of vertex B: 2
Degree of vertex C: 2
Degree of vertex D: 2
Degree of vertex E: 2
Degree of vertex F: 2
Degree of vertex G: 4
Degree of vertex H: 4

3. Number of edges: 5
Number of vertices (nodes): 3
Degree of vertex A: 2
Degree of vertex B: 4
Degree of vertex C: 4

4. Number of edges: 7
Number of vertices (nodes): 5
Degree of vertex A: 4
Degree of vertex B: 3
Degree of vertex C: 3
Degree of vertex D: 2
Degree of vertex E: 2

5. Number of edges: 10
Number of vertices (nodes): 4
Degree of vertex A: 5
Degree of vertex B: 5
Degree of vertex C: 5
Degree of vertex D: 5

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6. Number of edges: 7
Number of vertices (nodes): 4
Degree of vertex A: 2
Degree of vertex B: 5
Degree of vertex C: 2
Degree of vertex D: 5

7. Number of edges: 10
Number of vertices (nodes): 6
Degree of vertex A: 2
Degree of vertex B: 4
Degree of vertex C: 3
Degree of vertex D: 2
Degree of vertex E: 4
Degree of vertex F: 5

Page 5 cont...

8. Number of edges: 20
Number of vertices (nodes): 10
Degree of vertex A: 3
Degree of vertex B: 3
Degree of vertex C: 3
Degree of vertex D: 3
Degree of vertex E: 3
Degree of vertex F: 5
Degree of vertex G: 5
Degree of vertex H: 5
Degree of vertex I: 5
Degree of vertex J: 5

9. Number of edges: 12
Number of vertices (nodes): 6
Degree of vertex A: 4
Degree of vertex B: 4
Degree of vertex C: 4
Degree of vertex D: 4
Degree of vertex E: 4
Degree of vertex F: 4

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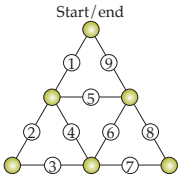
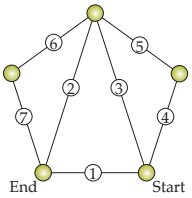
10.



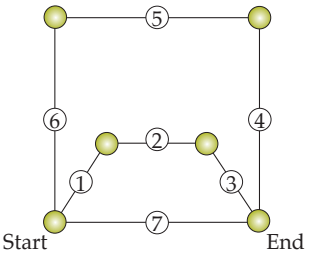
Euler circuit.

11.



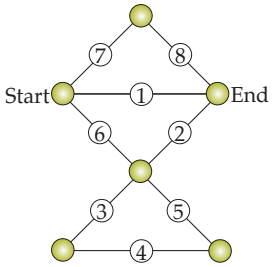
Euler path.

12.



Euler path.

13.



Euler path.

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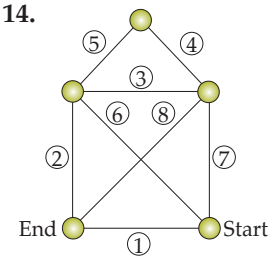
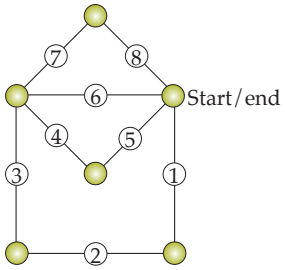
14.



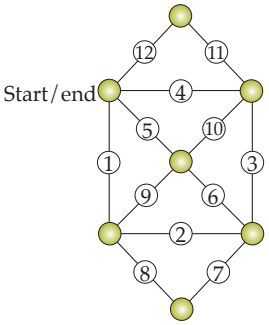
Euler path.

15.



Euler circuit.

16.



Euler circuit.

17. Neither.

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18. Even vertices: A, B, C, D, E
Odd vertices: None
Classification: Euler circuit
Reason: All vertices are even.

19. Even vertices: E
Odd vertices: A, B, C, D
Classification: Neither
Reason: More than two odd vertices so neither.

20. Even vertices: A, D
Odd vertices: B, C
Classification: Euler path
Reason: Has two odd vertices and some even vertices.

21. Even vertices: None
Odd vertices: A, B, C, D, E, F, G, H
Classification: Neither
Reason: More than two odd vertices so neither.

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22. Even vertices: A, B, C, D, E, F, G
Odd vertices: None
Classification: Euler circuit
Reason: All vertices are even.

23. Even vertices: A, C, E, F, G
Odd vertices: B, D
Classification: Euler path
Reason: Has two odd vertices and some even vertices.

24. Even vertices: A, B
Odd vertices: C, D
Classification: Euler path
Reason: Has two odd vertices and some even vertices.

25. Even vertices: C, D
Odd vertices: A, B, E, F
Classification: Neither
Reason: More than two odd vertices so neither.

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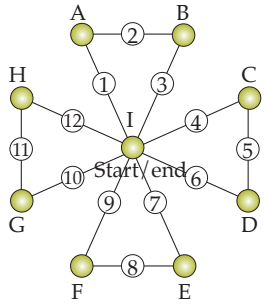
26. Even vertices: A, B, C, D, E, F, G H, I
Odd vertices: None
Traversable: Yes
Start and End points: Start I
End I



27. Even vertices: C, D, E, H
Odd vertices: A, B, F, G
Traversable: No
Start and End points: N/A

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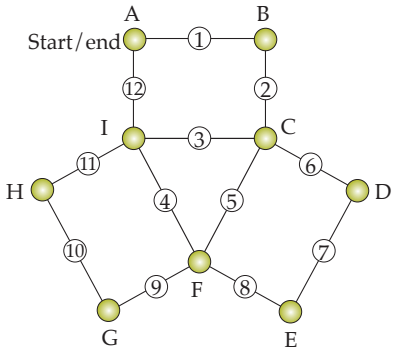
28. Even vertices: A, B, C, D, E, F, G H, I
Odd vertices: None
Traversable: Yes
Start and End points: Start A
End A



29. Even vertices: A, D, E, H
Odd vertices: B, C, F, G, I, J
Traversable: No
Start and End points: N/A

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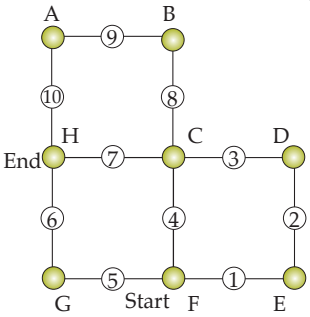
30. Even vertices: A, B, C, D, E, G,
Odd vertices: F, H
Traversable: Yes
Start and End points: Start F
End H



31. Even vertices: A, B, C, D, F, H, K
Odd vertices: E, G, I, J
Traversable: No
Start and End points: N/A

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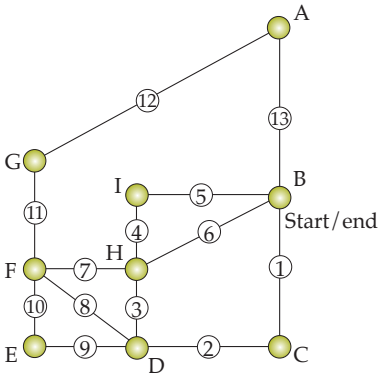
32. Even vertices: A, B, C, D, E, F, G H, I
Odd vertices: None
Traversable: Yes
Start and End points: Start B
End B



33. Even vertices: A, B, D, E, H, J
Odd vertices: C, F, G, I
Traversable: No
Start and End points: N/A

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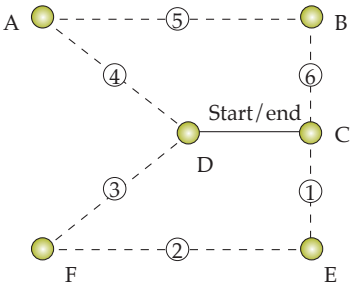
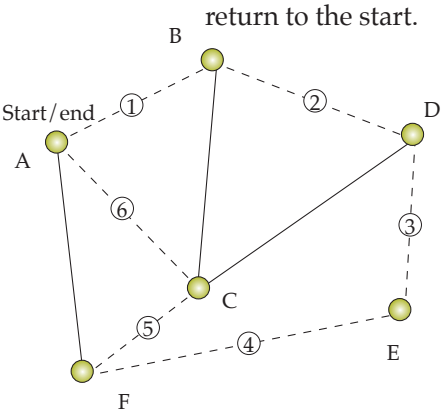
34. Circuit/path: Hamiltonian circuit
Justification: Possible to visit each vertex and return to the start.



Path: C, E, F, D, A, B, C.

35. Circuit/path: Hamiltonian circuit
Justification: Possible to visit each vertex and return to the start.



Path: A, B, D, E, F, C, A.

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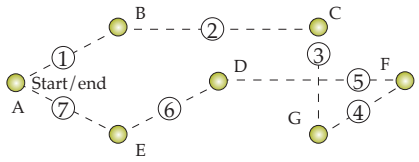
36. Circuit/path: Hamiltonian circuit

Justification: Possible to visit each vertex and return to the start.



Path: A, B, C, G, F, D, E, A.

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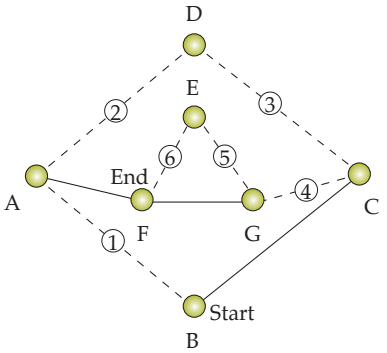
37. Circuit/path: Hamiltonian path

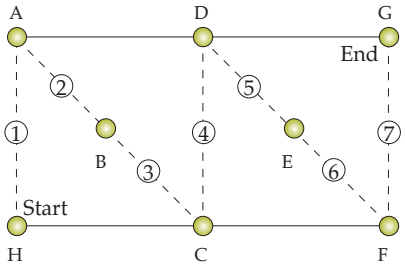
Justification: Possible to visit each vertex but not return to the start.



Path: B, A, D, C, G, E, F.

38. Circuit/path: Hamiltonian path

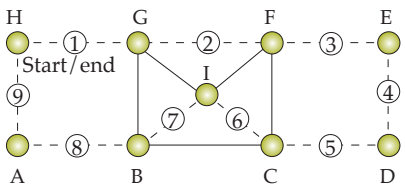
Justification: Possible to visit each vertex but not return to the start.



Path: H, A, B, C, D, E, F, G.

39. Circuit/path: Hamiltonian circuit

Justification: Possible to visit each vertex and return to the start.



Path: H, G, F, E, D, C, I, B, A, H.

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40. a) Circuit 1: A, C, D, E, B, A.

Circuit 2: A, C, E, D, B, A.

b) Dist. 1: 10 + 8 + 6 + 11 + 9
= 44 km

Dist. 2: 10 + 10 + 6 + 7 + 9
= 42 km

c) Optimal circuit is A, C, E, D, B, A with distance 42 km.

41. a) Circuit 1: A, D, E, B, C, A

Circuit 2: A, E, D, B, C, A.

Circuit 3: A, E, B, D, C, A.

Circuit 4: A, E, B, C, D, A.

b) Dist. 1: 11 + 8 + 10 + 28 + 27
= 84 km

Dist. 2: 6 + 8 + 11 + 28 + 27
= 80 km

Dist. 3: 6 + 10 + 11+ 25 + 27
= 79 km

Dist. 4: 6 + 10 + 28 + 25 + 11
= 80 km

c) Optimal circuit is A, E, B, D, C, A with distance 79 km.

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42. a) Circuit 1: A, D, B, C, A
Distance: 8 + 15 + 9 + 19
= 51 km

Circuit 2: A, D, C, B, A
Distance: 8 + 6 + 9 + 10
= 33 km

Circuit 3: A, C, D, B, A
Distance: 19 + 6 + 15 + 10
= 50 km

b) Optimal route is A, D, C, B, A with distance 33 km.

43. a) Circuit 1: A, B, C, F, D, E, A
Time: 10 + 11 + 14 + 17 + 18 + 12
= 82 min

Circuit 2: A, B, C, D, F, E, A
Time: 10 + 11 + 12 + 17 + 7 + 12
= 69 min

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Circuit 3: A, B, C, D, E, F, A
Time: 10 + 11 + 12 + 18 + 7 + 9
= 67 min

Circuit 4: A, B, F, C, D, E, A
Time: 10 + 8 + 14 + 12 + 18 + 12
= 74 min

Circuit 5: A, E, D, C, B, F, A
Time: 12 + 18 + 12 + 11 + 8 + 9
= 70 min

b) Optimal route is A, B, C, D, E, F, A with total travelling time 67 min.

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44. a) Circuit 1: A, E, D, B, C, A.
Time: 38 + 18 + 20 + 48 + 13
= 137 min

Circuit 2: A, B, E, D, C, A
Time: 8 + 28 + 18 + 8 + 13
= 75 min

Circuit 3: A, B, C, D, E, A
Time: 8 + 48 + 8 + 18 + 38
= 120 min

Circuit 4: A, C, B, E, D, A
Time: 13 + 48 + 28 + 18 + 10
= 117 min

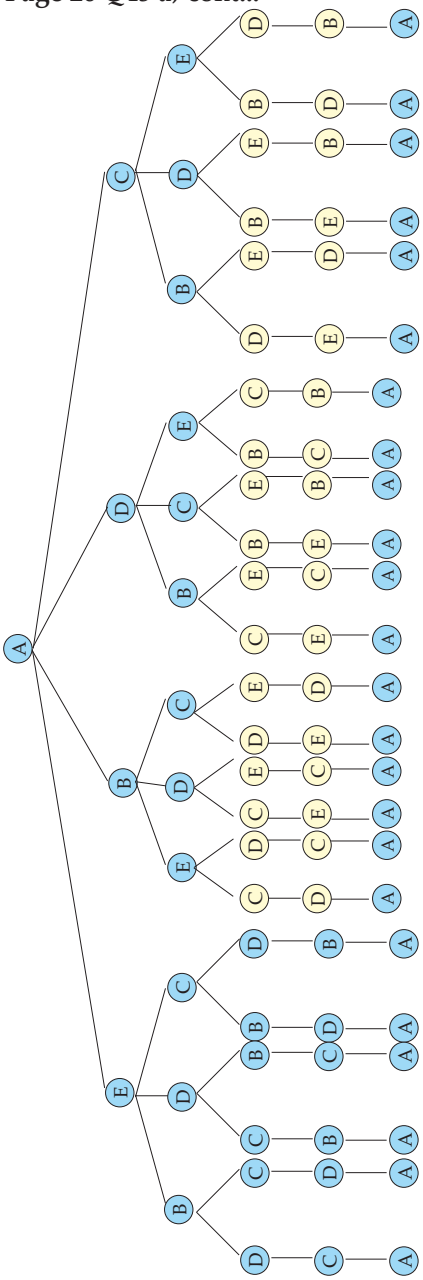
Circuit 5: A, C, D, B, E, A
Time: 13 + 8 + 20 + 28 + 38
= 107 min

Circuit 6: A, D, C, B, E, A
Time: 10 + 8 + 48 + 28 + 38
= 132 min

b) Optimal route is A, B, E, D, C, A with total travelling time 75 min.

45. a) A, B, E, D, C, A
A, B, E, C, D, A
A, B, D, E, C, A
A, B, D, C, E, A
A, B, C, E, D, A
A, B, C, D, E, A
A, D, B, C, E, A
A, D, B, E, C, A
A, D, C, B, E, A
A, D, C, E, B, A
A, D, E, B, C, A
A, D, E, C, B, A
A, C, B, D, E, A
A, C, B, E, D, A
A, C, D, B, E, A
A, C, D, E, B, A
A, C, E, D, B, A
A, C, E, B, D, A

Page 26 Q45 a) cont...



b) A, E, B, D, C, A
A, E, B, C, D A
A, E, D, C, B, A
A, E, D, B, C, A
A, E, C, B, D, A
A, E, C, D, B, A
A, B, E, D, C, A
A, B, E, C, D, A
A, B, D, E, C, A
A, B, C, E, D, A
A, D, B, E, C, A
A, D, E, B, C, A

c) A, E, B, D, C, A
4 + 21 + 22 + 10 + 7 = 64
A, E, B, C, D A
4 + 21 + 6 + 10 + 5 = 46
A, E, D, C, B, A
4 + 11 + 10 + 6 + 13 = 44

Page 26 Q45 c) cont...

A, E, D, B, C, A
4 + 11 + 22 + 6 + 7 = 50
A, E, C, B, D, A
4 + 13 + 6 + 22 + 5 = 50
A, E, C, D, B, A
4 + 13 + 10 + 22 + 13 = 62
A, B, E, D, C, A
13 + 21+ 11 + 10 + 7 = 62
A, B, E, C, D, A
13 + 21 + 13 + 10 + 5 = 62
A, B, D, E, C, A
13 + 22 + 11 + 13 + 7 = 66
A, B, C, E, D, A
13 + 6 + 13 + 11 + 5 = 48
A, D, B, E, C, A
5 + 22 + 21 + 13 + 7 = 68
A, D, E, B, C, A
5 + 11 + 21 + 6 + 7 = 50

Optimal route is
A, E, D, C, B, A with a
travelling time of
44 minutes.

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46. Shortest path is A, E, F with distance 11 km.

47. Shortest path is A, C, D, I with travelling time 9 hours.

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48. Shortest path is A, F, E, H with distance 21 km.

49. Shortest path is A, D, G, H with travelling time 45 minutes.

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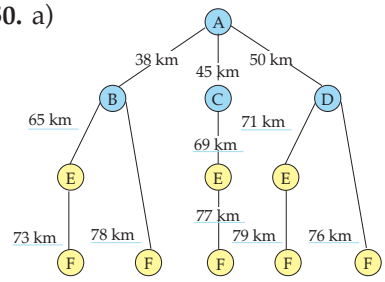
50. a)



b) A, B, E, F = 73 km
A, B, F = 78 km
A, C, E, F = 77 km
A, D, E, F = 79 km
A, D, F = 76 km

c) A, B, E, F with distance 73 km

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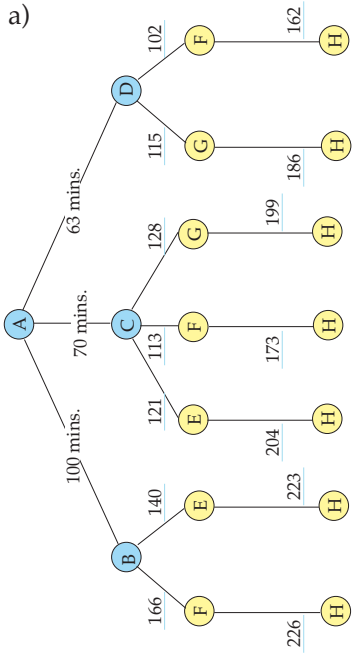
51. a)

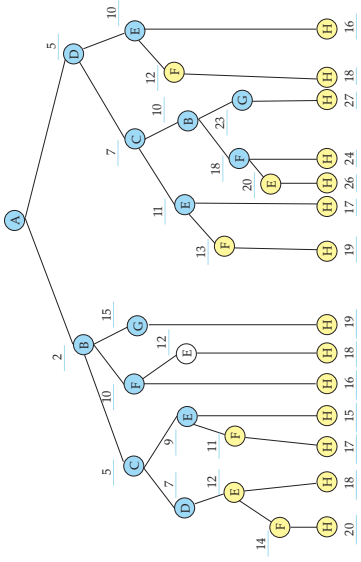


Page 32 Q51 cont...

b) A, B, F, H = 226
A, B, E, H = 223
A, C, E, H = 204
A, C, F, H = 173
A, C, G, H = 199
A, D, G, H = 186
A, D, F, H = 162

c) A, D, F, H with travelling time 162 minutes.

52. a)



b) A, B, C, D, E, F, H = 20 km
A, B, C, D, E, H = 18 km
A, B, C, E, F, H = 17 km
A, B, C, E, H = 15 km
A, B, F, H = 16 km
A, B, F, E, H = 18 km
A, B, G, H = 19 km
A, D, C, E, F, H = 19 km

Page 32 Q52 b) cont...

A, D, C, E, H = 17 km
A, D, C, B, F, E, H = 26 km
A, D, C, B, F, H = 24 km
A, D, C, B, G, H = 27 km
A, D, E, F, H = 18 km
A, D, E, H = 16 km

c) Shortest path is A, B, C, E, H with distance 15 km.

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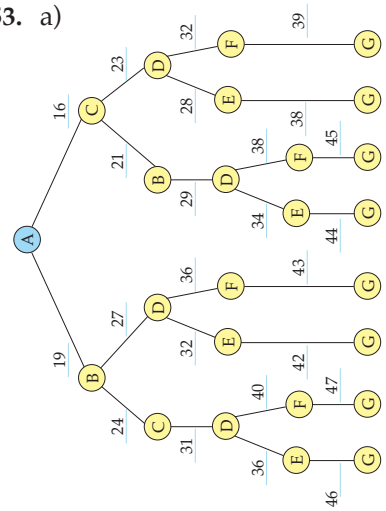
53. a)



b) A, B, C, D, E, G = 46
A, B, C, D, F, G = 47
A, B, D, E, G = 42
A, B, D, F, G = 43
A, C, B, D, E, G = 44
A, C, B, D, F, G = 45
A, C, D, E, G = 38
A, C, D, F, G = 39

c) A, C, D, E, G with distance 38 km.

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54. Shortest path A, C, D, E, F, H = 26 ($643 500)

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55. Shortest path A, B, F, C, D, H = 67 minutes.

Page 36

56. Shortest path A, B, G, E, I = 29 minutes.

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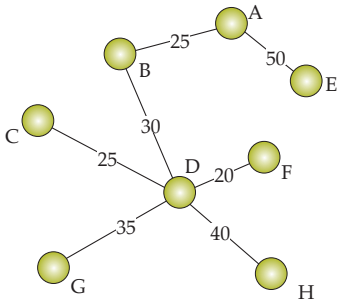
57. Distance = 225 units



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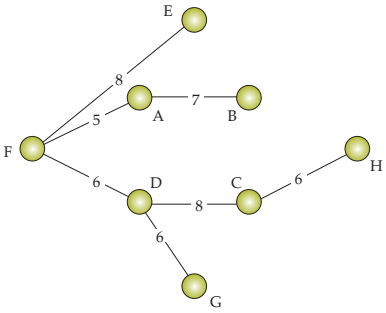
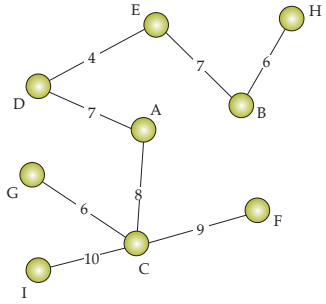
58. Distance = 46 units



59. Distance = 57 units



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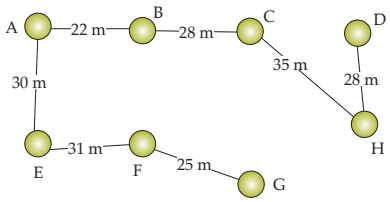
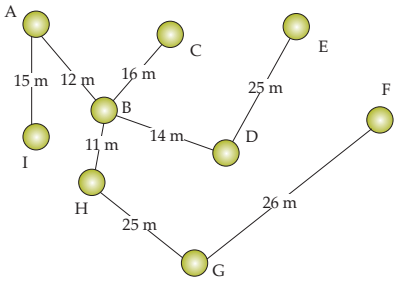
60. Length = 199 m



61. Length = 144 m



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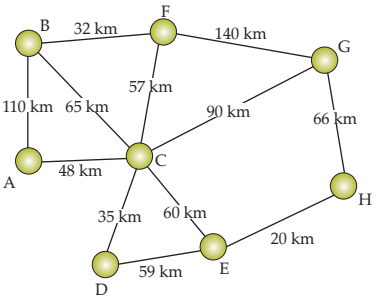
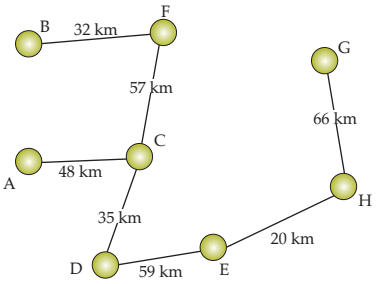
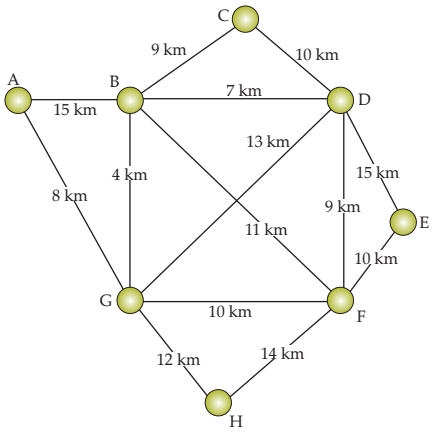
62.

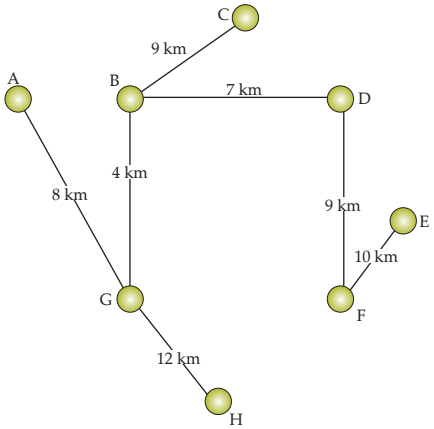




Distance to cover the daily route is 317 + 48 = 375 km (have to repeat the 48 km from A to C). Possible start and end points are B and G respectively.

63. Distance = 59 km





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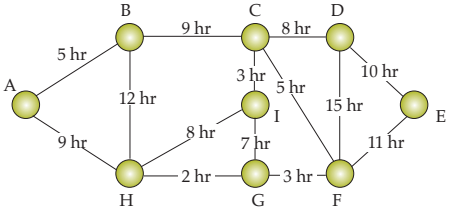
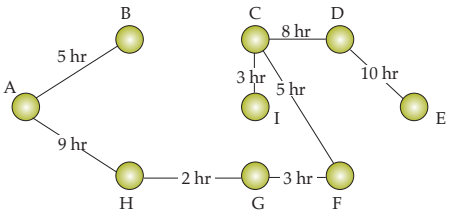
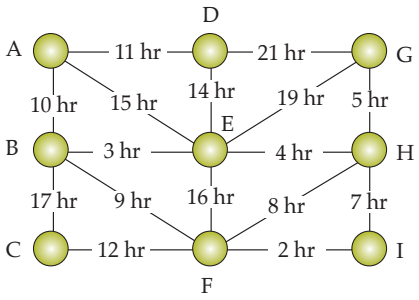
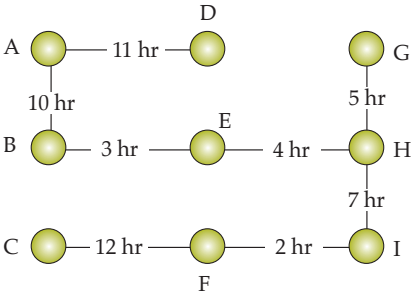
64. Laying time = 45 hours





65. Installing time = 54 hours





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66. a) A: Odd (3)
B: Even (4)
C: Even (4)
D: Even (4)
E: Even (2)
F: Even (4)
G: Even (2)
H: Odd (3)

b) Since the graph has two odd vertices and the rest even it contains an Euler Path.

It can be traversed because of this and either odd vertex will be the start point while the other odd vertex will be the end point.

c) Start point A.

End point H.

A, D, C, G, H, C, A, B, D, F, B, E, F, H.

d) From the endpoint the quickest return route is H, F, B, A with a distance of 145 m.

Total distance of the postman's route including returning to the start is

775 m + 145 m

= 920 m

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67. a) A: Odd (3)
B: Even (4)
C: Even (2)
D: Even (2)
E: Even (2)
F: Even (4)
G: Even (4)
H: Even (2)
I: Even (4)
J: Even (4)
K: Even (2)
L: Even (2)
M: Even (2)
N: Odd (1)

b) Since the graph has two odd vertices and the rest even it contains an Euler Path.

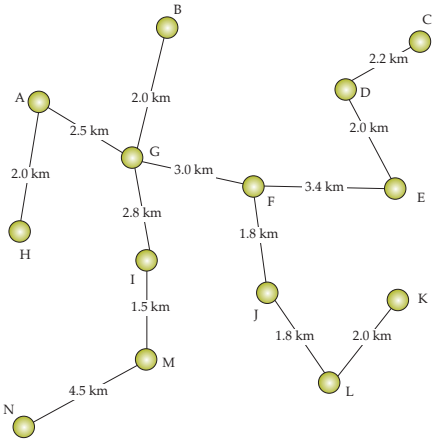
It can be traversed because of this and either odd vertex will be the start point while the other odd vertex will be the end point.

c) Start point A.

End point N.

A, B, G, F, B, C, D, E, F, J, K, L, J, I, G, A, H, I, M, N.

d)



Total distance 31.5 km

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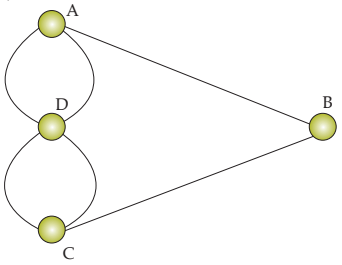
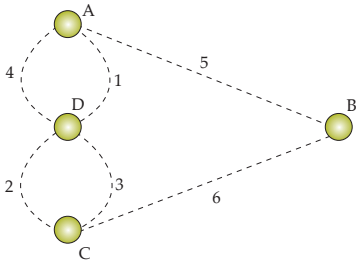
68. a) No, because there are more than two odd vertices. The graph cannot be traversed.

b)



c) Yes, because there are only two odd vertices.

Possible route:



Note: You cannot start and end at the same point.

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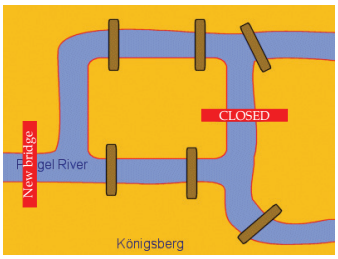
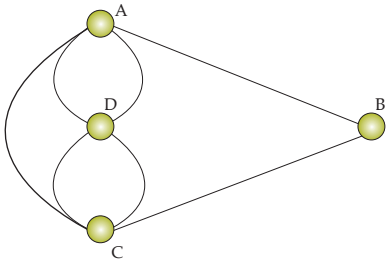
d)



Now all vertices are even so we can traverse the graph and start and end at the same point.



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69. a) Shortest path is A, C, B, E, F, I with a total travelling time of 23 minutes.

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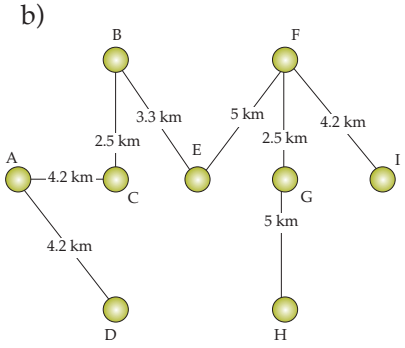
b)



Amount of piping = 30.9 km

c) Route A, B, C, A, D, C, E, D, H, E, B, F, E, G, H, I, G, F, I.

d) 101.7 + 19.2 = 120.9 km

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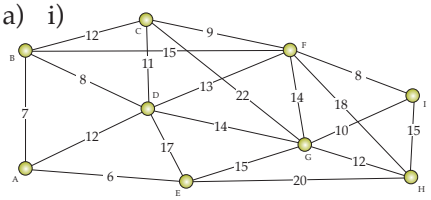
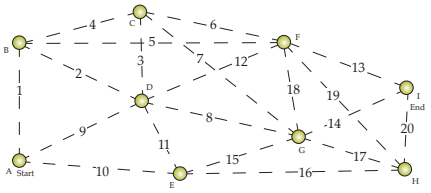
Practice Assessment Task

a) i)



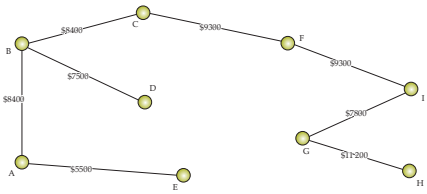
ii) The garden is traversable because there are two odd vertices and the rest are even. Either odd vertex (A or I) can be the start while the other odd vertex will be the end point.

iii) Other solutions possible.



b) Shortest path A, B, F, I with travelling time 30 minutes.

c) Total cost $67 400.



d) There are seven distinct Hamiltonian circuits.

ABFDGHEA = 87 mins.

ABDFGHEA = 80 mins.

ABDFHGEA = 79 mins.

ABDGFHEA = 87 mins.

ADBFGHEA = 87 mins.

ADBFHGEA = 86 mins.

ADGEHFBA = 101 mins.

The shortest route the gardener should take is ABDFHGEA at 79 minutes.

Achievement

The student has applied network methods in solving problems. The student correctly selects and uses network methods. They have demonstrated knowledge of concepts and terms and communicated using appropriate representations.

For example:

- Shortest path

The student finds the correct shortest path and the minimum distance from point A to I.

- Traversability

The student explains that the network is traversable and draws a possible route.

- Minimum spanning tree

The student develops a minimum spanning tree for the garden's irrigation pipes.

Merit

The student has applied network methods, demonstrating relational thinking in solving problems.

The student has related their findings to the context or communicated their thinking using appropriate mathematical statements.

Students will demonstrate an understanding of concepts.

For example:

- Shortest path

The student finds the correct shortest path and the minimum distance from point A to I. justifying the solution by a clear and logical approach with

evidence of an appropriate algorithm, for example, Dijkstra's algorithm or method of trees.

- Traversability

The student accurately uses the traversability condition to explain whether their network is traversable and draws a possible route.

- Minimum spanning tree

The student develops a minimum spanning tree for the garden's irrigation pipes with correct expected total cost and justifies the solution by a clear and logical approach and evidence of an appropriate algorithm.

Excellence

The student has applied networks, demonstrating extended abstract thinking in solving problems.

The student has used correct mathematical statements or communicated mathematical insight.

For example:

- Shortest path

The student develops the shortest path from point A to I and justifies their solution using an appropriate algorithm, for example, Djikstra's algorithm or method of trees.

- Traversability

The student accurately uses the traversability condition to explain whether their network is traversable and draws a possible route.

The student identifies the effect of the flood and the paths that can no longer be traversed and lists the seven possible distinct routes that are now possible and identifies the shortest route to visit the remaining vista points.

- Minimum spanning tree

The student develops a minimum spanning tree for the garden's irrigation pipes with correct expected total cost and justifies the solution by a clear and logical approach and evidence of an appropriate algorithm.

A student's final grade will be decided using professional judgement based on a holistic examination of the evidence provided against the criteria in the Achievement Standard.