Answers

Page 4

1. $y + 8 = -2(x + 2)$

$2x + y + 12 = 0$

2. $m = -3$

$y + 7 = -3(x - 4)$

$3x + y - 5 = 0$

3. $m = \frac{-3}{4}$

$3x + 4y = 0$

4. $m = \frac{3}{5}$

$3x - 5y + 31 = 0$

Page 5

5. $m = \frac{-3}{2}$

$3x + 2y + 1 = 0$

6. $m = \frac{-1}{4}$

$x + 4y + 6 = 0$

7. $m = \frac{4}{3}$

$4x - 3y - 13 = 0$

8. $m = \frac{1}{2}$

$x - 2y - 7 = 0$

9. $m = \frac{7}{5}$

$7x - 5y - 38 = 0$

10. $m = 2$

$2x - y - 1 = 0$

11. $m = -3$

$3x + y - 14 = 0$

12. $m = \frac{2}{3}$

$2x - 3y - 24 = 0$

Page 7 (other solutions are possible. For example the coefficient of t can be of opposite sign)

13. $x = -2 + 6t$, $y = 5 - 4t$ OR $x = 4 - 6t$, $y = 1 + 4t$

14. $x = 1 + t$, $y = -2 + 2t$ OR $x = 2 - t$, $y = -2t$

Page 7 cont...

15. $x = 5 + t$
$y = -2 + 2t$

16. $x = 2 + 3t$
$y = 5 - 2t$

17. $4x + y - 10 = 0$

18. $3x - y - 2 = 0$

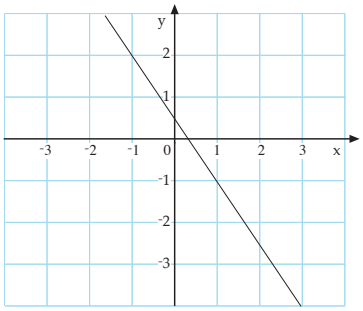
19. $y = \frac{-5x}{2} - 3$

$5x + 2y + 6 = 0$

20. $y = \frac{x - 13}{2}$

$x - 2y - 13 = 0$

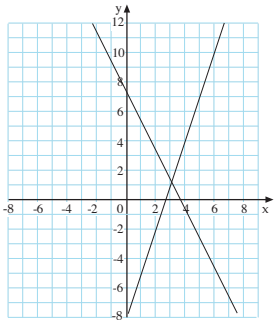
21. a) and b) Same line
$3x + 2y - 1 = 0$



Page 8

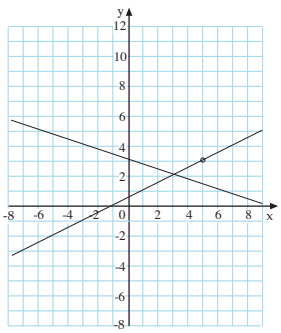
22. a) $2x + y = 7$
$3x - y = 8$
Pt of intersection (3, 1)

b)



23. a) $x - 2y = -1$
$x + 3y = 9$
Pt of intersection (3, 2)

b)



Page 11

24. a) $(x - 2)^2 + (y - 3)^2 = 4$
$x^2 - 4x + y^2 - 6y + 9 = 0$

b) $(x + 3)^2 + (y - 4)^2 = 16$
$x^2 + 6x + y^2 - 8y + 9 = 0$

c) $(x + 4)^2 + (y + 5)^2 = 5$
$x^2 + 8x + y^2 + 10y + 36 = 0$

d) $(x - a)^2 + (y - 2a)^2 = a^2$
$x^2 + y^2 - 2ax - 4ay + 4a^2 = 0$

25. Centre = (6, 4)
Radius = 16
$(x - 6)^2 + (y - 4)^2 = 16^2$

26. Centre = (2, -3)
Radius = 7.5 units
$(x - 2)^2 + (y + 3)^2 = 7.5^2$

27. Centre = (1, 3)
Radius = $2\sqrt{5}$ (≈ 4.472)
$(x - 1)^2 + (y - 3)^2 = 20$

28. Centre = (-1, -2)
Radius = $\sqrt{10}$ (≈ 3.162)
$(x + 1)^2 + (y + 2)^2 = 10$

Page 12

29. $(x - 1)^2 + (y + 1)^2 = 3^2$
$x^2 + y^2 - 2x + 2y - 7 = 0$
(0, 1.83) (0, -3.83), (3.83, 0) and (-1.83, 0)

30. $(x + 2)^2 + (y + 2.5)^2 = 2.5^2$
$x^2 + y^2 + 4x + 5y + 4 = 0$

31. $(x - 0.5)^2 + (y - 1)^2 = 3.5^2$
$x^2 + y^2 - x - 2y - 11 = 0$
(0, 4.46), (0, -2.46), (3.85, 0) and (-2.85, 0)

32. $(x + 2.5)^2 + (y - 1.5)^2 = 2.5$
Radius = $0.5\sqrt{10}$ (≈ 1.581)
$x^2 + y^2 + 5x - 3y + 6 = 0$

33. $x^2 + 2kx + y^2 + 2mx + n = 0$
$(x + k)^2 + (y + m)^2 - k^2 - m^2 + n = 0$
$(x + k)^2 + (y + m)^2 = k^2 + m^2 - n$
Centre $(-k, -m)$
Radius = $\sqrt{k^2 + m^2 - n}$

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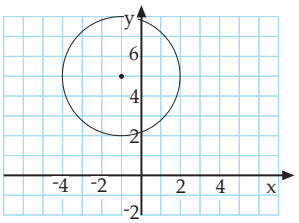
34. a) $(x + 1)^2 + (y - 5)^2 = 9$

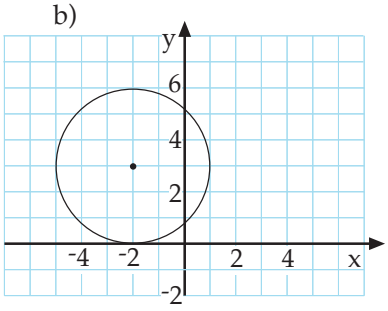
b)



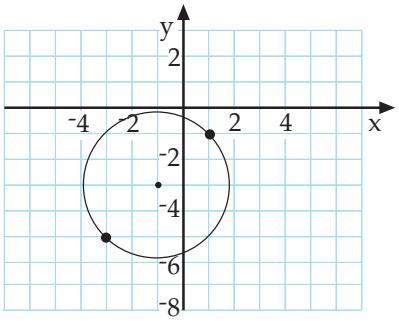
Page 14 cont...

35. a) $x = -2 + 3\cos t$
$y = 3 + 3\sin t$

b)

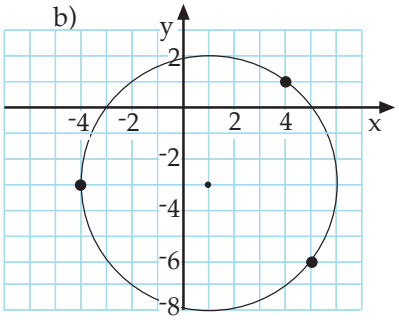


36. $x = -1 + 2\sqrt{2}\cos t$
$y = -3 + 2\sqrt{2}\sin t$



37. a) $x = 1 + 5\cos t$
$y = -3 + 5\sin t$

b)



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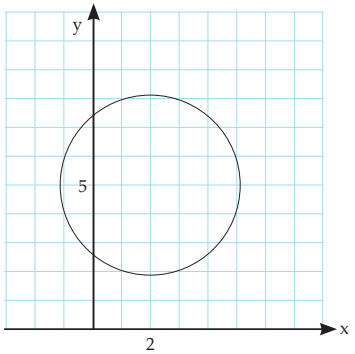
38. a) $(x-2)^2 + (y-5)^2 = 10$

b)



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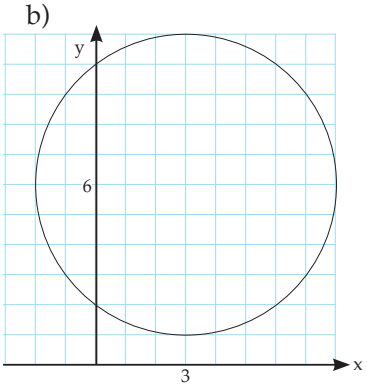
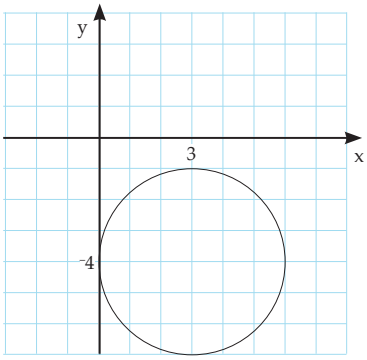
39. a) $x = 3 + 5\cos t$
$y = 6 + 5\sin t$

b)

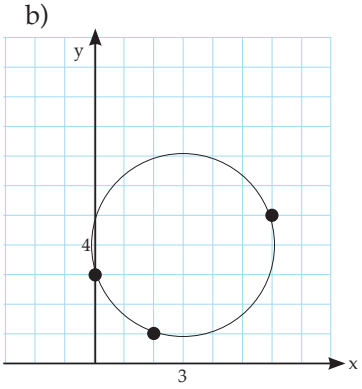


40. $x = 3 + 3\cos t$
$y = -4 + 3\sin t$



41. a) $x = 3 + \sqrt{10}\cos t$
$y = 4 + \sqrt{10}\sin t$

b)



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42. Centre (0, 0)
Major axis = 12
Minor axis = 10
Foci (-3.32, 0) and (3.32, 0)
End points (±6, 0) and (0, ±5)

Page 18 cont...

43. Centre (-2, 3)
Major axis = 10
Minor axis = 6
Foci (-6, 3) and (2, 3)
End points (-7, 3) to (3, 3)
End points (-2, 0) to (-2, 6)

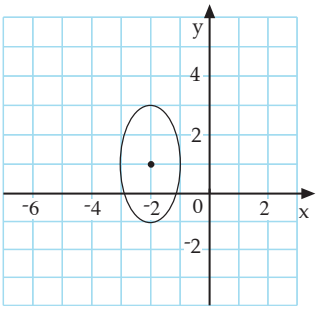
44. Centre (2, -1)
Major axis = 6
Minor axis = 2
Foci (-0.83, -1) and (4.83, -1)
End points (-1, -1) to (5, -1)
End points (2, 0) to (2, -2)

45. Centre (0, 0)
Major axis = $2\sqrt{5}$
Minor axis = 4
Foci (-1, 0) and (1, 0)
End points $(\pm\sqrt{5}, 0)$
End points (0, ±2)

46. Centre (-2, 5)
Major axis = 2
Minor axis = 1
Foci (-2.87, 5) and (-1.13, 5)
End points (-3, 5) to (-1, 5)
End points (-2, 4.5) to (-2, 5.5)

47. Centre (4, 0)
Major axis = 8
Minor axis = 6
Foci (1.35, 0) and (6.65, 0)
End points (0, 0) to (8, 0)
End points (4, -3) to (4, 3)

48. $\dfrac{(x+2)^2}{1} + \dfrac{(y-1)^2}{4} = 1$
Centre (-2, 1)
Foci (-2, -0.73) and (-2, 2.73)



49. $\dfrac{(x+1.5)^2}{3.5^2} + \dfrac{(y-1)^2}{1} = 1$

$x = -1.5 + 3.5\cos t$
$y = 1 + \sin t$

Foci (-4.85, 1) and (1.85, 1)

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50. $\frac{x^2}{16}+\frac{y^2}{4}=1$

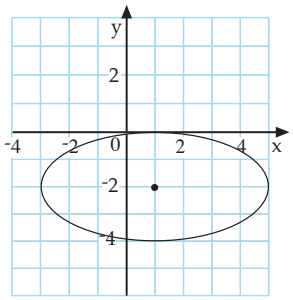
51. x = 4 cos t and y = 6 sin t

52. $\frac{(x+2)^2}{16}+\frac{(y-5)^2}{2}=1$

53. x = $^-$2 + 0.5 cos t and
y = 3 + $\sqrt{5}$ sin t

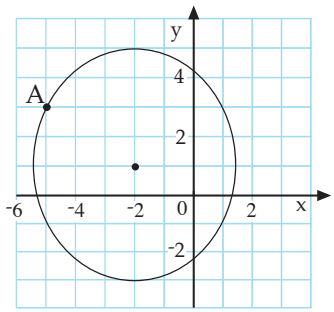
54. a) x = 1 + 4 cos t
y = $^-$2 + 2 sin t

b) Graph of
$x^2 + 4y^2 - 2x + 16y + 1 = 0$



55. a) t = $\frac{5\pi}{6}$ (2.618 to 4 sf)
or 150°

b)



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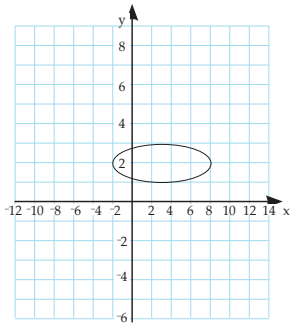
56. a) $\frac{(x-3)^2}{25}+\frac{(y-2)^2}{1}=1$
x = 3 + 5 cos t
y = 2 + sin t

b) Foci ($^-$1.9, 2) and (7.9, 2)

c)



d) (0, 1.2) and (0, 2.8)

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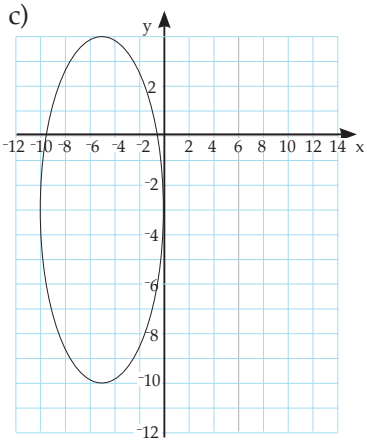
57. a) $\frac{(x+5)^2}{25}+\frac{(y+3)^2}{49}=1$
x = $^-$5 + 5 cos t
y = $^-$3 + 7 sin t

b) Foci ($^-$5, $^-$7.9) and ($^-$5, 1.9)

c)



d) (0, $^-$3), ($^-$0.48, 0), ($^-$9.52, 0)

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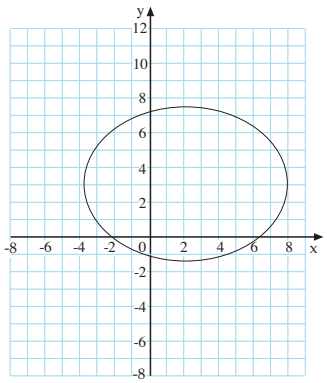
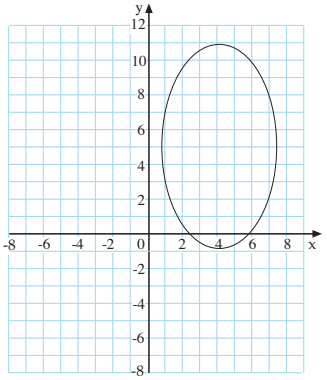
58. $\frac{(x-2)^2}{36}+\frac{(y-3)^2}{20}=1$
x = 2 + 6 cos t
y = 3 + $\sqrt{20}$ sin t



59. $\frac{(x-4)^2}{11}+\frac{(y-5)^2}{36}=1$
x = 4 + $\sqrt{11}$ cos t
y = 5 + 6 sin t



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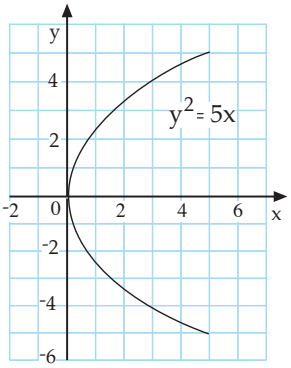
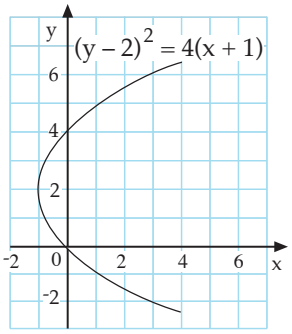
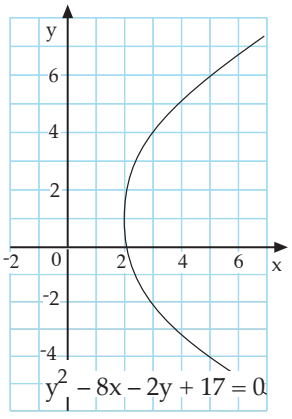
60. Vertex and x intercept (0, 0), focus is (1.25, 0)



61. Vertex ($^-$1, 2). Intercepts (0, 0) and (0, 4) focus is (0, 2).



62. Vertex (2, 1). Intercepts (2.125, 0), focus is (4, 1).



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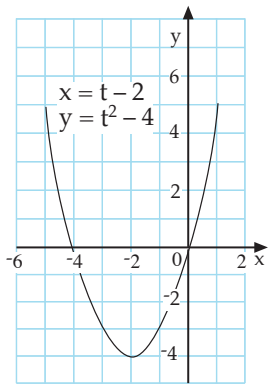
63. $y = x^2 + 4x$ Vertex $(-2, -4)$.
Intercepts $(0, 0)$ and $(-4, 0)$,
focus is $(-2, -3.75)$.



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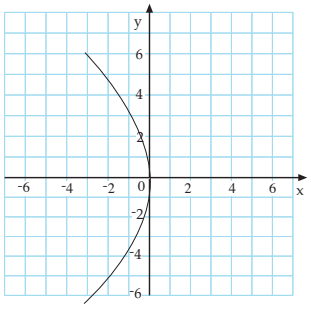
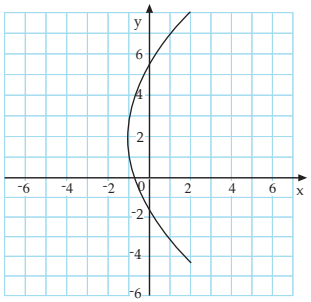
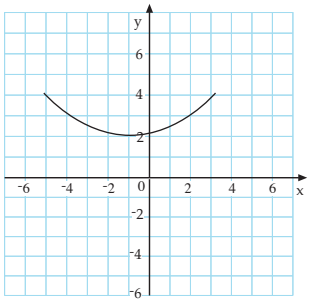
64. Vertex $(0, 0)$
Focus $(-3, 0)$
Directrix $x = 3$.



65. Vertex $(-1, 2)$, Focus $(2, 2)$
Directrix $x = -4$.



66. Vertex $(-1, 2)$, Focus $(-1, 4)$
Directrix $y = 0$.



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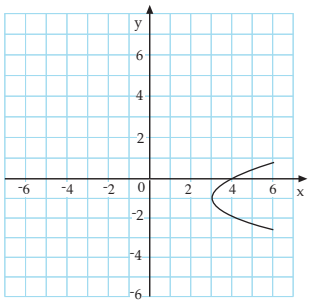
67. Vertex $(3, -1)$
Focus $(3.25, -1)$
Directrix $x = 2.75$.



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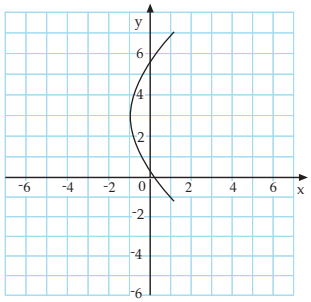
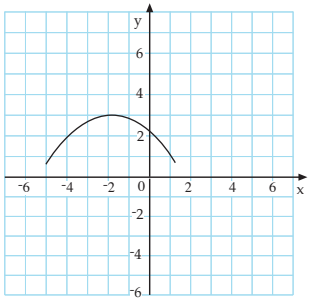
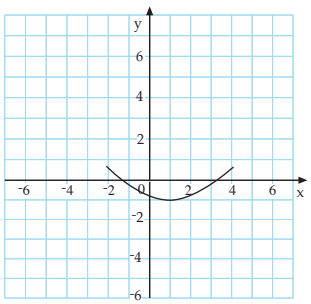
68. Vertex $(-1, 3)$
Focus $(1, 3)$
Directrix $x = -3$.



69. Vertex $(-2, 3)$, Focus $(-2, 2)$
Directrix $y = 4$.



70. Vertex $(1, -1)$, Focus $(1, 0.5)$
Directrix $y = -2.5$.



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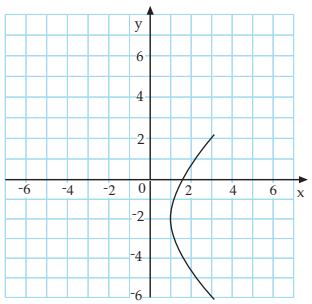
71. Vertex $(1, -2)$
Focus $(3, -2)$
Directrix $x = -1$.



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72. $(y + 2)^2 = 8(x + 7)$
Focal length 2

73. $(y + 1)^2 = 6(x + 3)$
Directrix $x = -4.5$

74. $(x + 2)^2 = 2(y - 6)$
Focal length 0.5

75. $(x + 1)^2 = -1(y + 3)$
Directrix $y = -2.75$

76. $y^2 = 12(x - 1)$

77. $(x - 3)^2 = -12(y - 5)$

78. $(x + 1)^2 = 4(y + 2)$

79. $(x + 1)^2 = 16(y + 1)$

80. $(y - 2)^2 = 6(x + 1)$
$x = -1 + 1.5t^2$
$y = 2 + 3t$

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81. Chain length
$= 3 + 2(8 + 6.2 + 4.8 + 3.8 + 3.2)$
$= 55$ m

82. Focus is 27.78 cm from the centre.

83. 2.1 m either side of the centre line the bridge is only 1.755 m up so NO there is not enough room (unless you lower the river level).

84. a) Distance = 45 mm
b) $y = 90t$
$x = 45t^2$

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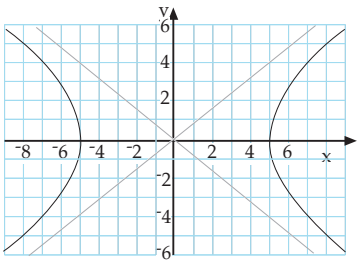
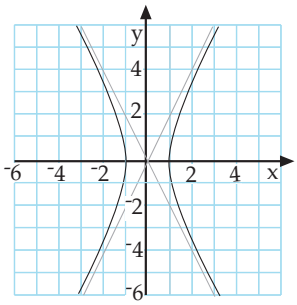
85. Vertices (±5, 0),
Asymptotes $y = \pm\frac{4}{5}x$
Foci $(\pm\sqrt{41}, 0)$ and $(\sqrt{41}, 0)$



86. Vertices (±1, 0),
Asymptotes $y = \pm 2x$
Foci $(\pm\sqrt{5}, 0)$ and $(\sqrt{5}, 0)$



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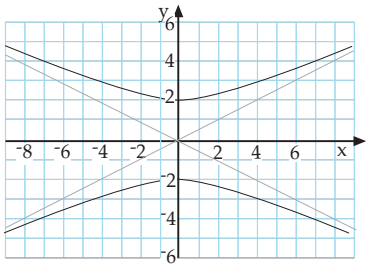
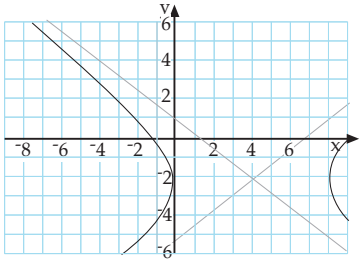
87. Vertices $(0, -2)$ and $(0, 2)$,
Asymptotes $y = 0.5x$
and $y = -0.5x$
Foci $(0, \pm\sqrt{20})$ and $(0, \sqrt{20})$



88. Vertices $(0, -2)$ and $(8, -2)$,
Asymptotes $4y - 3x + 20 = 0$
and $4y + 3x - 4 = 0$
Foci $(-1, -2)$ and $(9, -2)$



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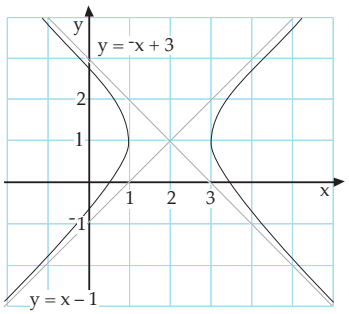
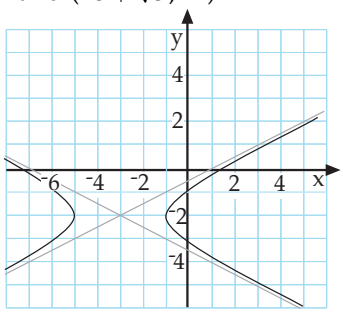
89. $x^2 - 4x + 2y - y^2 + 2 = 0$
$(x-2)^2 - (y-1)^2 = 1$
$\frac{(x-2)^2}{1} - \frac{(y-1)^2}{1} = 1$
Centre is (2, 1)
Vertices $(2 \pm a, 1) = (1, 1)$ and $(3, 1)$ and Asymptotes
$y = x - 1$ and $y = -x + 3$
Foci $(2-\sqrt{2}, 1)$ and $(2+\sqrt{2}, 1)$.



90. $x^2 - 4y^2 + 6x - 16y - 11 = 0$
$(x+3)^2 - 4(y+2)^2 = 4$
$\frac{(x+3)^2}{4} - \frac{(y+2)^2}{1} = 1$
Centre $(-3, -2)$
Vertices $(-1, -2)$ and $(-5, -2)$
Asymptotes $2y - x + 1 = 0$
and $2y + x + 7 = 0$
Foci $(-3-\sqrt{5}, -2)$
and $(-3+\sqrt{5}, -2)$.



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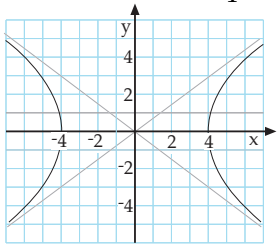
91. a) $\frac{x^2}{16} - \frac{y^2}{9} = 1$
Asymptotes $y = \pm\frac{3}{4}x$

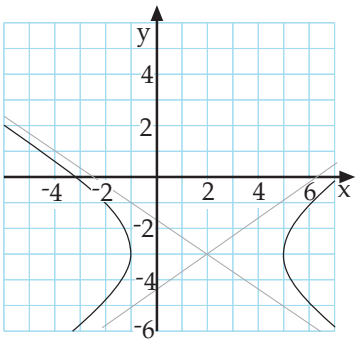
b)



Page 33 cont...

92. a) $x = 3\sec t + 2$ and $y = 2\tan t - 3$

b)



93. a) $\frac{(x-1)^2}{4} - \frac{(y+1)^2}{4} = 1$

b) $(5, 2\sqrt{3} - 1)$ or $(5, 2.464)$

c) $y = x - 2$ and $y + x = 0$

94. a) $x = 4\sec t$ and $y = 5\tan t$

b) $\frac{x^2}{16} - \frac{y^2}{25} = 1$

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95. $y^2 - 4x^2 = 1$
Foci $(0, \pm\sqrt{1.25})$ and $(0, -\sqrt{1.25})$

96. $\frac{(x-4)^2}{4} - \frac{(y+1)^2}{5} = 1$

97. $\frac{(x+1)^2}{225} - \frac{(y-4)^2}{36} = 1$
$x = -1 + 15\sec t$
$y = 4 + 6\tan t$

98. $\frac{x^2}{144} - \frac{y^2}{576} = 1$
$x = 12\sec t$
$y = 24\tan t$

99. $\frac{(x-1)^2}{9} - \frac{(y-2)^2}{36} = 1$
$x = 1 + 3\sec t$
$y = 2 + 6\tan t$

100. $\frac{y^2}{9} - \frac{(x-2)^2}{27} = 1$
$x = 2 + \sqrt{27}\tan t$
$y = 3\sec t$

101. $\frac{(y-3)^2}{16} - \frac{(x-3)^2}{9} = 1$
$x = 3 + 3\tan t$
$y = 3 + 4\sec t$

Page 34 cont...

102. $\frac{(x+1)^2}{36}-\frac{(y-4)^2}{64}=1$

$x = -1 + 6 \sec t$

$y = 4 + 8 \tan t$

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103. Equation of the boundary of the service area is:

$(x + 2)^2 + (y - 4)^2 = 400$

Distance between the centre $(-2, 4)$ and user at $(10, -13)$ is 20.8 km (1 dp) so outside the range.

104. Equation of parabola is:

$x^2 = 4(96.15)(y - 4)$

When $x = 50$, $y = 10.5$ m.

105. $a^2 = 40$, $b^2 = 22.5$

so $c^2 = 62.5$ and $c = 7.9$

Camera must be below mirror so lens at $(0, -7.9)$ and vertex at $(0, 6.32)$. The lens is 14.2 cm from the mirror.

106. Equation of the ellipse is:

$\frac{x^2}{74}+\frac{(y-1.8)^2}{49}=1$

When $x = 5$, $y = 7.496$

So height from floor is 7.5 m and height from whispering point (focus) to the ceiling is $7.5 - 1.8 = 5.7$ m (1 dp).

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107. Tangent $2x - 9y - 20 = 0$

Normal $9x + 2y - 5 = 0$

108. Tangent $4y + x - 9 = 0$

109. Tangent $24x - 25y - 22 = 0$

Normal $25x + 24y - 123 = 0$

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110. Normal $3x + y = 0$

111. Tangent $5x + y - 17 = 0$

112. Tangent $3x - 2y + 1 = 0$

Normal $2x + 3y - 8 = 0$

113. $y = -0.8165x + 2$

114. $\frac{dy}{dx}=\frac{3x}{4y}$

$= -1$ at $(8, -6)$

$= 1$ at $(8, 6)$

Normals $y - x + 14 = 0$

$y + x - 14 = 0$

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115. Gradient of the tangent at $(5, -2)$ is 2. The centre is at $(3, -1)$ so the gradient of the radius is $\frac{-1}{2}$.

As these multiply to give -1 then they are at right angles.

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116. Single point of contact $(3, -2)$ using simultaneous equations.

Gradient of $x + y = 1$ is -1

Gradient of $\frac{x^2}{3}-\frac{y^2}{2}=1$ at $(3, -2)$ is -1.

117. $\frac{dy}{dx}=\frac{-x}{4y}$

$\frac{-x}{4y}=\frac{-1}{2}$

$x = 2y$

substitute into ellipse gives

$y = \pm 2$

points are $(4, 2)$ and $(-4, -2)$

tangents are

$2y + x + 8 = 0$

$2y + x - 8 = 0$

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118. Tangent $x + 6y - 10 = 0$

Normal $6x - y + 51 = 0$

119. $3\sqrt{3}\,y - 12 + 2x = 0$

$y + 0.385x - 2.309 = 0$

120. a) Normal 1

$y = 1.557x + 4.115$

b) Normal 2

$y = -2.185x - 3.370$

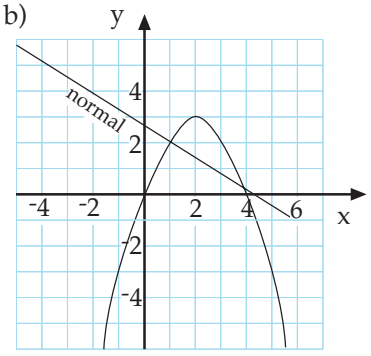
c) Intersect at $(-2, 1)$ the circle centre.

121. a) Normal

$y = -0.667x + 2.917$

$2x + 3y - 8.75 = 0$

b)



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122. $\theta = 0.9273$

$\frac{dy}{dx}=\frac{-5\cos\theta}{10\sin\theta}$

$=\frac{-3}{8}$ (-0.375)

$m_{Normal}=\frac{8}{3}$

$y-4=\frac{8}{3}(x-6)$

$8x - 3y = 36$

123. Point $(7, 6)$ $\frac{dy}{dx}=\frac{1}{2}$

Tangent $2y - x - 5 = 0$,

Normal $y + 2x - 20 = 0$

124. a) At $(2, 2)$ $y = 4x - 6$

b) $(4, -1)$ $y = -x + 3$

Intersect at $(1.8, 1.2)$

125. Substitute for y gives perfect square so must be a tangent.

$x^2 + 2(2x + 9)^2 = 18$

$x^2 + 8x^2 + 72x + 162 = 18$

$9x^2 + 72x + 144 = 0$

$9(x + 4)^2 = 0$

Point of contact $(-4, 1)$

126. $y = 5.000$ and $x = 3.524$

$\frac{dy}{dx}=\frac{12\cos 2\theta}{-4\sin\theta}$

$= -3.506$

$m_{Normal} = 0.2852$

$y - 5 = 0.2852(x - 3.524)$

$y = 0.2852x + 3.995$

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127. $\frac{dy}{dx}=\frac{-\sin t}{2\cos t}$

Tangent

$0.2887x + y - 5.443 = 0$

Normal

$3.464x - y - 2.062 = 0$

128. $\frac{dy}{dx}=\frac{b\cos t}{-a\sin t}$

Tangent

$y=\frac{-b}{a}\left(x-\frac{\sqrt{2}}{2}a\right)+\frac{\sqrt{2}}{2}b$

or $y=\frac{-b}{a}x+\sqrt{2}b$

129. $\frac{dy}{dx}=\frac{3\sin t}{4\cos t}$

Solve to find when gradient is undefined and equals 0 gives $(3, 1)$, $(3, 7)$ and $(-1, 4)$, $(7, 4)$.

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130. $\frac{dy}{dx} = \frac{-b^2x}{a^2y}$

Normal gradient at (X, Y)

$= \frac{a^2Y}{b^2X}$

Expression

$y - Y = \frac{a^2Y}{b^2X}(x - X)$

or $yXb^2 - xYa^2 + XY(a^2 - b^2) = 0$

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131. (10, -4) and (-6, 0)

132. (-3, 2) and (2, -3)

133. (5, 1) and (4, -2)

134. (3, -6) and (12, 12)

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135. (2, 3) and (6, 1)

136. (4, 4) and (4, -4)

137. (5, 1.5811), (5, -1.5811), (-9, 3.0822) and (-9, -3.0822)

138. $(\sqrt{20}, 4)$, $(\sqrt{20}, -4)$, $(-\sqrt{20}, 4)$ and $(-\sqrt{20}, -4)$

Note: $\sqrt{20} = 4.4721$ (to 4 dp)

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139. $y^2 = 8x$. Focus = (2, 0)

140. $a = 2$, $(y - 1)^2 = 8(x - 2)$

Focus = (4, 1)

141. $a = 3$, $(x - 2)^2 = 12(y + 3)$

X intercepts (-4, 0) and (8, 0)

142. $e = \frac{5}{6}$ (0.833)

$a = 6$

$b = \sqrt{11}$ (3.317)

Minor axes = 6.6332

143. $c = 3$

$a = 4.5$

$e = \frac{2}{3}$ (0.667)

144. $e = 2$

$a = 2$

$b = \pm\sqrt{12}$ (±3.354)

$\frac{x^2}{4} - \frac{y^2}{12} = 1$

Asym. $y = \pm\sqrt{3}x$ (±1.732x)

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145. $2y\frac{dy}{dx} = 12$

$\frac{dy}{dx} = 0.5$

$m_n = -2$

$y - 12 = -2(x - 12)$

intersects at (27, -18)

focus = (3, 0)

$m_{FG} = \frac{12}{9}$, $m_{FH} = \frac{-18}{24}$

$m_{FG}.m_{FH} = -1$ so at right angles

146. $\frac{dy}{dx} = \frac{b^2x_0}{a^2y_0}$

where (x_0, y_0) is the point P on the line.

$y - y_0 = \frac{b^2x_0}{a^2y_0}(x - x_0)$

rearranging this equation of the tangent gives

$$\frac{yy_0}{b^2} - \frac{(y_0)^2}{b^2} = \frac{xx_0}{a^2} - \frac{(x_0)^2}{a^2}$$

The tangent crosses the x-axis at (q, 0) so x = q when y = 0

$$\frac{(x_0)^2}{a^2} - \frac{(y_0)^2}{b^2} = \frac{qx_0}{a^2}$$

rearranging this so we can compare it to the hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$\frac{qx_0}{a^2} = 1$$

but at P, $x_0 = p$

$$\frac{qp}{a^2} = 1$$

so $qp = a^2$

147. $\frac{dy}{dx} = \frac{b\cos t}{-a\sin t}$

at $t = t_0$

$y - b\sin t_0 = \frac{b\cos t_0}{-a\sin t_0}(x - a\cos t_0)$

$\frac{y\sin t_0}{b} - \sin^2 t_0 = \frac{x\cos t_0}{-a} + \cos^2 t_0$

$\frac{y\sin t_0}{b} + \frac{x\cos t_0}{a} = \sin^2 t_0 + \cos^2 t_0$

$\frac{y\sin t_0}{b} + \frac{x\cos t_0}{a} = 1$

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148. a) centre = 12 million km

$a = 46 + 12$

$= 58.0$ million km

$12 = \sqrt{58^2 - b^2}$

$b = 56.745$ million km

$\frac{x^2}{58^2} + \frac{y^2}{56.745^2} = 1$

b) $e = \frac{\sqrt{a^2 - b^2}}{a}$

$e = 0.207$

149. a) $e = \frac{c}{a}$

$0.0167 = \frac{c}{(146.0 + c)}$

$c = 2.48$ million km

$a = 148.48$ million km

$b = 148.46$ million km

$\frac{x^2}{148.48^2} + \frac{y^2}{148.46^2} = 1$

b) Directrix

$x = \pm 8890$

The ratio of the distance that every point on the ellipse is to the centre c compared to this line is 0.0167.

150. $y^2 = 4ax$

$2y\frac{dy}{dx} = 4a$

$\frac{dy}{dx} = \frac{2a}{y}$

at (x_0, y_0) the gradient is

$\frac{dy}{dx} = \frac{2a}{y_0}$

$y - y_0 = \frac{2a}{y_0}(x - x_0)$

$yy_0 - y_0^2 = 2ax - 2ax_0$

$yy_0 = 2ax - 2ax_0 + 4ax_0$

$yy_0 = 2ax + 2ax_0$

$yy_0 = 2a(x + x_0)$

151. $a = 3$

$b = 4$

$\frac{y^2}{3^2} - \frac{x^2}{4^2} = 1$

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Practice Internal Assessment – Conics

Equation of parabolas

$y^2 = 4a(x - 5)$

$x = 70$ and $y = 43.75$

$(43.75)^2 = 4a(70 - 5)$

$a = 7.3617...$

Equation $y^2 = 29.45(x - 5)$

$y^2 = 4a(x + 5)$

$x = ^-70$ and $y = 43.75$

$(43.75)^2 = 4a(^-70 + 5)$

$a = ^-7.3617...$

Equation $y^2 = ^-29.45(x + 5)$

Equation of ellipse

$$\frac{x^2}{900} + \frac{y^2}{10074} = 1$$

Points of intersection

$$\frac{x^2}{900} + \frac{29.45(x-5)}{10074} = 1$$

$10074x^2 + 26505x - 9199125 = 0$

$x = 28.93, ^-31.56$ (ignore)

$y^2 = 29.45(28.93 - 5)$

$y = \pm 26.55$

Pts. (28.93, 26.55) and (28.93, $^-$26.55)

$$\frac{x^2}{900} + \frac{^-29.45(x+5)}{10074} = 1$$

$10074x^2 - 26505x - 9199125 = 0$

$x = ^-28.93, 31.56$ (ignore)

$y^2 = ^-29.45(^-28.93 + 5)$

$y = \pm 26.5$

Pts. ($^-$28.93, 26.55)
and ($^-$28.93, $^-$26.55)

Gradient of ellipse

$$\frac{2x}{900} + \frac{2y}{10074}\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{^-1679x}{150y}$$

At (28.93, 26.55) gradient = $^-$12.19..

Gradient of parabola

$y^2 = 29.45x - 147.25$

$2y\frac{dy}{dx} = 29.45$

$$\frac{dy}{dx} = \frac{29.45}{2y}$$

At (28.93, 26.5) gradient = 0.5546

Angle between panel and rod is
$180° - 85.32° - 29.01° = 65.7°$ (1 dp)

For Achievement the student needs to have applied the geometry of conic sections in solving problems. The student needs to have shown evidence of selecting and using methods, demonstrating their knowledge of concepts and terms, and communicating using appropriate representations.

Examples are:

Finding the equation of one or both parabolas.

Finding the equation of the ellipse.

Evidence of checking their model against the actual table's measurements.

For Merit the student needs to have applied the geometry of conic sections using relational thinking in solving problems. The student needs to have formed and used a model. The student has related their findings to the context or communicated thinking using appropriate mathematical statements.

Examples are:

Finding the point of intersection of the ellipse and parabola and then checking that they relate to the design.

Finding the gradient of both the parabola and ellipse at the required point and then identifying the angle created between them using an appropriate approach.

For Excellence the student needs to have applied the geometry of conic sections using extended abstract thinking in solving problems. The student has identified relevant concepts in context. The student has devised a strategy to solve the problem and developed a logical chain of reasoning. The student has used correct mathematical statements or communicated mathematical insight.

Examples are:

Using an ellipse above and below that joins the two parabolas.

Appropriate equations and points of intersection need to be given.

No credit for a circle in the centre that attaches to the two parabolas.

Teacher marked as answers will vary.