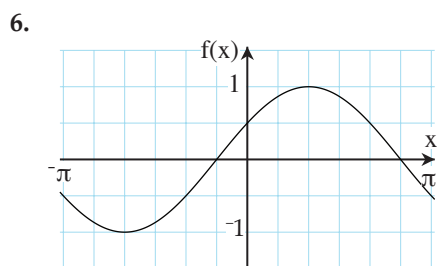
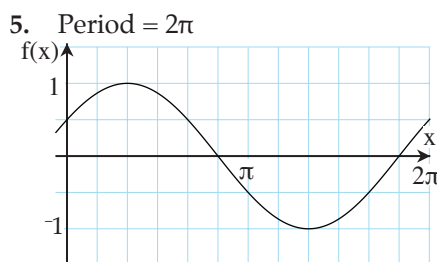
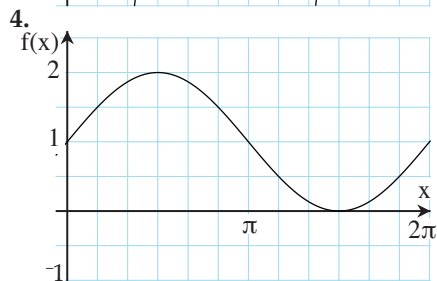
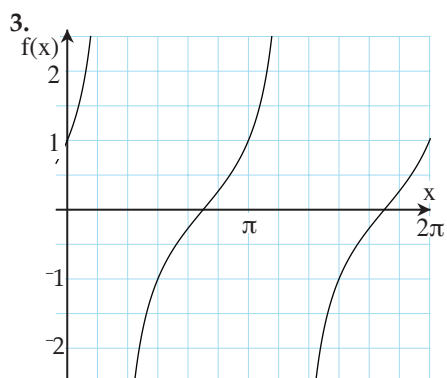
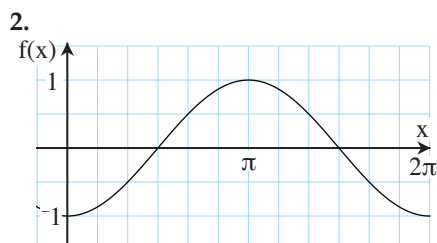
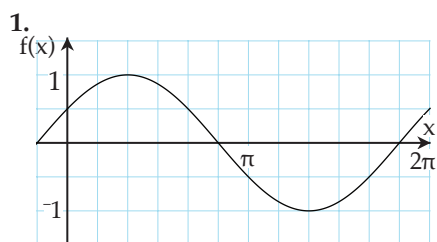
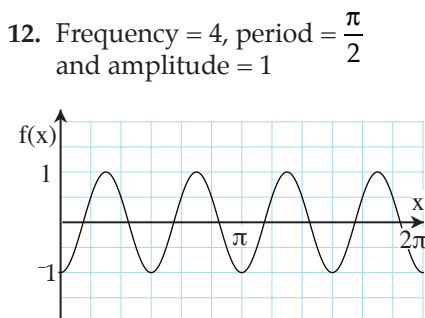
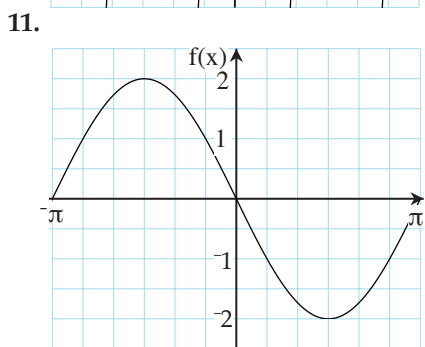
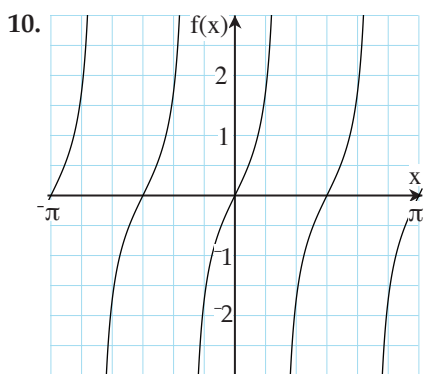
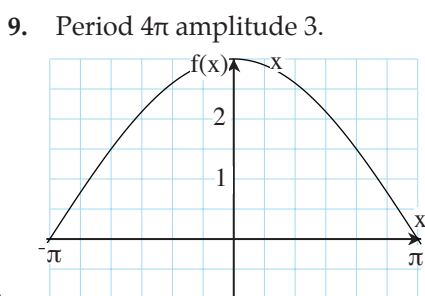
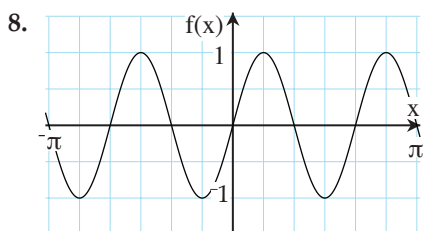
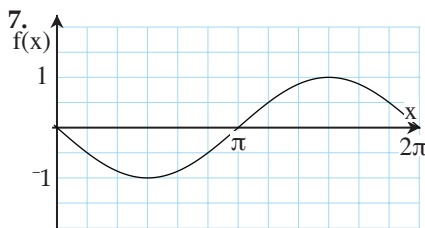


Answers

Page 9



Page 10



Page 13 (Other answers possible)

- 13. $x = 0.3514$ (4 sf)
- 14. $x = 2.3005$ (4 dp)
- 15. $x = 1.929$ (4 sf)
- 16. $4x = -0.22079$
 $x = -0.0552$ (4 dp)
- 17. $x = 0.5773$ (4 dp)
- 18. $x = -1.1398$ or
 $x = 0.1990$ (4 dp)
- 19. $x = -0.4343$ or
 $x = 2.7073$ (4 dp)
- 20. $x = 0.6261$ or
 $x = 3.1185$ (4 dp)
- 21. $x = 2.3562$ or
 $x = -2.3562$ (4 dp)
- 22. $x = 2.3562$ or
 $x = -0.7854$ (4 dp)
- 23. $x = -2.8798$ or
 $x = -1.3090$ (4 dp)

24. $x = 0$

Page 15

- 25. $x = \frac{\pi}{4}$
- 26. $x = \frac{-\pi}{3}$
- 27. $\frac{x}{2} = \frac{-\pi}{3}$
 $x = \frac{-2\pi}{3}$
- 28. $x + \frac{\pi}{6} = \frac{-\pi}{6}$
 $x = \frac{-\pi}{3}$
- 29. $x = \frac{5\pi}{6}$
- 30. $x = \frac{\pi}{4}$
- 31. $3x = \frac{\pi}{3}$
 $x = \frac{\pi}{9}$
- 32. $4x = \frac{5\pi}{6}$
 $x = \frac{5\pi}{24}$

Page 19

$$33. \quad 2x + 1.426 = n\pi - 1.1365$$

$$x = 1.571n\pi - 1.281$$

$$34. \quad x + 1.571 = 2n\pi \pm 1.231$$

$$x = 2n\pi \pm 1.231 - 1.571$$

$$35. \quad 3x - \frac{\pi}{2} = n\pi + \frac{\pi}{4}$$

$$3x = n\pi + \frac{3\pi}{4}$$

$$x = \frac{n\pi}{3} + \frac{\pi}{4}$$

$$36. \quad x - \frac{\pi}{4} = n\pi + (-1)^n \left(\frac{\pi}{6} \right)$$

$$x = n\pi + (-1)^n \left(\frac{\pi}{6} \right) + \frac{\pi}{4}$$

$$= n\pi + (-1)^n \times 0.5236 + 0.7854$$

$$37. \quad x = 0.547, 2.594, 3.689, 5.736$$

$$38. \quad x = -1.869, -1.272, 0.225, 0.822, 2.319, 2.917$$

$$39. \quad x = 3.4814, 5.9433$$

$$40. \quad x = 1.112, 2.029, 4.254, 5.171$$

Page 20

$$41. \quad x = 0.262, 1.309, 3.403, 4.451$$

$$\left(\frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12} \right)$$

$$42. \quad x = 0.262, 0.524, 1.833, 2.094, 3.404, 3.665, 4.974, 5.236$$

$$\left(\frac{\pi}{12}, \frac{\pi}{6}, \frac{7\pi}{12}, \frac{2\pi}{3}, \frac{13\pi}{12}, \frac{7\pi}{6}, \frac{19\pi}{12}, \frac{5\pi}{3} \right)$$

$$43. \quad x = 0.654, 1.702, 3.796, 4.843$$

$$\left(\frac{5\pi}{24}, \frac{13\pi}{24}, \frac{29\pi}{24}, \frac{37\pi}{24} \right)$$

$$44. \quad 2x - \pi = 2n\pi \pm 0.5236$$

$$x = 1.309, 1.833, 4.451, 4.974$$

$$\left(\frac{5\pi}{12}, \frac{7\pi}{12}, \frac{17\pi}{12}, \frac{19\pi}{12} \right)$$

$$45. \quad x = 3.712, 5.713$$

$$46. \quad x = -2.744, -0.398, 0.398, 2.744$$

Page 21

$$47. \quad x = -2.718, -1.995, 0.424, 1.147$$

$$48. \quad x = 0.417, 2.724$$

$$49. \quad x = 0.424, 1.147, 1.995, 2.718, 3.566, 4.288$$

$$50. \quad x = -0.937, 0.152, 0.633, 1.723, 2.204, 3.294, 3.775, 4.864$$

Page 23

$$51. \text{ a) } h = 2.25 \sin \left(\frac{2\pi}{11}(t - 1.75) \right) + 4.35$$

$$\text{ b) } t = 12.236 \text{ hours (12:14 pm)}$$

$$\text{ c) } \text{time} = 6.5 \text{ hours}$$

Page 24

$$52. \text{ a) } s = \text{sales in 000}$$

$$s = 3.5 \sin \left(\frac{2\pi}{26}(t - 2.5) \right) + 8.5$$

$$\text{ b) } t = 11 \text{ 100 (3 sf)}$$

$$\text{ c) } \text{time} = 4.3 \text{ weeks}$$

$$53. \text{ a) } c = 4 \sin \left(\frac{2\pi}{8}(t - 4.5) \right) + 28$$

$$\text{ b) } \text{Exceeds 30 MWH 8 hours a day}$$

Page 24-25

$$54. \text{ a) } w = \text{consumpt. in 000 L/min.}$$

$$w = 9 \sin \left(\frac{2\pi}{15}(t - 3.25) \right) + 36$$

$$\text{ b) } w = 27 \text{ 200 litres/minute}$$

$$\text{ c) } \text{time} = 8 \times 4 = 32 \text{ minutes}$$

Page 25

55. a) Period every 14 days.
Maximum effect = 1 at day 9 and 23. That is 14th and 28th January.
Minimum effect = 0 at day 2 and day 16. That is 7th and 21st of January.

$$\text{ b) } H(x) = 1.2 \sin \left(\frac{2\pi}{0.52}(x - 0.17) \right) + 2$$

c) Spring tide is on day 9.
Closest lunar tide to day 9 is day 9.14. This is 14th January at 3:22 am.

d) Must relate model to reality.
Other factors that affect timing and height of tides are storm surges and atmospheric pressure. Also the lunar tide effect is not a constant height but depends upon whether the moon is on the near or far side of the earth.

Page 27

$$56. \quad \frac{1}{\tan 1.257} = 0.3245$$

$$57. \quad \frac{1}{\sin 0.2187} = 4.6091$$

$$58. \quad \frac{1}{\cos 0.5478} = 1.1714$$

$$59. \quad \frac{1}{\cos \left(\frac{\pi}{3} \right)} = 2$$

$$60. \quad \frac{1}{\sin \left(\frac{4\pi}{5} \right)} = 1.701$$

$$61. \quad \frac{1}{\tan \left(\frac{2\pi}{3} \right)} = -0.5774$$

Page 28

$$62. \quad \text{LHS} = \cot x \sin x$$

$$= \frac{\cos x}{\sin x} \sin x$$

$$= \cos x$$

$$= \text{RHS}$$

$$63. \quad \text{LHS} = \sec x \sin x$$

$$= \frac{1}{\cos x} \sin x$$

$$= \frac{\sin x}{\cos x}$$

$$= \tan x$$

$$= \text{RHS}$$

$$64. \quad \text{LHS} = \frac{\sec x}{\csc x}$$

$$= \frac{1}{\cos x} \div \frac{1}{\sin x}$$

$$= \frac{1}{\cos x} \times \frac{\sin x}{1}$$

$$= \tan x$$

$$= \text{RHS}$$

$$65. \quad \text{LHS} = \cos^2 x - \sin^2 x$$

$$= \cos^2 x - (1 - \cos^2 x)$$

$$= \cos^2 x - 1 + \cos^2 x$$

$$= 2\cos^2 x - 1$$

$$= \text{RHS}$$

Page 29

66. LHS = $\tan x \cos x$
 $= \frac{\sin x}{\cos x} \cos x$
 $= \sin x$
 $= \text{RHS}$
67. LHS = $\frac{1}{\operatorname{cosec} x \tan x}$
 $= \frac{1}{\frac{1}{\sin x} \times \frac{\sin x}{\cos x}}$
 $= \frac{1}{\frac{1}{\cos x}}$
 $= \cos x$
 $= \text{RHS}$
68. LHS = $\cos x \tan x \operatorname{cosec} x$
 $= \cos x \times \frac{\sin x}{\cos x} \times \frac{1}{\sin x}$
 $= 1$
 $= \text{RHS}$
69. LHS = $\operatorname{cosec}^2 x - 1$
 $= \frac{1}{\sin^2 x} - 1$
 $= \frac{1}{\sin^2 x} - \frac{\sin^2 x}{\sin^2 x}$
 $= \frac{1 - \sin^2 x}{\sin^2 x}$
 $= \frac{\cos^2 x}{\sin^2 x}$
 $= \cos^2 x \operatorname{cosec}^2 x$
 $= \text{RHS}$
70. LHS = $\frac{1}{\cos \theta} \times \frac{\cos \theta}{\sin \theta} \times \frac{1}{\sin \theta}$
 $= \frac{1}{\sin^2 \theta}$
 $= \operatorname{cosec}^2 \theta$
 $= \text{RHS}$
71. LHS = $\operatorname{cosec}^2 x - 1$
 $= \cot^2 x$
 $= \text{RHS}$
72. LHS = $1 - \sin^2 \theta$
 $= \cos^2 \theta$
 $= \text{RHS}$
73. LHS = $\sec^2 x - \tan^2 x$
 $= 1$
 $= \text{RHS}$

Page 30

74. LHS = $\cos^2 x - \sin^2 x$
 $= (1 - \sin^2 x) - \sin^2 x$
 $= 1 - 2\sin^2 x$
 $= \text{RHS}$
75. LHS = $\frac{\cos x}{\sin x} + \frac{\sin x}{1 + \cos x}$
 $= \frac{\cos x + \cos^2 x + \sin^2 x}{\sin x(1 + \cos x)}$
 $= \frac{\cos x + 1}{\sin x(1 + \cos x)}$
 $= \frac{1}{\sin x}$
 $= \text{RHS}$
76. LHS = $\frac{\cot x}{\cot^2 x}$
 $= \frac{1}{\cot x}$
 $= \tan x$
 $= \text{RHS}$
77. LHS = $\frac{1}{1 + \sin x} + \frac{1}{1 - \sin x}$
 $= \frac{1 + \sin x + 1 - \sin x}{(1 + \sin x)(1 - \sin x)}$
 $= \frac{2}{1 - \sin^2 x}$
 $= \frac{2}{\cos^2 x}$
 $= 2 \sec^2 x$
 $= \text{RHS}$
78. RHS = $\frac{\operatorname{cosec} \theta - \sin \theta}{\operatorname{cosec} \theta}$
 $= 1 - \frac{\sin \theta}{1}$
 $= \frac{\sin \theta}{\sin \theta}$
 $= 1 - \sin^2 \theta$
 $= \cos^2 \theta$
 $= \text{LHS}$
79. LHS = $\frac{\sec x}{\sin x} - \frac{\sin x}{\cos x}$
 $= \frac{\sec x \cos x - \sin^2 x}{\sin x \cos x}$
 $= \frac{1 - \sin^2 x}{\sin x \cos x}$
 $= \frac{\cos^2 x}{\sin x \cos x}$
 $= \frac{\cos x}{\sin x}$
 $= \cot x$
 $= \text{RHS}$

Page 33

80. $-\cos x$
81. $\cos x$
82. $-\tan x$
83. $-\sin x$

Page 34

84. $\tan x$
85. $\sin x$
86. $\cos \theta$
87. $\frac{1}{\sqrt{2}}(\cos \theta - \sin \theta)$
88. $\sin \frac{\pi}{3} \cos \frac{\pi}{4} - \cos \frac{\pi}{3} \sin \frac{\pi}{4}$
 $= \frac{\sqrt{6} - \sqrt{2}}{4}$
89. $\cos \frac{\pi}{4} \cos \frac{\pi}{3} - \sin \frac{\pi}{4} \sin \frac{\pi}{3}$
 $= \frac{\sqrt{2} - \sqrt{6}}{4}$
90. $\frac{\tan \frac{\pi}{3} + \tan \frac{\pi}{4}}{1 - \tan \frac{\pi}{3} \tan \frac{\pi}{4}} = -2 - \sqrt{3}$
91. $\sin \frac{\pi}{3} \cos \frac{\pi}{3} + \sin \frac{\pi}{3} \cos \frac{\pi}{3} = \frac{\sqrt{3}}{2}$
92. $\cos(45^\circ - 60^\circ) = \frac{\sqrt{2} + \sqrt{6}}{4}$
93. $\tan(60^\circ - 45^\circ) = 2 - \sqrt{3}$

Page 35

94. LHS = $\frac{\cos A \cos B - \sin A \sin B}{\cos A \cos B}$
 $= 1 - \tan A \tan B$
 $= \text{RHS}$
95. LHS = $\frac{\cos x \cos z}{\sin z \cos z} - \frac{\sin x \sin z}{\sin z \cos z}$
 $= \frac{\cos x \cos z - \sin x \sin z}{\sin z \cos z}$
 $= \frac{\cos(x + z)}{0.5(\sin z \cos z + \sin z \cos z)}$
 $= \frac{2 \cos(x + z)}{\sin(z + z)}$
 $= \text{RHS}$
96. LHS = $\sin(x + y) + \cos(x - y)$
 $= \sin x \cos y + \sin y \cos x + \cos x \cos y + \sin x \sin y$
 $= \sin x \sin y + \sin x \cos y + \cos x \sin y + \cos x \cos y$
 $= (\sin x + \cos x)(\sin y + \cos y)$
 $= \text{RHS}$

Page 35 cont...

97. $LHS = \sin(x + y) \cdot \sin(x - y)$
 $= (\sin x \cos y + \sin y \cos x)(\sin x \cos y - \sin y \cos x)$
 $= \sin^2 x \cdot \cos^2 y - \sin^2 y \cdot \cos^2 x$
 $= \sin^2 x \cdot (1 - \sin^2 y) - \sin^2 y \cdot (1 - \sin^2 x)$
 $= \sin^2 x - \sin^2 x \cdot \sin^2 y - \sin^2 y + \sin^2 y \cdot \sin^2 x$
 $= \sin^2 x - \sin^2 y$
 $= RHS$

98. $LHS = \tan\left(A - \frac{\pi}{4}\right) \tan\left(A + \frac{\pi}{4}\right)$
 $= \frac{\left(\tan A - \tan \frac{\pi}{4}\right)\left(\tan A + \tan \frac{\pi}{4}\right)}{\left(1 + \tan A \tan \frac{\pi}{4}\right)\left(1 - \tan A \tan \frac{\pi}{4}\right)}$
 $= \frac{(\tan A - 1)(\tan A + 1)}{(1 + \tan A)(1 - \tan A)}$
 $= \frac{-(1 - \tan A)}{(1 - \tan A)}$
 $= -1$
 $= RHS$

99. $LHS = \cot(A + B)$
 $= \frac{1}{\tan(A + B)}$
 $= \frac{1 - \tan A \tan B}{\tan A + \tan B}$
 Divide top and bottom by $\tan A \cdot \tan B$
 $= \frac{\frac{1}{\tan A \tan B} - \frac{\tan A \tan B}{\tan A \tan B}}{\frac{\tan A}{\tan A \tan B} + \frac{\tan B}{\tan A \tan B}}$
 $= \frac{\cot A \cot B - 1}{\cot B + \cot A}$
 $= RHS$

Page 37

100. $\cos^2 4x - 0.5 = 0.5(2\cos^2 4x - 1)$
 $= 0.5\cos(8x)$

101. $\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right) = 0.5\sin x$

102. $LHS = (\sin A - \cos A)^2$
 $= \sin^2 A - 2 \sin A \cos A + \cos^2 A$
 $= 1 - 2 \sin A \cos A$
 $= 1 - \sin 2A$
 $= RHS$

Page 37 cont...

103. $RHS = \frac{2\cos 2x}{\sin 2x}$
 $= \frac{2(\cos^2 x - \sin^2 x)}{2\sin x \cos x}$
 $= \frac{\cos^2 x - \sin^2 x}{\sin x \cos x}$
 $= \frac{\cos x}{\sin x} - \frac{\sin x}{\cos x}$
 $= \cot x - \tan x$
 $= LHS$

104. $\sin 3A = \sin(A + 2A)$
 $= \sin A \cos 2A + \cos A \sin 2A$
 $= \sin A (1 - 2 \sin^2 A) + 2 \sin A \cos^2 A$
 $= \sin A - 2 \sin^3 A + 2 \sin A (1 - \sin^2 A)$
 $= 3 \sin A - 4 \sin^3 A$

105. $\cos 3A = \cos(A + 2A)$
 $= \cos A \cos 2A - \sin A \sin 2A$
 $= \cos^3 A - \cos A \sin^2 A - 2 \sin^2 A \cos A$
 $= \cos^3 A - 3 \cos A \sin^2 A$
 $= 4 \cos^3 A - 3 \cos A$

Page 39

106. $x = 2n\pi \pm 2.0944$
 or $x = n\pi + (-1)^n \cdot 0.3398$
 $x = 0.3398, 2.0944, 2.8018, 4.1888$

107. $x = n\pi + (-1)^n \cdot 0.5236$
 or $x = n\pi - (-1)^n \cdot 0.7297$
 $x = 0.5236, 2.6180, 3.8713, 5.5535$

108. $x = 2n\pi \pm 0.8411$
 or $x = 2n\pi \pm 2.4189$
 $x = 0.8411, 2.4189, 3.8643, 5.4421$

109. $x = n\pi + 1.2490$
 or $x = n\pi - 1.1071$
 $x = 1.2490, 2.0344, 4.3906, 5.1760$

110. $(2\sin x - 1)(\sin x + 2) = 0$
 $x = n\pi + (-1)^n \cdot 0.5236$ only
 $x = 0.5236, 2.6180$

111. $x = 2n\pi \pm 0.8411$
 or $x = 2n\pi \pm 3.1416$
 $x = 0.8411, 3.1416, 5.4421$

112. $\tan x = 0$
 $x = 0, \pi, 2\pi$

113. $x = n\pi + (-1)^n \cdot 0.3398$
 or $x = n\pi - (-1)^n \cdot 0.2526$
 $x = 0.3398, 2.8018, 3.3943, 6.0305$

Page 41

114. $P = 4 \sin 9\theta + 4 \sin \theta$

115. $Q = \frac{1}{2}(\cos 4A + \cos (2A + 2B))$

116. $R = 5 \cos 10\theta + 5 \cos 2\theta$

117. $S = 2(\cos(3A + 2B) - \cos 5A)$

118. $T = 3(\cos 2x - \cos 4x)$

119. $U = \cos\left(2x + \frac{\pi}{2}\right) + \cos \frac{\pi}{2}$
 $= \cos\left(2x + \frac{\pi}{2}\right)$

120. $V = \sin 4x + \sin \frac{\pi}{2}$
 $= \sin 4x + 1$

121. $W = 0.5 \cos \pi - 0.5 \cos 2x$
 $= -0.5 - 0.5 \cos 2x$

122. $X = 3(\cos 2\pi + \cos 2x)$
 $= 3 + 3 \cos 2x$

123. $Y = 5(\cos^{-2}x - \cos\left(\frac{\pi}{4} + \frac{\pi}{4}\right))$
 $= 5 \cos 2x$

Page 42

124. $4\left(\cos\left(\frac{\pi}{3}\right) - \cos(2x)\right) = 4$
 $2x = 2n\pi \pm 2.0944$
 $x = n\pi \pm 1.0472$
 $x = 1.0472, 2.0944, 4.1888, 5.2359$
 or $x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

125. $\cos 2x = 1$
 $2x = 2n\pi$
 $x = n\pi$
 $x = 0, \pi, 2\pi$

126. $\sin(2x - 0.5) + \sin(0.5) = 2 \times 0.4567$
 $2x = n\pi + (-1)^n \cdot 0.4489 + 0.5$
 $x = 0.5n\pi + (-1)^n \cdot 0.2244 + 0.25$
 $x = 0.4744, 1.596, 3.616, 4.738$

127. $2x = n\pi + (-1)^n \cdot 0.4058 - 0.5$
 $x = 0.5n\pi + (-1)^n \cdot 0.2029 - 0.25$
 $x = 1.118, 3.095, 4.259, 6.236$

128. $2(\cos(4x + \pi) + \cos \pi) = -3$
 $4x + \pi = 2n\pi \pm 2.0944$
 $x = 0.5n\pi \pm 0.5236 - 0.7854$
 $x = 0.2618, 1.3090, 1.8326, 2.8798,$
 $3.4034, 4.4506, 4.9742, 6.0214$

129. $(2x - 0.5267) = 2n\pi \pm 1.8140$
 $x = n\pi \pm 0.9070 + 0.2634$
 $x = 1.1703, 2.4980, 4.3120, 5.6396$

Page 44

130. $T = 2 \sin 4A \cos 2A$

131. $U = -2 \sin 2x \sin x$

132. $V = 8 \cos 4x \sin x$

133. $W = -4 \sin A \sin B$
or $= 4 \sin A \sin (-B)$

134. $2 \sin 3x \cos x = 0$

$x = \frac{n\pi}{3}$ or

$x = 2n\pi \pm \frac{\pi}{2}$

$x = 0, \frac{\pi}{3}, \frac{\pi}{2}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, \frac{3\pi}{2}, \frac{5\pi}{3}, 2\pi$

135. $-2 \sin 2x \sin x = 0$

$x = \frac{n\pi}{2}$

$x = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$

136. $2 \cos 2x \cos \left(x + \frac{\pi}{3}\right) = 0$

$x = n\pi \pm \frac{\pi}{4}$

$x = 2n\pi \pm \frac{\pi}{2} - \frac{\pi}{3}$

$x = \frac{\pi}{6}, \frac{\pi}{4}, \frac{3\pi}{4}, \frac{7\pi}{6}, \frac{5\pi}{4}, \frac{7\pi}{4}$

$x = 0.5236, 0.7854, 2.356, 3.665, 3.927, 5.498$

137. $2 \sin x \cos \frac{\pi}{4} = \sqrt{2}$

$\sin x = 1$

$x = n\pi + (-1)^n \frac{\pi}{2}$

$x = \frac{\pi}{2}$

Page 48

138. a) $AB = 101 \text{ m}$

b) Use the cosine rule
Angle $ABC = 48.4^\circ$

c) Angle $ATC = 43.6^\circ$

Page 49

139. Look for an isosceles triangle $h = a \sin 2x + 1.8$

140. $AY = 5.23$

$AW = 3.59$

$WY = \sqrt{3.59^2 + 5.23^2 - 2 \times 3.59 \times 5.23 \cos 33^\circ}$

$WY = 3.0 \text{ km (2 sf)}$

Page 50

141. a) Height $= 7.50 + 1.65$
 $= 9.15 \text{ m (3 sf)}$

b) Using sine rule

$\frac{h - 1.65}{\sin A} = \frac{x \sin B}{\sin((90 - A) + (90 - B))}$

$h = \frac{x \sin A \sin B}{\sin(180 - (A + B))} + 1.65$

$h = \frac{x \sin A \sin B}{\sin(A + B)} + 1.65$

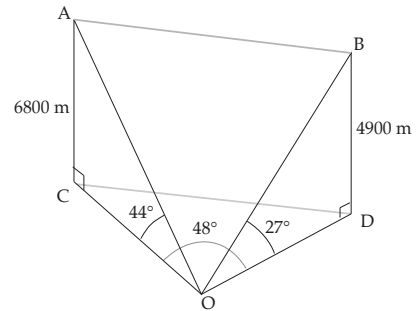
or $h = \frac{x \sin A \sin B}{\sin A \cos B + \cos A \sin B} + 1.65$

$h = \frac{x \tan A \tan B}{\tan A + \tan B} + 1.65$

142. $h = 54.9 \text{ m (3 sf)}$

Page 51

143. a)



b) $OC = 6800 \cot 44^\circ$
 $OD = 4900 \cot 27^\circ$
 $CD^2 = OC^2 + OD^2 - 2OC \times OD \cos 48$
 $CD = 7172 \text{ metres (7170 m 3 sf)}$

144. a) $x + 23 = h \cot A$ and $x = h \cot B$

$h \cot B + 23 = h \cot A$

$23 = h \cot A - h \cot B$

$23 = h(\cot A - \cot B)$

$h = \frac{23}{\cot A - \cot B}$

b) $h = 12 \text{ metres}$

Pages 52 – 55

Practice Internal Assessment – Trigonometry

Possible student responses for Achievement.

City at latitude 37.0° S

$$\text{Amplitude (A)} = (14.5 - 9.5) \div 2 = 2.5$$

$$B = 2\pi \div 365 = 0.01721\dots$$

$$\text{Shift (C)} = -(356 - 0.25(365)) = -264.75$$

$$\text{Vertical translation (D)} = (14.5 + 9.5) \div 2 = 12$$

$$\text{Equation } 2.5 \sin 0.0172(t - 264.75) + 12$$

City at latitude 49.0° N

$$\text{Amplitude (A)} = (16.0 - 8.0) \div 2 = 4$$

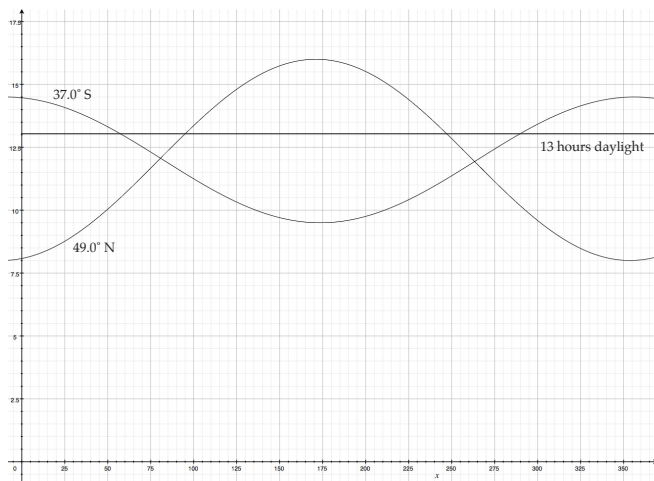
$$B = 2\pi \div 365 = 0.01721\dots$$

$$\text{Shift (C)} = -(171 - 0.25(365)) = -79.75$$

$$\text{Vertical translation (D)} = (16.0 + 8.0) \div 2 = 12$$

$$\text{Equation } 4 \sin 0.0172(t - 79.75) + 12$$

Graphs for both latitudes (see below).



For achievement the student needs to have applied trigonometric methods in solving problems. The student needs to have correctly selected and used methods, demonstrated knowledge of concepts and terms and communicated using appropriate representations.

Evidence from both features of trigonometric functions and solving trigonometric equations is required.

Possible student responses for Merit.

Students have formed two correct models for both cities and used the models to identify how many days through the year both cities receive in excess of 13 hours of daylight hours and then identify which receives most and by how much.

For city at latitude 37.0° S

$$\text{Solve } 2.5 \sin 0.0172(t - 264.75) + 12 = 13$$

to get $x = 58.2$ and 288.7

Number of days with 13 hours of daylight or more per year is $58.2 + 365 - 288.7 = 134.5$

For City at latitude 49.0° N

$$\text{Solve } 4 \sin 0.0172(t - 79.75) + 12 = 13$$

to get $x = 94.4$ and 247.7

Number of days with 13 hours of daylight or more per year is $247.7 - 94.4 = 153.3$

City at 49.0° N receives 18 more days of 13 or more hours of daylight than city at 37.0°S.

For merit the student needs to have applied trigonometric methods, using relational thinking, in solving problems. The student needs to have formed and used a model and related findings in context or communicated thinking using appropriate mathematical statements.

Possible student responses for Excellence.

Effect of latitude on the trigonometric graph parameters.

The amplitude (A) of both curves is different. A increases the further we move away from the equator. At the equator A would equal 0.

B is constant for both graphs ($2\pi \div 365$)

C remains the same for northern hemisphere cities.

C remains the same for southern hemisphere cities.

The difference between C for northern and southern hemispheres is 6 months (approximately 185 days).

D is constant for both graphs, i.e. 12.

For excellence the student needs to have applied trigonometric methods, using extended abstract thinking, in solving problems. The student needs to have devised a strategy to investigate the situation and used correct mathematical statements or communicated mathematical insight.